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Geometric Modeling SS2010

Assignment sheet 3 (due May 18th, before the lecture)

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(1) Curvature [5 points]

a. Derive the curvature function $\kappa(t)$ for the following functions:

$$f_1(t) = \binom{r\cos t}{r\sin t} \qquad f_2(t) = \binom{t}{t^2} \qquad f_3(t) = \begin{pmatrix}\cos t\\ t\\ \sin t\end{pmatrix}$$

b. Given the surface $f(x,y) = 3 + xy$, what is the curvature $\kappa(\alpha)$ at point $\binom{0}{0}$ in direction α (use polar coordinates)? In which direction is it minimal / maximal?

(2) Compactly Supported Basis Functions [8 points]

a. We want to interpolate n regularly spaced points (x_i, y_i), $i = 1 \dots n-1$ with $x_i = ih$ sampled from a 1D function $f(x) = y$. To avoid oscillation artifacts, we want to use *piecewise* polynomial basis functions $b_i(x)$ which have the following properties :

$$b_i(x) = \begin{cases} 1 & x = ih \\ 0 & x \le (i-1)h \\ 0 & x \ge (i+1)h \end{cases} \qquad b_i \in \mathbb{C}^1, i = 1 \dots n-2$$

Derive basis functions b_i which satisfy the properties.

b. Use a plotting tool to draw the function you get by interpolating the sample points

$$s(0) = 1 \quad , \quad s(1) = 2 \quad , \quad s(2) = 3 \quad , \quad s(3) = 4$$

on the interval $[0,3]$ using the basis functions derived in (a) with $h = 1$.

c. How do you need to change the basis functions b_i to make them reproduce constant functions $f(x) = c$ faithfully not only on the sample points but on the whole interval? Write down the new basis functions.

(3) Least-Squares Approximation [4 points]

Given n sample points $\binom{x_0}{y_0} \dots \binom{x_n}{y_n} \in \mathbb{R}^2$, how do you find the center $\binom{a}{b}$ and radius r of the best fitting circle in an algebraic sense?

Hint: Represent the circle as an implicit function.
Substituting $c = a^2 + b^2 - r^2$ may be helpful to reduce the degree of the error function (this means effectively solving for (*a*,*b*,*c*) instead of (*a*,*b*,*r*).

(4) Bernstein Polynomials [2 points]

Prove the following two identities for Bernstein Polynomials:

$$x^n = \sum_{i=k}^{n} \frac{\binom{i}{k}}{\binom{n}{k}} \mathrm{B}_i^n(x)\,, \qquad \frac{d}{dx}\mathrm{B}_i^n(x) = n\left(\mathrm{B}_{i-1}^{n-1}(x) - \mathrm{B}_i^{n-1}(x)\right)$$

(5) Pricipal Component Analysis [1point]

Sketch the normal, tangent and the PCA ellipsoid for the marked points and their neighbours in the following 3 diagrams (no calculations necessary, a rough sketch is sufficient) :