Geometric Modeling

Summer Semester 2012

Summary

UNIVERSITÄT DES SAARLANDES

CLUSTER OF EXCELLENCE

max planck institut informatik

Final Exam / Re-exam

Final Exam

Written exam ("final exam"):

- Tuesday July 31st 2012
- Lecture Hall 001, Building E1 3 (CS building)
- 14h – 16h: starts at 14:00 s.t., duration 2h

Permitted Tools

You must bring:

- Yourself.
- *Pens/Pencils*, *paper* for writing down the solutions.
- A *ruler* for drawing stuff.

Additionally, you are allowed to bring:

- Up two 4 handwritten pages (A4) of notes in your own handwriting (double sided, if you want).
- A non-programmable calculator, no wireless access.
- Turned off any cell phones.

Written Exam Rules

Rules

- You can take either the final exam or the re-exam.
- In case you fail the final, you have a second chance in the re-exam.
- In case you take both exams, the better of the two grades will be your final grade

Re-exam

Re-exam

- Friday September 21st 2012
- Lecture Hall 001, Building E1 3 (CS building)
- 14h – 16h: starts at 14:00 s.t., 2h duration

Registration

- Within *14 days of the announcements* of the grades for the first exam, please *write me an email* saying whether you want to participate in the re-exam or not.
  - Please write the email in *either case*.
  - This helps with our internal planning – thanks.

Grading

Grading of the final written exam:

- You need to obtain at least 50% of the score of the exam to pass the lecture.
  - The bonus score from the homework does not count for this bound. Bonus score can only improve your grade, it does not affect passing.
- Bonus score:
  - You may obtain a bonus for the written exam.
  - For 50% homework $\rightarrow$ 0% bonus
  - For 100% homework $\rightarrow$ 10% bonus

} added to the score of the exam (linear in between)

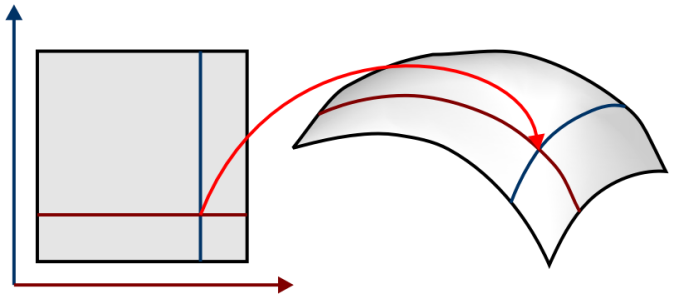
Summary

Overview...

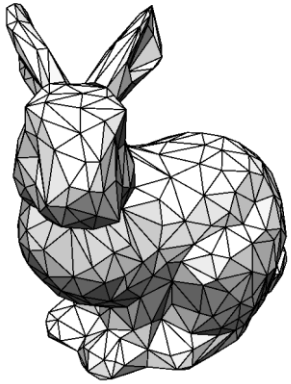
Topics:

- Introduction
- Mathematical Background
- Interpolation & Approximation
- Polynomial Spline Curves
- Blossoming and Polars
- Rational Spline Curves
- Spline Surfaces
- Point-Based Modeling
- Triangle Meshes & Multi-Resolution
- Subdivision Surfaces
- Implicit Functions
- Variational Modeling
- Summary

