Optics

Projects

List available now

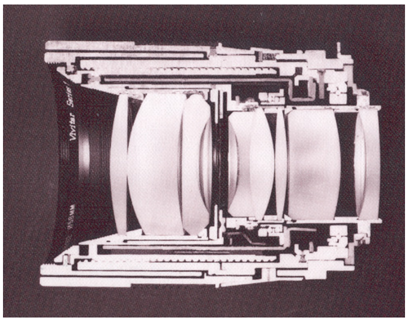
Project proposal (2 pages): 1st of June

Project idea presentation: 8th of June

Final Project presentation: 20th of July

Project report

Real Lens



Cutaway section of a Vivitar Series 1 90mm f/2.5 lens
Cover photo, Kingslake, *Optics in Photography*

Optics

Outline

- Refraction, focusing, formulas
- Field of view, sensor format
- Aperture and depth of field
- Aberrations

Acknowledgements for slides

- Steve Marschner, Bennett Wilburn, Pat Hanrahan, Marc Levoy

Pinhole Camera

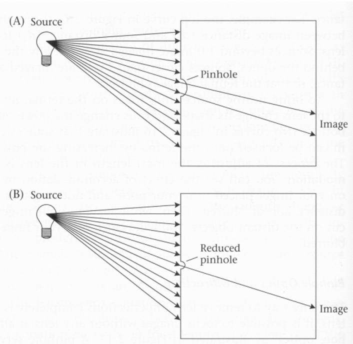


image: Wandell

Pinhole camera

Large pinhole gives geometric blur

Small pinhole gives diffraction blur

Optimal pinhole gives very little light

- for 35mm format is around f/200

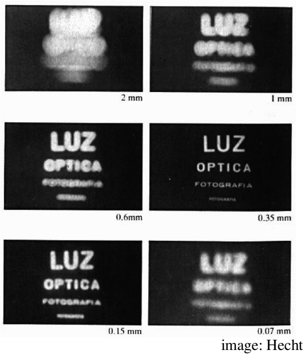
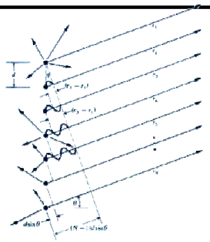
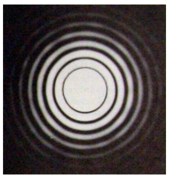


image: Hecht

Diffraction





diffraction from a circular aperture: Airy rings

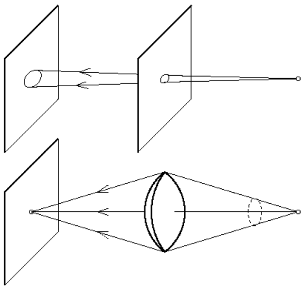
Huygens: every point on a wavefront can be considered as a source of spherical wavelets

Fresnel: the amplitude of the optical field is the superposition of these waves, considering amplitude and phase

Fraunhofer: resulting far-field diffraction pattern

images: Hecht 1987

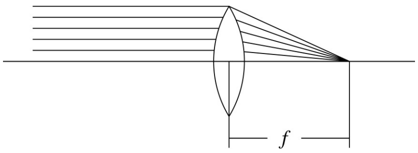
The Reason for Lenses



Purpose of lens

Produce bright but still sharp image

Focus rays emerging from a point to a point



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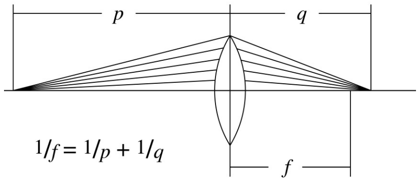
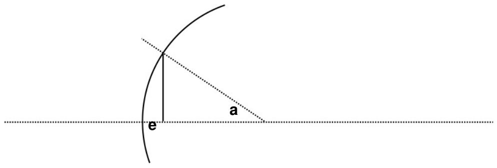


