Eric Berberich, Michael Kerber **WS 2013**

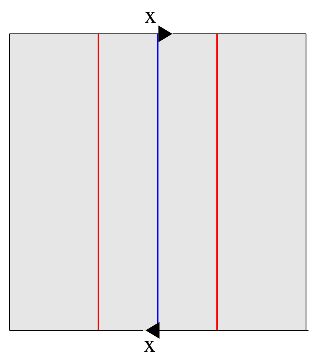
Excercises
Computational Geometry

http://www.mpi-inf.mpg.de/departments/d1/teaching/ws13/ComputationalGeometry/

Sheet 13 — Deadline: 28.01.2013, 10:00am

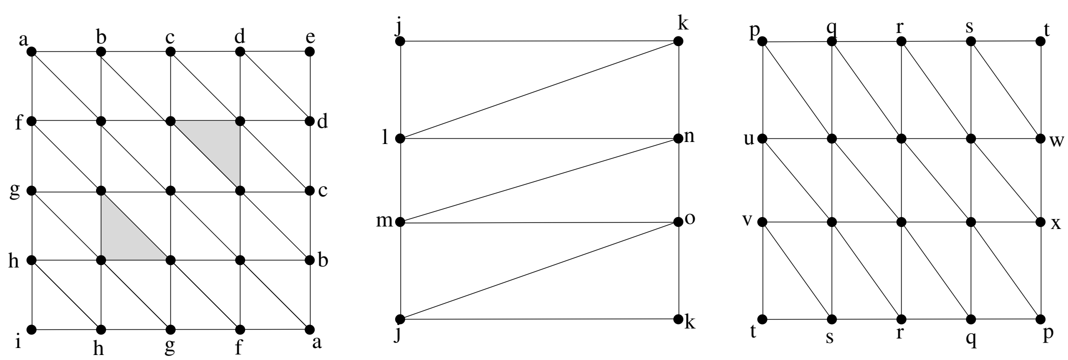
Rules: Until the end of the semester you have to reach 50% of the achievable points to be admitted to the exam. 40 points correspond to 100%; you can get up to 10 bonus points.

Exercise 1 (*10 pts*)[Fun with Möbius strips]



Consider the fundamental polygon of the Möbius strip as in the above figure. The blue and the union of the red line segments describe two different cycles on the Möbius strip. What shapes do we get if we cut the strip (a) along the blue curve (b) along the red curve? (Hint: if you have no idea what is going on, you can take paper and scissors and try it out)

Exercise 2 (*10 pts*)[Euler characteristic of 2-manifolds]



a) Consider the triangulation of the sphere as depicted on the left. Verify that the Euler characteristic is 2.

b) Now, consider the cylinder in the middle. Compute its Euler characteristic. Observe that its boundary cycles are triangles. We attach it to the sphere by removing the two shaded triangles and glueing the cylinder there. Show that the Euler characteristic of the resulting shape (a torus) is 0.

c) Show that the Euler characteristic of a sphere with g handles is $2 - 2g$.

d) Develop a similar argument for the sphere wth g crosscaps, using the triangulation of the Möbius strip on the right.

Exercise 3 (*10 pts*)[Homeomorphism and Homotopy]

a) Show that $\mathbb{R}^2$ and the open disk $D = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 < 1\}$ are homeomorphic.

b) Show that D and $\{(0, 0)\}$ are homotopically equivalent (that is, D is contractible).

c) Is $\mathbb{R}^2$ also contractible?

Exercise 4 (*10 pts*)[Simplicial complex] Let K be an (abstract) simplicial complex. A *chain* is an increasing sequence of simplices, $\sigma_1 \subset \sigma_2 \subset \ldots \subset \sigma_p$, where each containment is proper. Let $B(K)$ denote the set of chains over K.

a) Show that $B(K)$ is an (abstract) simplicial complex. What are its vertices?

b) Draw an embedding of $B(K)$, assuming K consists of a single triangle and its faces.

c) Draw an embedding of $B(B(K))$, assuming K consists of a single triangle and its faces.

d) Let K consist of a single tetrahedron and its faces. Count the simplices in $B(K)$.

Exercise 5 (*10 pts*)[Nerves]

a) Show by a counter-example that the Nerve theorem does not hold for non-convex sets.

b) Let r_1, r_2, r_3 be the minimal scales such that the Rips-, the Cech- and the alpha-complex become connected, respectively. Show that $r_1 = r_2 = r_3$.

c) (Bonus) Give an algorithm to compute r_1 for given set of n points in $\mathbb{R}^d$. You should be able to find a solution with complexity $O(n^2(d + \log n))$. Can you find something better? (there is no "correct" answer for this question – be creative!)