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Geometric Modeling

Assignment sheet 5 (due June 8th 2010, before the lecture)

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(1) Bézier Curves [4 points]

Find a cubic Bézier curve P(u), $P:[0,1]\rightarrow R^2$ with:

$$P(0)=\begin{pmatrix}0\\0\end{pmatrix},\quad P(1)=\begin{pmatrix}9\\0\end{pmatrix}$$

which intersects itself at $P(\frac{1}{4})=P(\frac{3}{4})$ orthogonally.

(2) **De Casteljau algorithm and subdivision [1+2+2 points]**

Given the cubic polynomial curve

$$P(u)=-\begin{pmatrix}7/8\\5/8\end{pmatrix}u^3+\begin{pmatrix}9\\15/4\end{pmatrix}u^2-\begin{pmatrix}57/2\\9/2\end{pmatrix}u+\begin{pmatrix}30\\-1\end{pmatrix}$$

(a) Find the polar form $p(u_1,u_2,u_3)$ of P(u), as well as the Bézier points (the vertices of the control polygon) P_0,P_1,P_2,P_3 of P(u) w.r.t. the interval [2,4]. Sketch the the control polygon. (Hint: use a full A4 paper and a meaningful scale, like 4 cm = 1 unit)

(b) Evaluate the polynomial P(u) using the De Casteljau algorithm at the sample points $u\in\{5/2,3,7/2\}$ and draw it into the same graph. Hint: Result of a :

$$P_0=\begin{pmatrix}2\\0\end{pmatrix},\quad P_1=\begin{pmatrix}0\\2\end{pmatrix},\quad P_2=\begin{pmatrix}3\\4\end{pmatrix},\quad P_3=\begin{pmatrix}4\\1\end{pmatrix}$$

(c) Use the result from (b) for subdividing P(u) at $u=3$ and subdivide the right part of the curve again at its midpoint $u=7/2$. Add this control polygon to the same graph like before and sketch the curve described by P(u).

(3) **Polar forms and derivatives [1+2+4 points]**

Given is the cubic polynomial curve

$$F(u) = \begin{pmatrix} 15 \\ -6 \end{pmatrix} u^3 + \begin{pmatrix} 27 \\ 10 \end{pmatrix} u^2 - \begin{pmatrix} 9 \\ 9 \end{pmatrix} u$$

w.r.t. the parameter interval [0,1].

(a) Find the first and second derivative of F.

(b) Find the polar form $f(u_1,u_2,u_3)$ of F as well as the polar forms of the derivatives F' and F''. show that they are equal to

$3\, f(u_1,u_2,\vec{1})$ and $6f(u_1, \vec{1},\vec{1})$

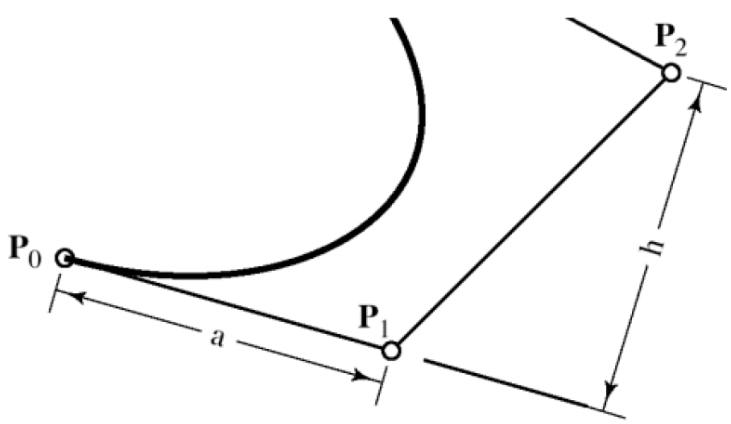
respectively.

Note: $f(u_1,u_2,\vec{1})$ is short for $f(u_1,u_2,1)- f(u_1,u_2,0)$

(c) Prove that the curvature of a Bézier curve at the starting point P_0 is given by

$$\kappa 2(P_0) = 2\frac{n-1}{n}\frac{area(P_0,P_1,P_2)}{dist^3(P_0,P_1)} = \frac{n-1}{n}\frac{h}{a^2}$$

where n is the degree of the curve and a and h are shown below.



(4) DeBoor algorithm [2+2 points]

a. Given the uniform B-spline defined by the points

$$P_0 = \begin{pmatrix} -2 \\ -10 \end{pmatrix}, \quad P_1 = \begin{pmatrix} -4 \\ 2 \end{pmatrix}, \quad P_2 = \begin{pmatrix} 6 \\ 5 \end{pmatrix}, \quad P_3 = \begin{pmatrix} 4 \\ -7 \end{pmatrix}$$

and the knot vector [0,1,2,3,4,5]. Evaluate the position of the curve at parameter t=2.5 using DeBoor's algorithm. Sketch the control polygon and the points constructed by the algorithm.

b. For the B-spline from (a), compute the corresponding Bézier control points which describe the same cubic curve. Sketch the points and the resulting Bézier curve.