$$1/f = 1/p + 1/q$$

Paraxial Refraction

"First order" (or Gaussian) optics

1. assume e = 0
2. assume sin a = tan a ~ a

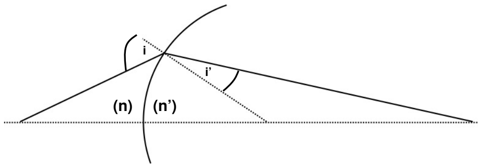


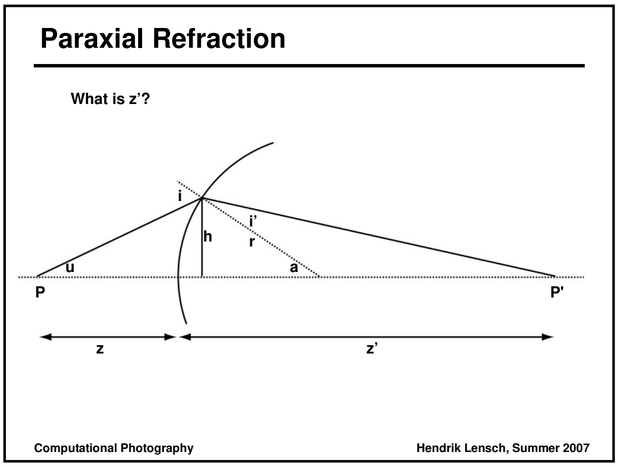
Paraxial Refraction

Refraction governed by Snell's Law

n sin i = n' sin i'

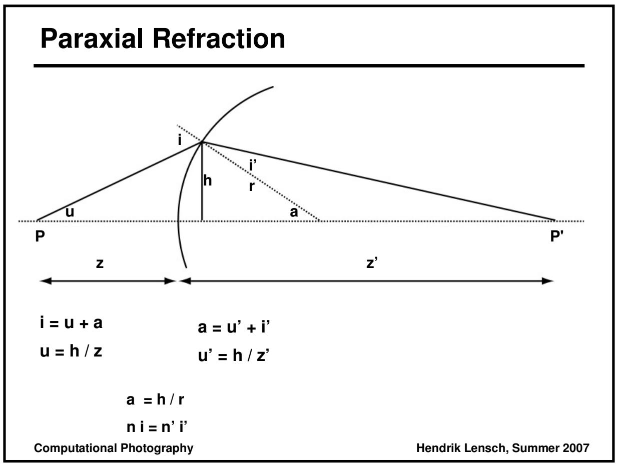
n i ≈ n' i' (Gaussian optics for small angles)





Paraxial Refraction

What is z'?



