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Excercises
Computational Geometry

http://www.mpi-inf.mpg.de/departments/d1/teaching/ws13/ComputationalGeometry/

Sheet 5 Deadline: 19.11.2013, 10:00am

Rules: Until the end of the semester you have to reach 50% of the achievable points to be admitted to the exam. 40 points correspond to 100%; you can get up to 10 bonus points.

Exercise 1 (*10 pts*)

a) Show that the medial axis of a convex polygon with n vertices could have a vertex of degree n.

b) What is the maximum and minimum number of edges the medial axis tree $M(P)$ can have for a convex polygon with n vertices?

c) Is there a nonconvex polygon whose medial axis is composed entirely of straight segments?

Exercise 2 (*10 pts*)
We compute the medial axis as in the lecture.

a) Prove that the first two bisectors of a convex polygon P to meet are adjacent.

b) Show how to implement the algorithm to run in $O(n \log n)$ time.

Exercise 3 (*10 pts*)

a) Show that the maximum number of edges of the $S(P)$ tree is $2n - 3$ for a polygon with n vertices.

b) Design an algorithm to construct the straight skeleton in $O(n^3)$ time.

Exercise 4 (*10 pts*) Let P and Q be convex polygons in the plane with m an n vertices respectively. Let $\hat{P}$ denote $\mathbb{R}^2 \setminus P$, namely the entire plane with P carved out. What is the shape of the Minkowski sum $M = \hat{P} \oplus Q$? What is the maximum combinatorial complexity of M? Describe an efficient algorithm to compute it, and analyze its time and storage requirements. [Exercise by Dan Halperin]

please see Exercise 5 on other side

Exercise 5 (*10 pts*) We use the unit circle $\mathcal{S}^1$ (the circle of radius 1 centered at the origin) to represent all the directions in the plane in the following way: A point u on $\mathcal{S}^1$ represents the direction from the origin to u. For a polygonal object O in the plane, the farthest point in direction $\vec{u}$ is the point $x \in O$ that maximizes the scalar product $\vec{x} \cdot \vec{u}$.

a) Given two convex interior-disjoint polygons in the plane P, Q, we assign each point $u \in \mathcal{S}^1$ to the polygon that contains the farthest point of $P \cup Q$ in the direction $\vec{u}$. (A point may be assigned to both P and Q.) Show that the points of $\mathcal{S}^1$ assigned to one polygon constitute a contiguous arc.

Let $\mathcal{P} = \{P_1, P_2, \ldots, P_n\}$ be a set of pairwise interior-disjoint convex polygons in the plane. Let R be a convex polygon in the plane. Let $Q_i := P_i \oplus R$ for $i = 1, \ldots, n$.

b) Prove that the set $\mathcal{Q} = \{Q_1, Q_2, \ldots, Q_n\}$ is a set of pseudo discs, namely, for every pair of distinct objects in the set, $\partial Q_i \cap \mathrm{interior}(Q_j)$ is connected and $\partial Q_j \cap \mathrm{interior}(Q_i)$ is connected.[1]

[Exercise by Dan Halperin]

[1]Such a set is referred to as a set of pseudo discs since in non-degenrate situations the condition is simply that the boundaries of every pair intersect in at most two points.