max planck institut informatik

UNIVERSITÄT DES SAARLANDES

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Excercises
Computational Geometry

http://www.mpi-inf.mpg.de/departments/d1/teaching/ws13/ComputationalGeometry/

Sheet 9 Deadline: 17.12.2013, 10:00am

Rules: Until the end of the semester you have to reach 50% of the achievable points to be admitted to the exam. 40 points correspond to 100%; you can get up to 10 bonus points.

Note: In all exercises, the constants hidden in O-notation are allowed to depend exponentially on d.

Exercise 1 (*10 pts*)[Approximating MST]
Let P be a set of n points in $\mathbb{R}^d$. Give an algorithm to compute, in time $O(n \log n + n\epsilon^{-d})$, a spanning tree $\mathcal{T}$ of P such that

$$w(\mathcal{T}) \leq (1+\epsilon)w(\mathcal{M})$$

where $\mathcal{M}$ is the minimum spanning tree (MST) of P. (by "spanning tree for P", we mean a spanning tree for the complete weighted graph with vertex set P, where edge weights are given by the Euclidean distance between the points. The weight of a graph G, $w(G)$, is the sum of the weights of all edges). You can use without proof that the MST of a graph with m edges can be computed in $O(m \log m)$ time.

Exercise 2 (*10 pts*)[Closest pair reloaded]

a) Let $p, q \in P$ be such that $\|p - q\| = CP(P)$, that is, p and q realize the closest distance in P. Show that any ϵ^{-1}-WSPD with $\epsilon = 1/2$ contains the pair $(\{p\}, \{q\})$ or the pair $(\{q\}, \{p\})$.

b) Give a *deterministic* algorithm to compute the closest pair in $O(n \log n)$.

Exercise 3 (*10 pts*)
The weight of a WSPD W is given by $\Sigma_{(u,v)\in W}|P_u| + |P_v|$. Show that there exists a set of n points (even with all points on a common line) such that any ϵ^{-1}-WSPD has weight $\Omega(n^2)$, for $\epsilon = 1/4$.

Exercise 4 (*10 pts*)
Let P be a set of n points in $\mathbb{R}^d$ and consider

$$U = \{i \mid 2^i \leq \|p - q\| < 2^{i-1}\}$$

Prove that $|U| = O(n)$. Intuitively, that means that although there are up to $\Theta(n^2)$ different distances, these distances are only distributed over $O(n)$ different scales.

Exercise 5 (*10 pts*)
We prove that an $(1/n)$-ring tree for a point set P can be computed in $O(n \log n)$ time. Let $b(p, r)$ denote the disk centered at p with radius r. and $\bar{b}(p, r)$ denote the complement of the disk $b(p, r)$ and $a(p, r_1, r_2)$ be the annulus at p with respect to radii r_1 and r_2, that is, $a(p, r_1, r_2) = b(p, r_2) \cap \bar{b}(p, r_1)$.

a) Show how to compute, in linear time, a point p and a radius α, such that $b(p, \alpha)$ contains at least n/c points (for some sufficiently large constant c), and $\bar{b}(p, 8\alpha)$ contains at least $n/2$ points.

b) Show how to compute, in linear time, a radius r between α and $e\alpha$ (where $e = 2.718\ldots$) such that annulus $a(p, r, (1 + 1/n)r)$ is empty.

c) Give a recursive construction of a $(1/n)$-ring tree in $O(n \log n)$ time.