Paraxial Refraction

$i = u + a$

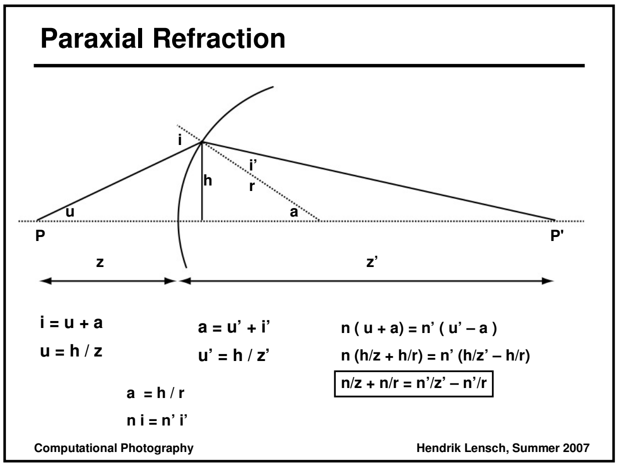
$u = h / z$

$a = u' + i'$

$u' = h / z'$

$a = h / r$

$n\, i = n'\, i'$



Paraxial Refraction

$i = u + a$

$u = h / z$

$a = u' + i'$

$u' = h / z'$

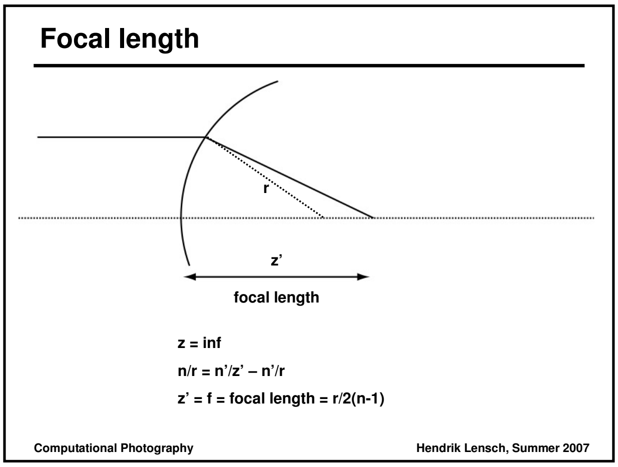
$n\,(u + a) = n'\,(u' - a)$

$n\,(h/z + h/r) = n'\,(h/z' - h/r)$

$\boxed{n/z + n/r = n'/z' - n'/r}$

$a = h / r$

$n\, i = n'\, i'$

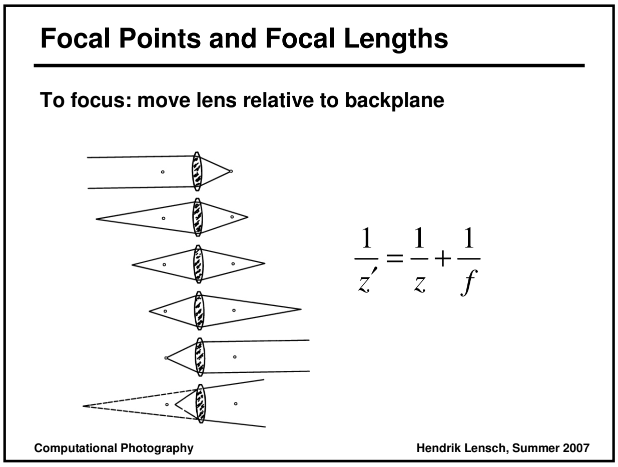


Focal length

$z = \text{inf}$

$n/r = n'/z' - n'/r$

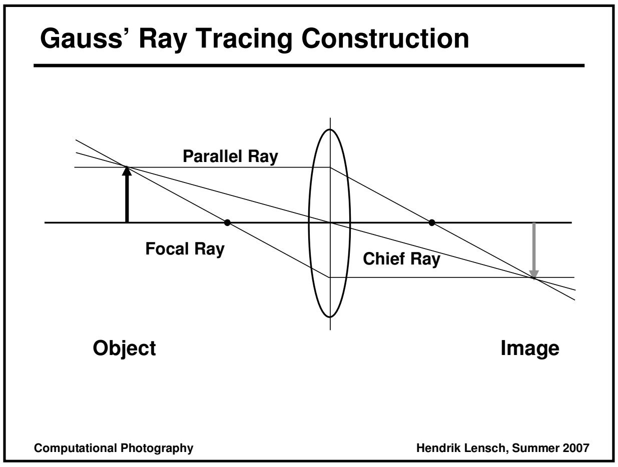
$z' = f = \text{focal length} = r/2(n-1)$



Focal Points and Focal Lengths

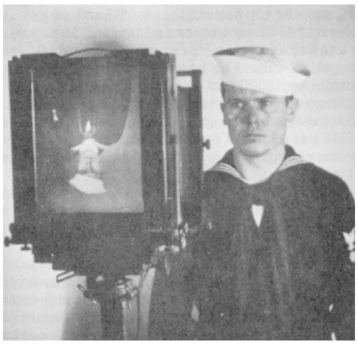
To focus: move lens relative to backplane

$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f}$$

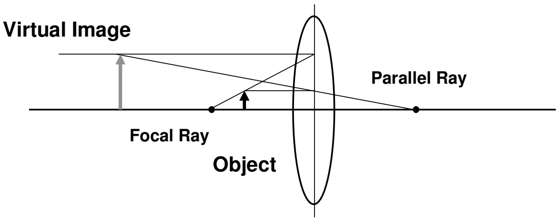


Gauss' Ray Tracing Construction

Real Image



Magnifying Glass



Virtual Image

Parallel Ray

Focal Ray

Object

Thick lenses

Complex optical system is characterized by a few numbers

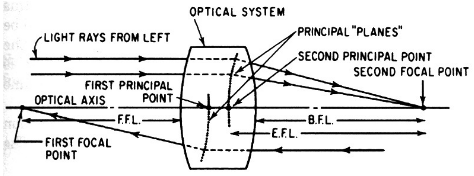
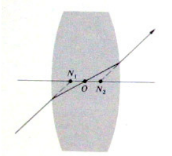


Figure 2.1 Illustrating the location of the focal points and principal points of a generalized optical system.

image: Smith 2000

The "center of perspective"



In a thin lens, the *chief ray* traverses the lens (through its optical center) without changing direction

In a thick lens, the intersections of this ray with the optical axis are called the *nodal points*

For a lens in air, these coincide with the principal points

The first nodal point is the center of perspective

image: Hecht 1987

Focal length and magnification

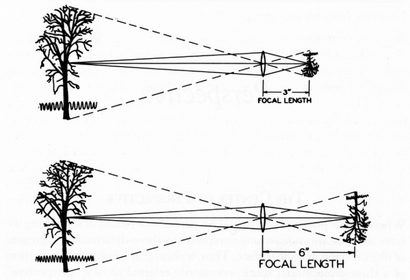


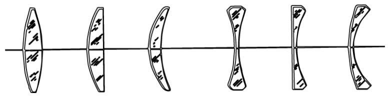
Figure 1.2. A lens of long focus produces a larger image than one of short focus.

