max planck institut informatik

UNIVERSITÄT DES SAARLANDES

Eric Berberich, Michael Kerber **WS 2013**

Excercises
Computational Geometry

http://www.mpi-inf.mpg.de/departments/d1/teaching/ws13/ComputationalGeometry/

Sheet 6 Deadline: 26.11.2013, 10:00am

Rules: Until the end of the semester you have to reach 50% of the achievable points to be admitted to the exam. 40 points correspond to 100%; you can get up to 10 bonus points.

Exercise 1 (*10 pts*)
Prove: The Voronoi diagram of n points in the plane consists of $n-1$ parallel lines if all points are collinear. Otherwise, the diagram is connected and its edges are either segments or rays.

Exercise 2 (*10 pts*)

a) For each $n \leq 3$, is it possible to construct an example of a point set with n sites having no Voronoi vertices? How about having exactly one Voronoi vertex?

b) Prove that for any $n > 3$ there is a set of n point sites S in the plane such that one of the cells of $Vor(S)$ has $n-1$ vertices. You could also prove that the point set has a Delaunay triangulation where one vertex has degree $n-1$.

c) For any point set S, prove that the average number of vertices of a Voronoi region of S is less than 6.

Exercise 3 (*10 pts*) Suppose we are given the Delaunay triangulation of a point set S with n points. Design an algorithm that constructs the Delaunay triangulation of the remaining $n-1$ sites if a site from S is deleted. How does this algorithm change if the deleted site was on the hull of S or in the interior of $\mathrm{CH}(S)$?

please see Exercise 4+5 on other side

Exercise 4 (*10 pts*)

a) Given four points p, q, r, s in the plane. Prove that point s lies in the interior of the circle through p, q and r if and only if the following condition holds: Assume that p, q r form the vertices of a triangle in clockwise-order. $\begin{vmatrix} p_x & p_y & p_x^2 + p_y^2 & 1 \\ q_x & q_y & q_x^2 + q_y^2 & 1 \\ r_x & r_y & r_x^2 + r_y^2 & 1 \\ s_x & s_y & s_x^2 + s_y^2 & 1 \end{vmatrix} > 0$

b) Show how to use this property to test if an edge in a triangulation is legal.

c) Interchange the role of q and r. What happens?

Exercise 5 (*10 pts*)

A *Euclidian Minimum Spanning Tree* (EMST) of a set of points P in the plane is a tree with vertex set P of minimum total length connecting all points. Prove that the set of edges of a Delaunay triangulation of P contains an EMST for P. Use this result to given an $O(n \log n)$ algorithm to compute an EMST for P.