max planck institut informatik

UNIVERSITÄT DES SAARLANDES

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Excercises
Computational Geometry

http://www.mpi-inf.mpg.de/departments/d1/teaching/ws13/ComputationalGeometry/

Sheet 4 Deadline: 12.11.2013, 10:00am

Rules: Until the end of the semester you have to reach 50% of the achievable points to be admitted to the exam. 40 points correspond to 100%; you can get up to 15 bonus points.

Exercise 1 (*10 pts*)
In the lecture we have only discussed planar subdivisions. But the earth is a globe[1]. How would you define a *spherical subdivision*? And give a point-location structure for such a subdivision.

Exercise 2 (*10 pts*)
Give an example of a set of n line segments with an order on them that makes the RIC trapezoidal point location algorithm create a search structure of size $\Theta(n^2)$ and worst-case query-time $O(n)$.

Exercise 3 (*10+5 pts*)
A polygon P is given as an array of n vertices in sorted order along its boundary.

a) Given that P is convex, show that, given a query point q, it can be tested in time $O(\log n)$ whether q lies inside the bounded area of P.

b) Now assume that P is y-monotone. Can you generalize the solution to y-monotone polygons?

c) Finally assume that P is *star-shaped* (i.e., there exists p in P such that the line segment $\overline{pq} \in P$ for all $q \in P$). Show that, given p and a query point q, it can be tested in time $O(\log n)$ whether q lies inside the bounded area of P. What if p is not given?

Exercise 4 (*10 pts*)

a) Kd-trees are dynamic structures. Discuss the addition and removal of single elements.

b) Range trees are dynamic structures. Discuss the addition and removal of single elements.

[1] at least more a globe than a plane: `http://en.wikipedia.org/wiki/Figure_of_the_Earth`

Exercise 5 (*10 pts*)
Kd-trees can be used for partial match queries. A 2-dimensional partial match query specifies a value for one of the coordinates and asks for all points that have that value for the specified coordinate. In higher dimensions we specify values for a subset of the coordinates. Here we allow multiple points to have equal values for the coordinates.

a) Show that 2-dimensional kd-trees can answer partial match queries in $O(\sqrt{n} + k)$ where k is the number of reported answers.

b) Explain how to use a 2-dimensional range tree to answer partial match queries. What is the resulting query time?

c) Describe a data structure that uses linear storage and solves 2-dimensional partial match queries in $O(\log n + k)$ time.

d) Show that with a d-dimensional kd-tree (d fixed) we can solve a d-dimensional partial match query in $O(n^{1-s/d} + k)$ time, where s (with $s < d$) is the number of specified coordinates.