image: Kingslake 1992

Lens-makers Formula

Refractive Power

$$P = (n' - n)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f} \quad \left[\frac{1}{m} = diopters\right]$$



Biconvex | Pos. Meniscus | Plano concave |

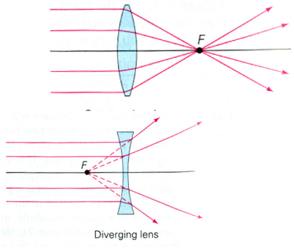
Plano-convex Biconcave Neg. meniscus

Convex = Converging **Concave = Diverging**

image: Smith 2000

Convex and Concave Lenses

- positive vs. negative focal length



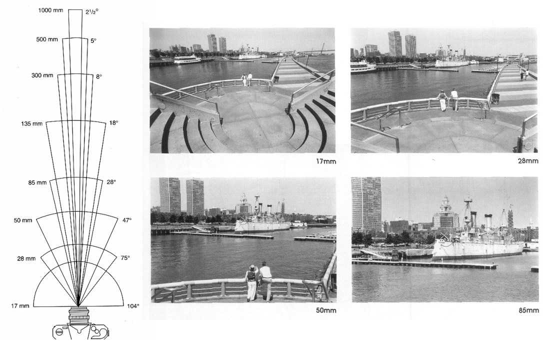
Focal length and field of view

Changing the magnification lets us move back from a subject, while maintaining its size on the image
Moving back changes perspective relationships

From (a) to (c), we've moved back from the subject and employed lenses with longer focal lengths

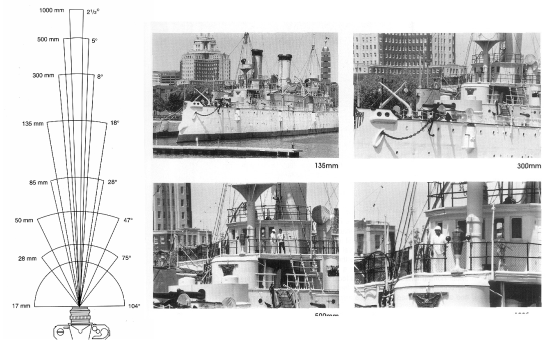
image: Kingslake 1992

Field of View



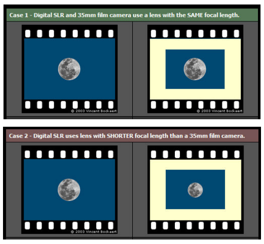
images: London and Upton

Field of View



images: London and Upton

Effects of image format



Field of view

$$\tan\frac{fov}{2} = \frac{filmsize}{2f}$$

Types of lenses

- Film camera
  - 36mm x 24mm filmsize
  - 50mm focal length = 40º field of view
- Digital camera
  - field of view is 2/3 of film for given focal length

images: dpreview.com

Effects of image format

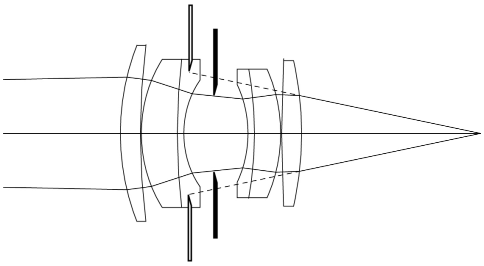
Smaller formats have...

- shorter focal length for same field of view, as we've seen
- smaller aperture size for same f-number
  - leads to larger depth of field
- lighter, smaller lens for same design
  - enables use of bulkier designs

Beware: diffraction does not scale down!

- smaller apertures suffer more from diffraction

Aperture: Stops and Pupils



- Principal effect: changes exposure
- Side effect: depth of field

Aperture

Irradiance on sensor is proportional to

- **square of aperture diameter A**
- **inverse square of sensor distance (~ focal length)**

Aperture N therefore specified relative to focal length

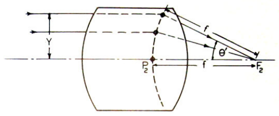
$$N = \frac{f}{A}$$

- **numbers like "f/1.4" – for 50mm lens, aperture is ~35mm**
- **exposure proportional to square of F-number, and independent of actual focal length of lens!**

Doubling series is traditional for exposure

- **therefore the familiar (rounded) sqrt(2) series**
- **1.4, 2.0, 2.8, 4.0, 5.6, 8.0, 11, 16, 22, 32, …**

How low can N be?