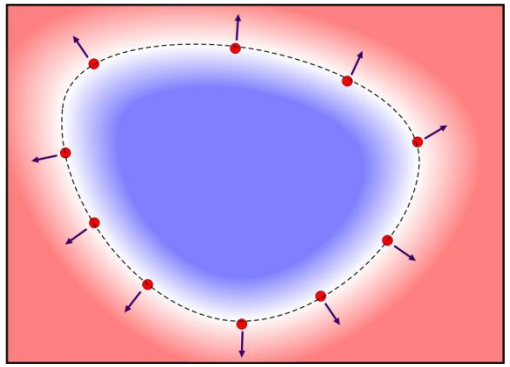
Introduction



Parametric Models



Primitive Meshes



Implicit Models

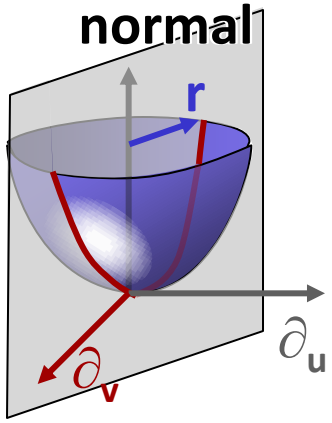
Particle Models

Mathematical Background

Analysis, Numerics & Linear Algebra

Differential Geometry:

- Curve length, curvature, torsion etc.
- Fundamental forms
- Principal curvatures, normal curvature
- Gauss and mean curvature
- Global theorems

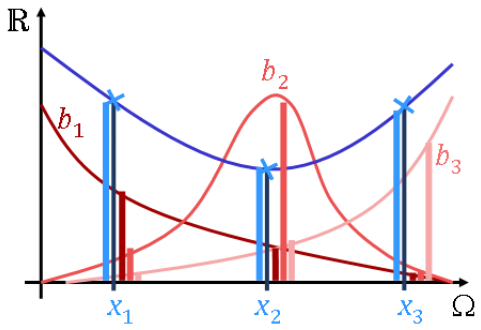


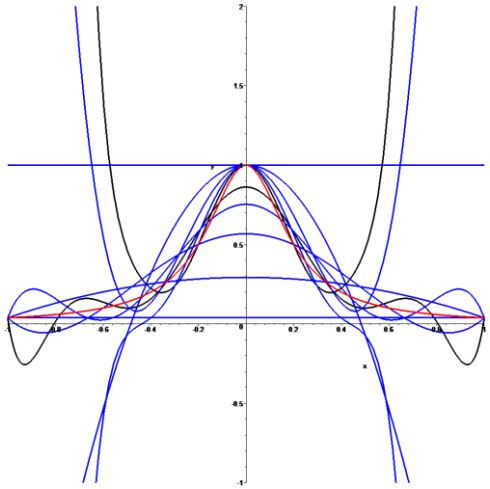
Interpolation & Approximation

Interpolation

Approximation

- Least squares
- Normal equations
- Total least squares / PCA





Normal equations:

$$\begin{pmatrix} \langle \mathbf{b}_1, \mathbf{b}_1 \rangle & \cdots & \langle \mathbf{b}_1, \mathbf{b}_k \rangle \\ \vdots & \ddots & \vdots \\ \langle \mathbf{b}_k, \mathbf{b}_1 \rangle & \cdots & \langle \mathbf{b}_k, \mathbf{b}_k \rangle \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \vdots \\ \lambda_k \end{pmatrix} = \begin{pmatrix} \langle \mathbf{y}, \mathbf{b}_1 \rangle \\ \vdots \\ \langle \mathbf{y}, \mathbf{b}_k \rangle \end{pmatrix}$$

with:

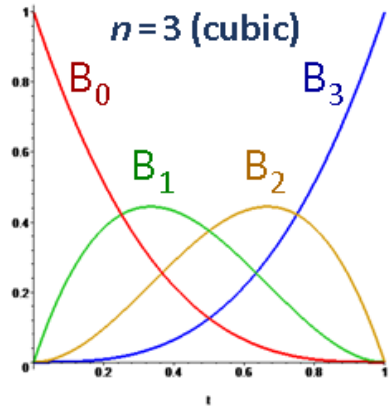
$$\langle \mathbf{b}_i, \mathbf{b}_j \rangle := \sum_{t=1}^{n} b_i(x_t) \cdot b_j(x_t)$$

