Michael Wand

Geometric Modeling

Assignment sheet 1 (Math recap, due May 4th 2010 before the lecture)

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(1) Gram-Schmidt Orthogonalization [4 points]

a. Calculate an orthogonal basis for $\mathbb{R}^3$ from the vectors v_1, v_2, v_3 using Gram-Schmidt Orthogonalization.

$$v_1 = \begin{pmatrix}1\\1\\0\end{pmatrix}, v_2 = \begin{pmatrix}0\\2\\0\end{pmatrix}, v_3 = \begin{pmatrix}3\\0\\1\end{pmatrix}$$

b. Calculate an orthogonal basis for the functional basis $[\,1\,, x\,, x^2]$ on the interval $[0\,, 1]$ using Gram-Schmidt Orthogonalization. (Remember to use the inner product for function spaces)

c. Sketch the resulting functions from b.

(2) Eigenvectors and –values in $\mathbb{R}^3$ [10 points]

a. Show that u is an eigenvector of the matrix uu^t and has an eigenvalue of $\|u\|^2$.

b. Show that uu^t has only one non-zero eigenvalue.

c. Given a vector v with $\|v\|^2 = 1$ show that $\mathrm{I} - vv^t$ has two non-zero eigenvalues with value 1. (Hint: What are eigenvectors of I ?)

d. Given eigenvectors $e_1, e_2, e_3 \in \mathbb{R}^3$ with corresponding eigenvalues d_1, d_2, d_3 , how do you reconstruct the source matrix M from which they were calculated?

e. The Fibonacci sequence can be written as:

$$\begin{pmatrix}F_n\\F_{n+1}\end{pmatrix} = \begin{pmatrix}0 & 1\\1 & 1\end{pmatrix}^n \begin{pmatrix}F_0\\F_1\end{pmatrix}$$

Derive a closed form solution for the nth Fibonacci number F_n.

(Hint: Diagonal form TDT^{-1} of $\begin{pmatrix}0 & 1\\1 & 1\end{pmatrix}$ is $T = \begin{pmatrix}1 & 1\\ \frac{1+\sqrt{5}}{2} & \frac{1-\sqrt{5}}{2}\end{pmatrix}, D = \begin{pmatrix}\frac{1+\sqrt{5}}{2} & 0\\ 0 & \frac{1-\sqrt{5}}{2}\end{pmatrix}$).

(3) Integral transformation [6 points]

Show that the surface area A of a sphere given by $f(x, y) = \pm\sqrt{r^2 - x^2 - y^2}$ is given by the formula $A = 4\pi r^2$ (Hint: Transform the integral to polar coordinates)