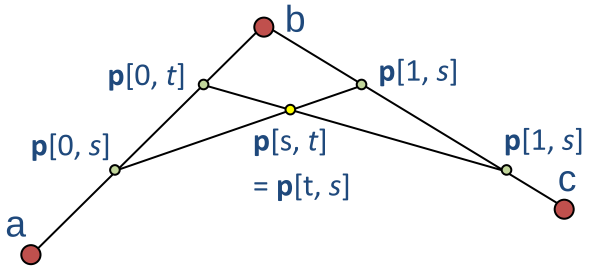
Geometric Modeling 2010

Assignment sheet 8 (Blossoms, Curves & All the Rest, due July 6th 2010)

Some more insight into Blossoming, a preview of subdivision surfaces, and some additional exercises promised earlier in the lecture...

(1) Why Blossoms are symmetric [4+1 points]



a. Consider a de Casteljau scheme as shown in the figure above. Prove that the following two operations yield the same result:

   1. Affine interpolation between **a** and **b** with ratios *s*, (1 – *s*) to get “**p**[0, *s*]” and between **b**, **c** with ratios *s*, (1 – *s*) to get “**p**[1, *s*]”. Then interpolate again between the resulting points with ratios t, (1 – *t*) to obtain the point “**p**[*s, t*]”.

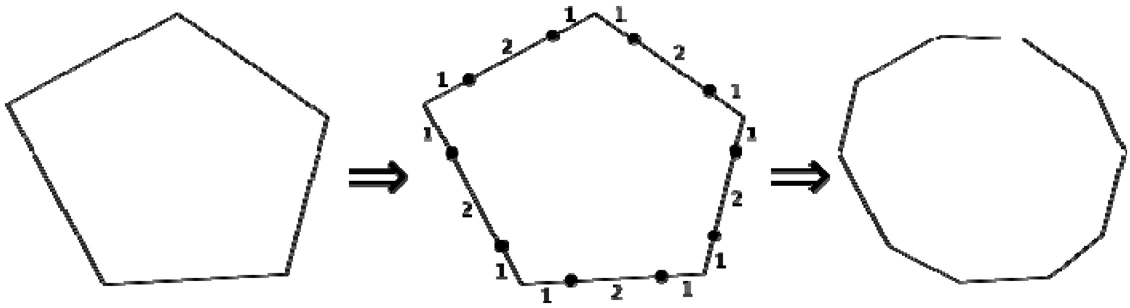
   2. Affine interpolation between **a** and **b** with ratios *t*, (1 – *t*) to get “**p**[0, *t*]” and between **b**, **c** with ratios *t*, (1 – *t*) to get “**p**[1, *t*]”. Then interpolate again between the resulting points with ratios s, (1 – s) to obtain the point “**p**[*t, s*]”.

   In other words, show that the points we have labeled p[s,t] and p[t,s] are identical. Use only the properties of affine interpolation between points in $\mathbb{R}^n$ (we do not yet know that this corresponds to Blossoms). This result is known as Menelaos' theorem.

b. What are the consequences for our Blossoming formalism (~2 sentences)?

(2) Chaikin’s Corner Cutting [5 points]
Consider a closed control polygon. Chaikin’s algorithm can be formulated as subdividing each linear segment 1:2:1 and using the arising points as the control points of the refined control polygon (cf. illustration). Show that the limit curve is a C1-continuous, piecewise quadratic Bézier curve.

Hint: Remember the De Casteljau algorithm.



Remark: This scheme is a simple example for a subdivision curve that is created by repeated weighted averaging of polygons. We will discuss other schemes for subdivision curves and surfaces more in detail in the lecture.

(3) Euler Curvature Formula [4 Points]

Let $k_n(\varphi) = k_{\max}\cos^2(\varphi) + k_{\min}\sin^2(\varphi)$ be the curvature associated with the angle between the current tangent and the tangent of the maximal principal curvature.
Proof that the mean curvature is actually the mean curvature:

$$H = \frac{1}{2\pi}\int_0^{2\pi} k_n(\varphi)d\varphi$$

In other words: The mean of all directional curvatures is the mean of the principal curvatures (k_{max} + k_{min})/2.

(4) Convex Sets (...as promised...) [6 points]
There are two definitions of the convex hull of a point set $P = \{\mathbf{p}_1,...,\mathbf{p}_n\} \subset \mathbb{R}^d$:

a. All convex combinations of points from *P* (reminder: convex combinations are linear combinations with non-negative weights that sum to one).

b. A minimal convex set with respect to set inclusion "$\subseteq$" that contains *P*. A set *C* is convex if and only if it contains all straight line segments between any two points from *C*.

Prove that the two definitions are equivalent. *Hint:* Consider two sets C_1 and C_2 that are the convex hull of P with respect to definition (a) and (b) and show separately that $C_1 \subseteq C_2$ and $C_2 \subseteq C_1$, by contradiction.