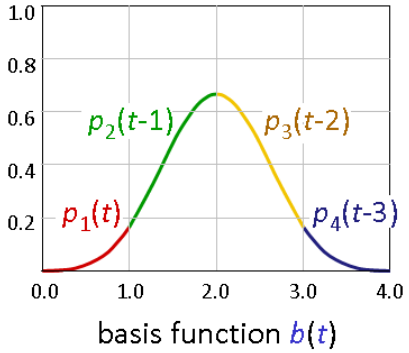
$$\langle \mathbf{y}, \mathbf{b}_i \rangle := \sum_{t=1}^{n} b_i(x_t) \cdot y_t$$

Polynomial Spline Curves

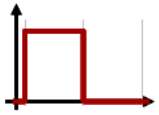
Spline Curves

- General properties
  (affine invariance, convex hull...)
- Bezier Curves
- B-Splines (Uniform/Non-uniform)
- Other splines (Hermite)

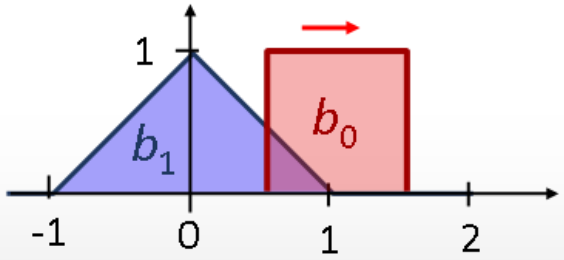




$$N_i^0(t) = \begin{cases} 1, & \text{if } t_{i-1} \le t < t_i \\ 0, & \text{otherwise} \end{cases}$$



$$N_i^d(t) = \frac{t - t_{i-1}}{t_{i+d-1} - t_{i-1}} N_i^{d-1}(t) + \frac{t_{i+d} - t}{t_{i+d} - t_i} N_{i+1}^{d-1}(t)$$



Polar Forms

Polar Forms / Blossoms:

A *polar form* or *blossom* f of a polynomial F of degree d is a function in d variables:

$$F: \quad \mathbb{R} \rightarrow \mathbb{R}$$

$$f: \quad \mathbb{R}^d \rightarrow \mathbb{R}$$

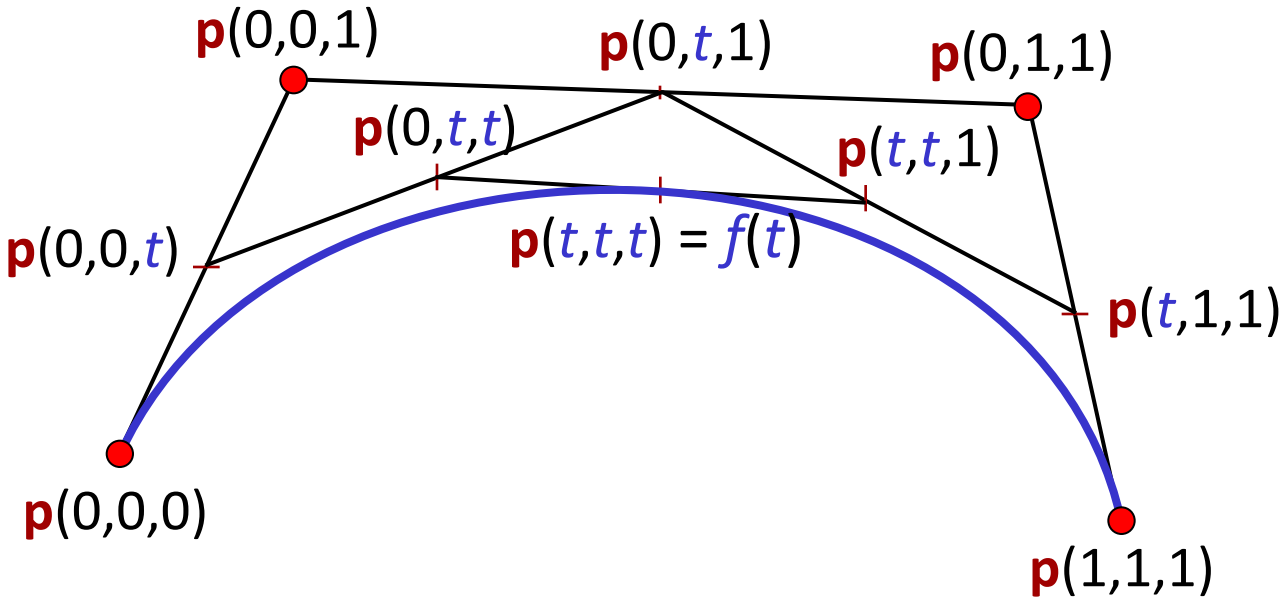
with the following properties:

