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Excercises
Computational Geometry

http://www.mpi-inf.mpg.de/departments/d1/teaching/ws13/ComputationalGeometry/

Sheet 7 Deadline: 03.12.2013, 10:00am

Rules: Until the end of the semester you have to reach 50% of the achievable points to be admitted to the exam. 40 points correspond to 100%; you can get up to 10 bonus points.

Exercise 1 (*10+5 pts*)
Weighted Voronoi Diagrams/Influence Diagram.

a) In an additively weighted Voronoi diagram, the "distance" from a point p to a site s_i is

$$d(p, s_i) = \|p - s_i\| - \omega_i$$

Thus, the boundary between two sites has the general form

$$d(x, p_i) = d(x, p_j) + \omega$$

b) Suppose the influence function is not additive but multiplicative. That is, the distance from a point to a site is defined as

$$d(p, s_i) = \|p - s_i\| / \omega_i$$

For both cases, describe the bisectors and show how these weighted Voronoi diagram can be computed.

c) The bisectors in parts a) and b) are, in general, non-linear. Is it possible to construct a weighted Voronoi diagram that consists, for given point sites s_i and weights w_i, of linear edges only?

Exercise 2 (*10 pts*)
Prove Lemma 3 from the lecture: For every edge e belonging to $\mathrm{DT}(S)$ there are real numbers $\alpha_{\min}(e), \alpha_{\max}(e) \in \mathbb{R} \cup \{\pm\infty\}$, such that e is an edge of the α-shape of S if and only if $\alpha_{\min}(e) \leq \alpha \leq \alpha_{\max}(e)$.

Exercise 3 (*10 pts*)
Given a graph on a subset of a point set S. Show how to determine whether G is an α-shape for some α. Which running time has your decision algorithm?

please see Exercises 4+5 on other side

Exercise 4 (*10 pts*)
What is the smallest doorway through which a convex polygon may pass? The doorway is a gap in an infinite line in the plane. The polygon may translate and rotate.

Exercise 5 (*10 pts*)

a) What is the maximal number of shortest paths connecting two fixed points among a set of n triangular obstacles in the plane?

b) Design an algorithm to find a shortest path between two points inside a simple polygon. Try to find an algorithm that runs in $o(n^2)$.