Canon EOS 50mm f/1.0
(discontinued)

Principal planes are the paraxial approximation of a spherical "equivalent refracting surface"

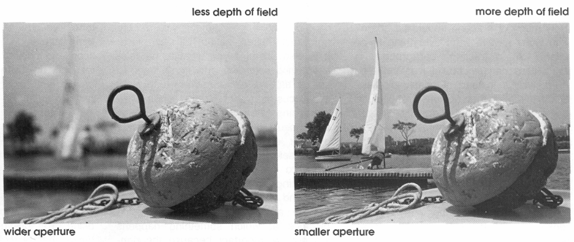
$$N = \frac{1}{2\sin\theta'}$$

Lowest N (in air) is f/0.5

Lowest N in SLR lenses is f/1.0

image: Kingslake 1992

Depth of Field

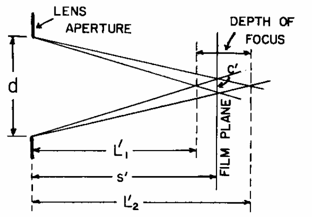


images: London and Upton

Depth of focus

(in image space)

tolerance for placing the focus plane



C' - circle of confusion

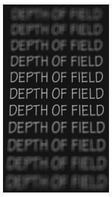
Note that distance from (in-focus) film plane to front versus back of depth of focus differ

image: Kingslake 1992

Depth of Field

(in object space)

the range of depths where the object will be in focus



www.cambridgeincolour.com

Depth of field
(in object space)

total depth of field (i.e. both sides of in-focus plane)

$$D_{tot} = \frac{2\,N\,C\,U^2}{f^2}$$

where (from Goldberg)

- N = F-number of lens
- C = size of circle of confusion (on image)
- U = distance to focused plane (in object space)
- f = focal length of lens

hyperfocal distance

- back focal depth becomes infinite when U = f^2 / C N

Numerical Aperture

$$NA = n \sin\theta$$

- The size of the finest detail that can be resolved is proportional to λ/NA.
- larger numerical aperture ⇔ resolve finer detail

Numerical Aperture vs. F-Number

$$f\,/\# \approx \frac{1}{2NA}$$

$$f\,/\#_w = \frac{1}{2NA} \approx (1-m)\,f\,/\#$$

working f-number: $f\,/\#_w$

distance-related magnification: m

relevant for systems with high magnification (microscopes or marco lenses)

Examples

$$D_{tot} = \frac{2\,N\,C\,U^2}{f^2}$$

N = f/4, C = 8μ, U = 1m, f = 50mm

- D_{tot} = 13mm

N = f/16, C = 8μ, U = 9mm, f = 65mm

- Canon MP-E at 5:1 (macro lens)
- use N' = (1+M)N at short distances (M=5 here)
- D_{tot} = 0.05mm !

image: Charles Chien

Tilt and Shift Lens

Lens shift simply moves the optical axis with regard to the film.

- change of perspective (sheared perspective)

Tilt allows for applying Scheimpflug principle

- all points on a tilted plane in focus

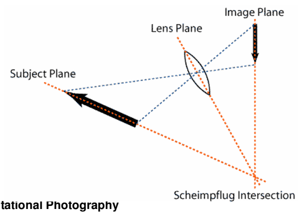
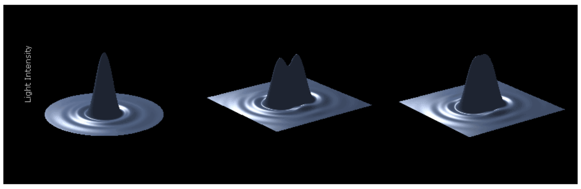


image: wikipedia

Diffraction Limit

Diameter d of 70% radius of the Airy disc

$$d = 1.22\lambda \frac{f}{a}$$



single spot barely resolved no longer resolved

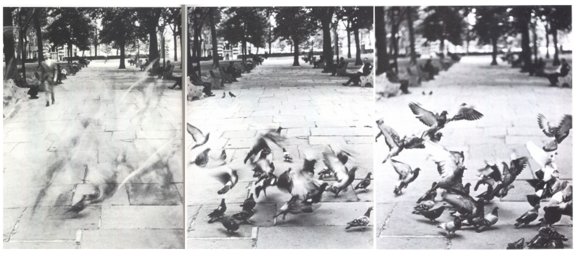
Camera Exposure

$$H = E \times T$$

Exposure overdetermined

- **Aperture: f-stop - 1 stop doubles *H***
  - **Interaction with depth of field**
- **Shutter: Doubling the effective time doubles *H***
  - **Interaction with motion blur**