- *Diagonality*: $f(t, t, \ldots, t) = F(t)$
- *Symmetry*: $f(t_1, t_2, \ldots, t_d) = f(t_{\pi(1)}, t_{\pi(2)}, \ldots, t_{\pi(d)})$
  for all permutations of indices π.
- *Multi-affine*: $\Sigma \alpha_k = 1$

$$\Rightarrow f(t_1, t_2, \ldots, \Sigma \alpha_k t_i^{(k)}, \ldots, t_d)$$

$$= \alpha_1 f(t_1, t_2, \ldots, t_i^{(1)}, \ldots, t_d) + \ldots + \alpha_n f(t_1, t_2, \ldots, t_i^{(n)}, \ldots, t_d)$$

Casteljau / De Boor

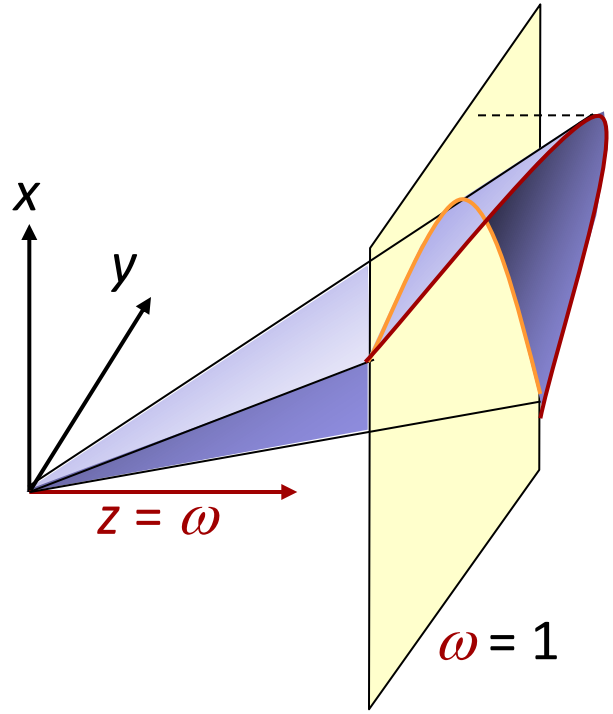


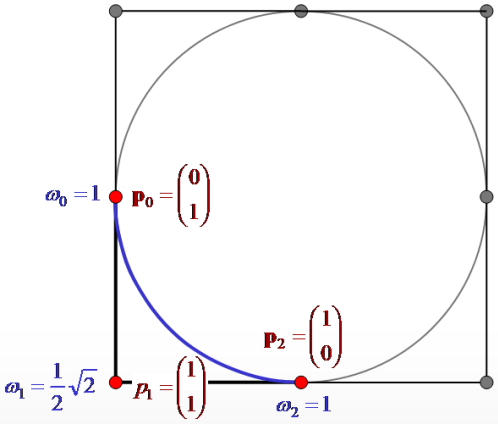
Bezier control points: **p**(0,0,0), **p**(0,0,1), **p**(0,1,1), **p**(1,1,1)

