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Geometric Modeling

Assignment sheet 1 (Math recap, due April 29th 2008 before the lecture)

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(1) Gram-Schmidt Orthogonalization [4 points]

a. Calculate an orthogonal basis for $\mathbb{R}^3$ from the vectors

$$v_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, v_2 = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$$

using Gram-Schmidt Orthogonalization.

b. Calculate an orthogonal basis for the functional basis $[\,1\,, x\,, x^2]$ on the interval $[0\,,1]$ using Gram-Schmidt Orthogonalization. (Remember to use the inner product for function spaces)

c. Sketch the resulting functions from b.

(2) Eigenvectors and –values in $\mathbb{R}^3$ [8 points]

a. Show that u is an eigenvector of the matrix uu^t and has an eigenvalue of $\|u\|^2$.

b. Show that uu^t has only one non-zero eigenvalue.

c. Given a vector v with $\|v\|^2 = 1$ show that $\mathrm{I} - vv^t$ has two non-zero eigenvalues with value 1. (Hint: What are eigenvectors of I ?)

d. Given eigenvectors $e_1, e_2, e_3 \in \mathbb{R}^3$ with corresponding eigenvalues d_1, d_2, d_3 , how do you reconstruct the source matrix M from which they were calculated?

(3) Integral transformation [4 points]

Show that the surface area A of a sphere given by $f(x, y) = \pm\sqrt{r^2 - x^2 - y^2}$ is given by the formula $A = 4\pi r^2$. (Hint: Transform the integral to polar coordinates)

(4) Curvature [4 points]

a. Derive the curvature function $\kappa(t)$ for the following functions:

$$f_1(t) = \begin{pmatrix} r\cos(t) \\ r\sin(t) \end{pmatrix} \qquad f_2(t) = \begin{pmatrix} t \\ t^2 \end{pmatrix}$$

b. Given the surface $f(x, y) = 3 + xy$, what is the curvature $\kappa(\alpha)$ at point $\binom{0}{0}$ in direction α (use polar coordinates)? In which direction is it minimal / maximal?