Aperture vs Shutter



f/16 1/8s **f/4 1/125s** **f/2 1/500s**

images: London and Upton

Describing sharpness

Point spread function (PSF)

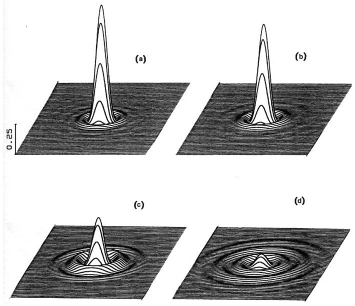


image: Smith 2000

Describing sharpness

Modulation transfer function (MTF)

- **Modulus of Fourier transform of PSF**

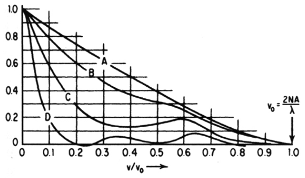


image: Smith 2000

Lens Aberrations

Spherical aberration

Coma

Astigmatism

Curvature of field

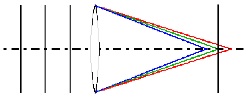
Distortion

Chromatic Aberration

Index of refraction varies with wavelength

For convex lens, blue focal length is shorter

Can correct using a two-element "achromatic doublet", with a different glass (different n') for the second lens



Achromatic doublets only correct at two wavelengths...

Why don't humans see chromatic aberration?

Chromatic aberrations

Longitudinal chromatic aberration (change in focus with wavelength)

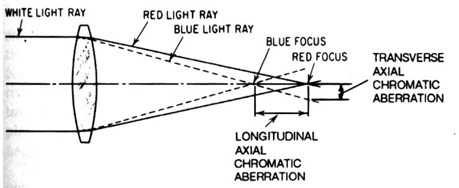


Figure 3.10 The undercorrected longitudinal chromatic aberration of a simple lens is due to the blue rays undergoing a greater refraction than the red rays.

image: Smith 2000

Chromatic aberrations

Lateral color (change in magnification with wavelength)

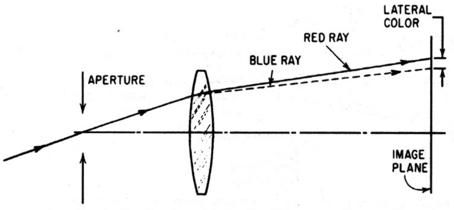
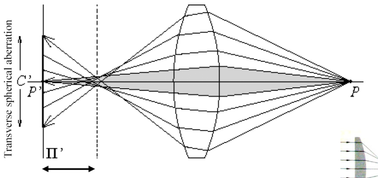


Figure 3.11 Lateral color, or chromatic difference of magnification, results in different-sized images for different wavelengths.

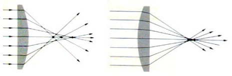
image: Smith 2000

Spherical Aberration

Focus varies with position on lens.



images: Forsyth&Ponce and Hecht 1987



- Depends on shape of lens
- Can correct using an aspherical lens
- Can correct for this and chromatic aberration by combining with a concave lens of a different n'

Oblique Aberrations

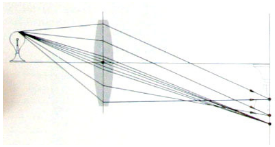
Spherical and chromatic aberrations occur on the lens axis. They appear everywhere on image.

Oblique aberrations do not appear in center of field and get worse with increasing distance from axis.

Aberrations

Coma

- **off-axis will focus to different locations depending on lens region**
- **(magnification varies with ray height)**



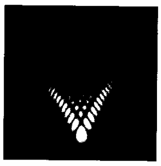


Figure 2.16. A typical comatic star image.

images: Smith 2000 and Hecht 1987

Astigmatism

The shape of the lens for an of center point might look distorted, e.g. elliptical

- **different focus for tangential and sagittal rays**

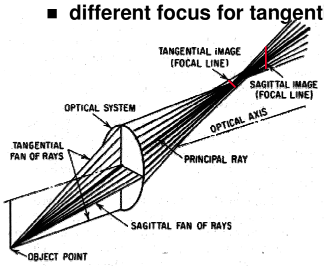


Figure 3.7 Astigmatism.

image: Smith 2000

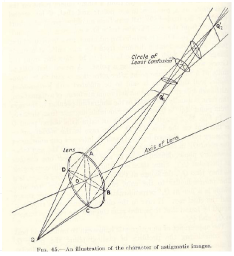


Fig. 45.—An illustration of the character of astigmatic images.