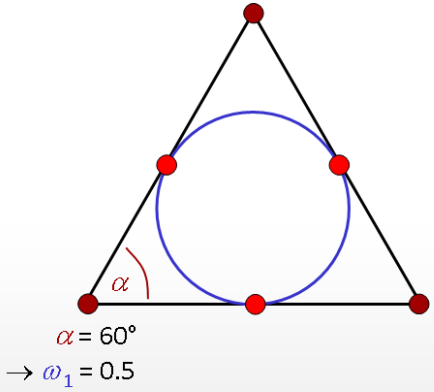
Rational Curves

Rational Curves:

- Conics & Quadrics
- Rational Bezier Splines
- NURBS
- How to create circles



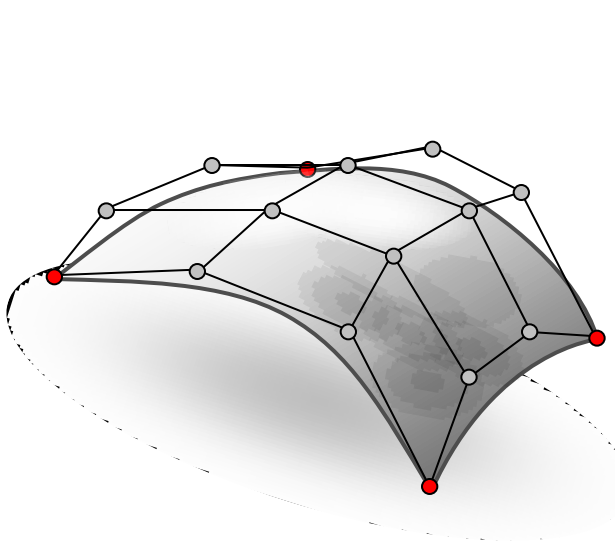


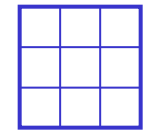


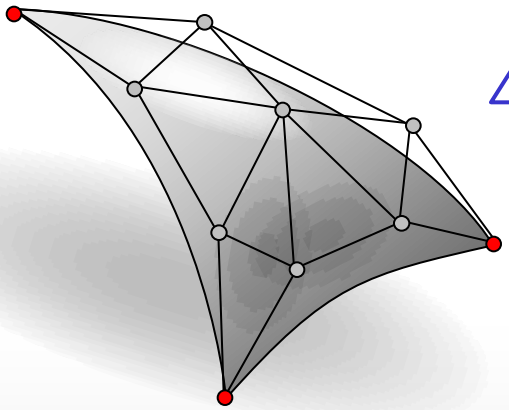
Spline Surfaces

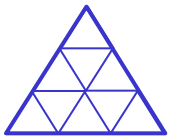
Spline Surfaces

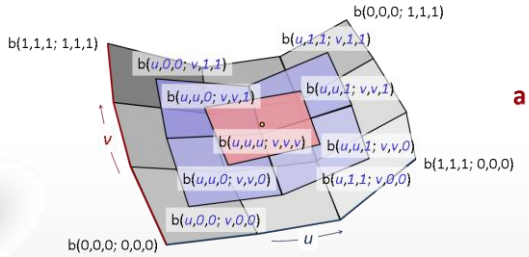
- Tensor product surfaces
- Total degree surfaces
- Surfaces of revolution
- Trimmed patches

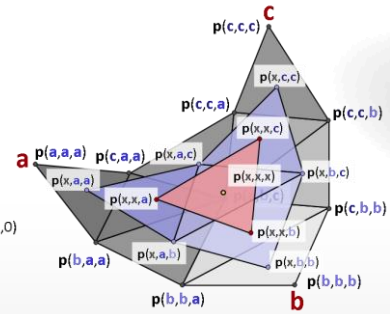








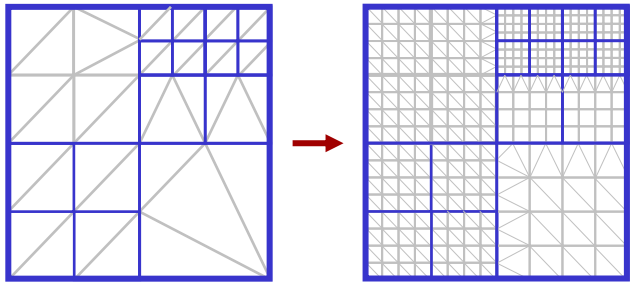




Triangle Meshes & Multires

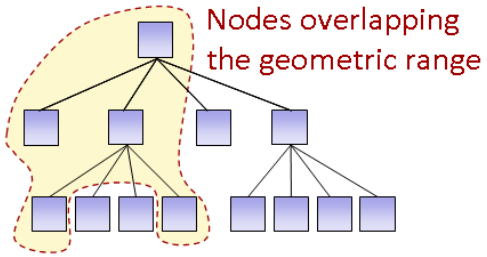
Triangle Meshes:

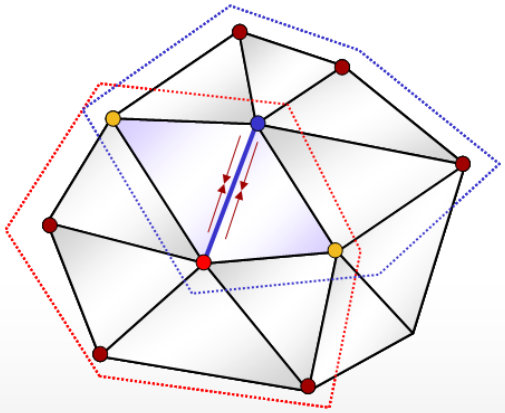
- Data structures
- Triangulations



Multi-Resolution Techniques:

- Hierarchical data structures & algorithms
- Mesh simplification
- Adaptive triangulation

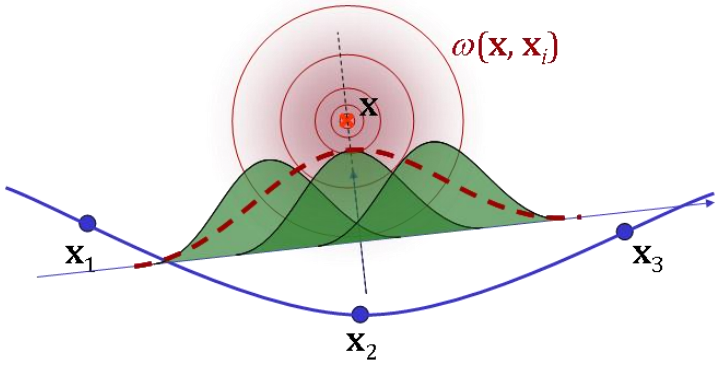


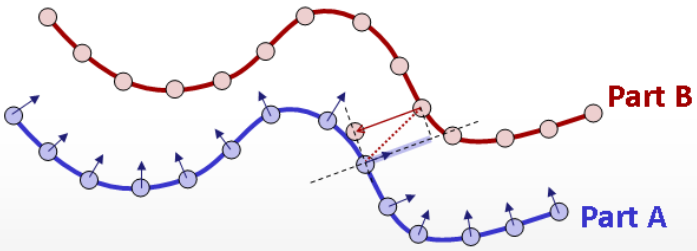


Point-Based Modeling

Point-based modeling:

- 3D scanners
- Surface smoothing & reconstruction
- Surface matching (ICP)
- Moving least squares
- Point-based modeling