Hardy&Perrin

Astigmatic Lenses

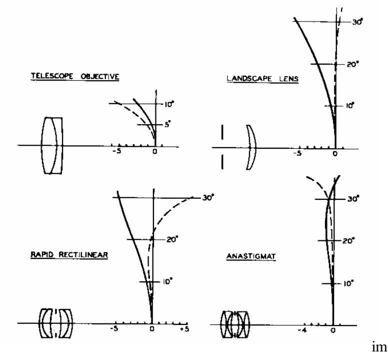
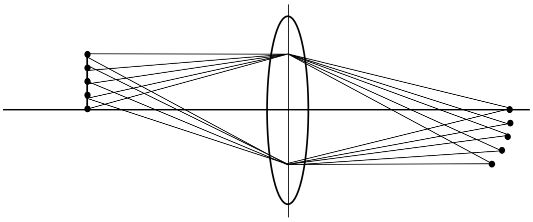


image: Smith 2000

Curvature of Field

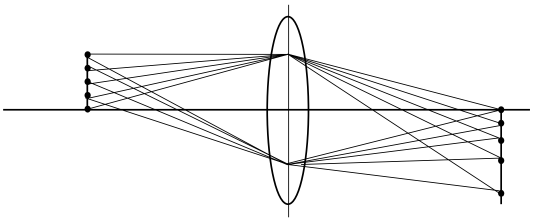
focus "plane" is actually curved



Object **Image**

Distortion

Ratios of lengths are no longer preserved.



Object **Image**

Geometric distortion

Change in magnification with image position

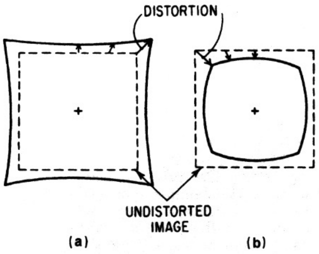


image: Smith 2000

Radial Distortion

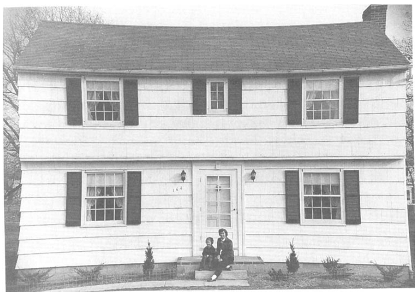


image: Kingslake

Flare

Artifacts and contrast reduction caused by stray reflections

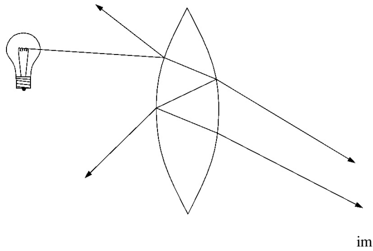
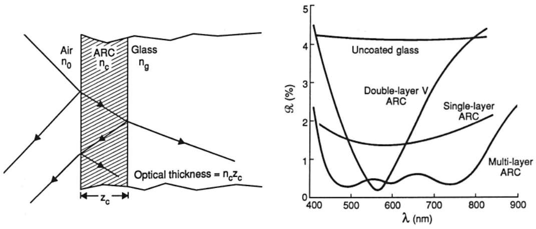


image: Curless notes

Flare

Artifacts and contrast reduction caused by stray reflections

Can be reduced by antireflection coating (now universal)



images: Curless notes

Ghost Images

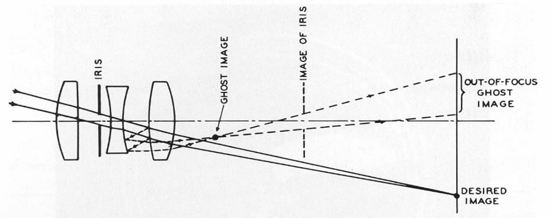


Figure 6.14. The formation of ghost images by light reflected from the internal surfaces in a lens.

image: Kingslake 1992

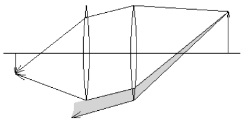
Ghost Images

Figure 6.13. A typical family of ghost images, formed by an uncoated high-aperture lens.

image: Kingslake 1992

Radial Falloff

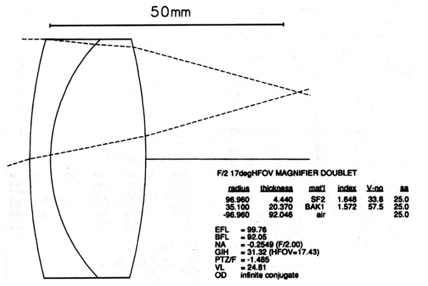
Vignetting – your lens is basically a long tube.



Cos^4 falloff.

- **At an angle, area of aperture reduced by cos(a)**
- **1/r^2: Falls off as 1/cos(a)^2 (due to increased distance to lens)**
- **Light falls on film plane at an angle, another cos(a) reduction.**

Real lens designs



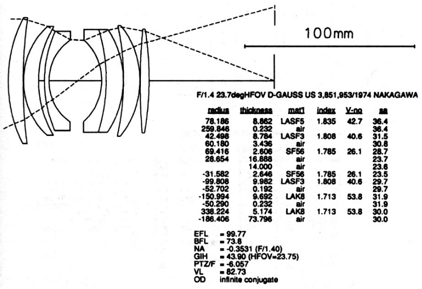
F/2 17degHFOV MAGNIFIER DOUBLET

radius	thickness	mat'l	index	V-no	sa
96.960	4.440	SF2	1.648	33.8	25.0
35.100	20.370	BAK1	1.572	57.5	25.0
-96.960	92.046	air			25.0

EFL = 99.76
BFL = 92.05
NA = -0.2549 (F/2.00)
GIH = 31.32 (HFOV=17.43)
PTZ/F = -1.485
VL = 24.81
OD infinite conjugate

image: Smith 2000

Real lens designs



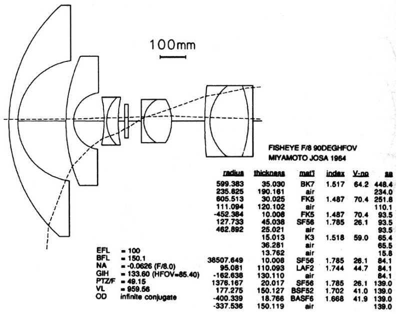
F/1.4 23.7degHFOV D-GAUSS US 3,851,953/1974 NAKAGAWA

radius	thickness	mat'l	index	V-no	sa
78.186	8.862	LASF5	1.835	42.7	36.4
259.846	0.232	air			36.4
42.498	8.784	LASF3	1.808	40.6	31.5
60.180	3.436	air			30.8
69.416	2.606	SF56	1.785	26.1	28.7
28.654	16.888	air			23.7
	14.000	air			23.6
-31.582	2.646	SF56	1.785	26.1	23.5
-99.808	9.982	LASF3	1.808	40.6	29.7
-52.702	0.192	air			29.7
-150.994	9.692	LAK8	1.713	53.8	31.9
-50.290	0.232	air			31.9
338.224	5.174	LAK8	1.713	53.8	30.0
-186.406	73.796	air			30.0

EFL = 99.77
BFL = 73.8
NA = -0.3531 (F/1.40)
GIH = 43.90 (HFOV=23.75)
PTZ/F = -6.057
VL = 82.73
OD infinite conjugate

image: Smith 2000

Real lens designs



FISHEYE F/8 90DEGHFOV
MIYAMOTO JOSA 1964

EFL = 100
BFL = 150.1
NA = -0.0626 (F/8.0)
GIH = 133.60 (HFOV=85.40)
PTZ/F = 49.15
VL = 959.56
OD infinite conjugate

radius	thickness	mat'l	index	V-no	sa
599.383	35.030	BK7	1.517	64.2	448.4
235.825	190.161	air			234.0
605.513	30.025	FK5	1.487	70.4	251.8
111.094	120.102	air			110.1
-452.384	10.008	FK5	1.487	70.4	93.5
127.733	45.038	SF56	1.785	26.1	93.5
462.892	25.021	air			93.5
	15.013	K3	1.518	59.0	65.4
	36.281	air			65.5
	13.762	air			15.8
38507.649	10.008	SF56	1.785	26.1	84.1
95.081	110.093	LAF2	1.744	44.7	84.1
-162.638	130.110	air			84.1
1376.167	20.017	SF56	1.785	26.1	139.0
177.275	150.127	BSF52	1.702	41.0	139.0
-400.339	18.766	BASF6	1.668	41.9	139.0
-337.536	150.119	air			139.0

image: Smith 2000

Real lens designs

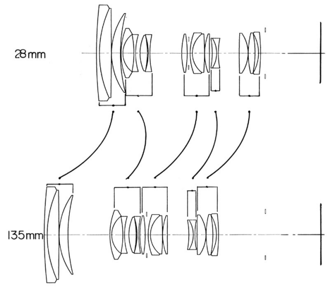


Figure 7.26 The Minolta zoom lens, 28–135 mm at f/4 to f/4.5.

image: Kingslake 1992

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