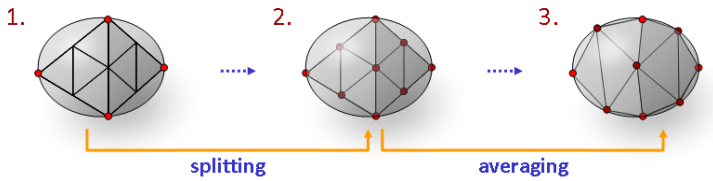


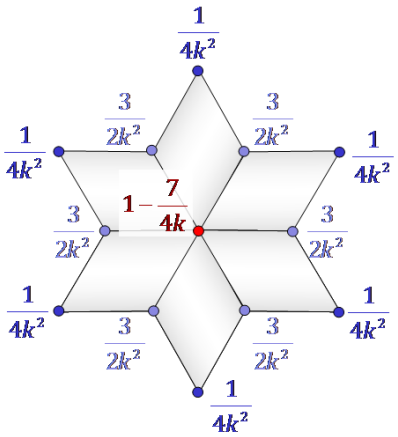


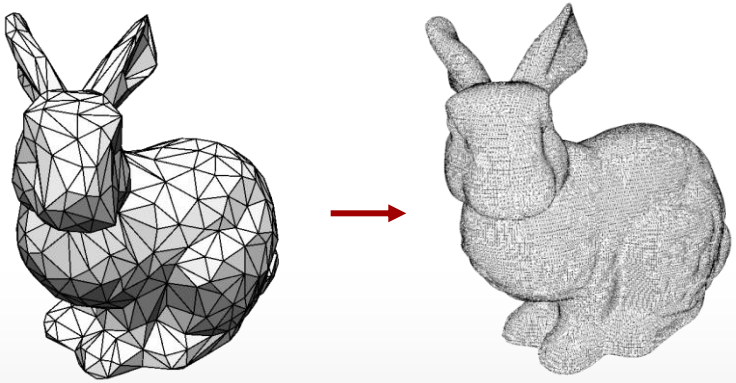
Subdivision Surfaces

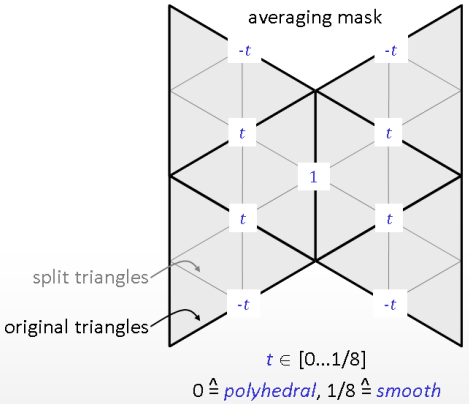
Subdivision Surfaces

- B-Spline subdivision
- Spectral analysis of subdivision
- Extraordinary vertices
- General subdivision rules





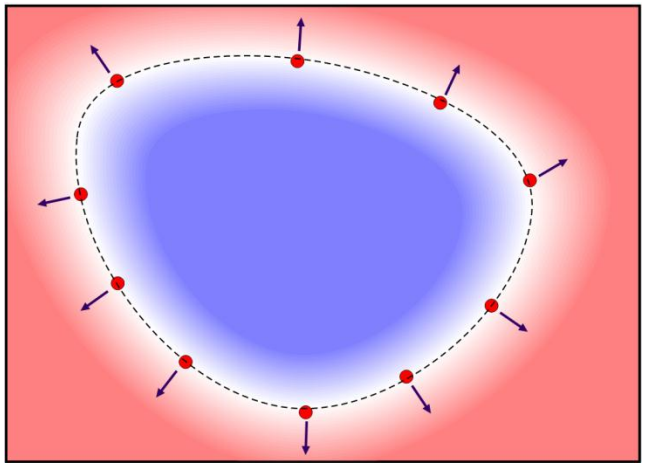


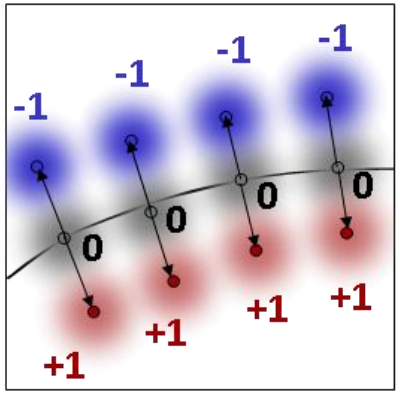


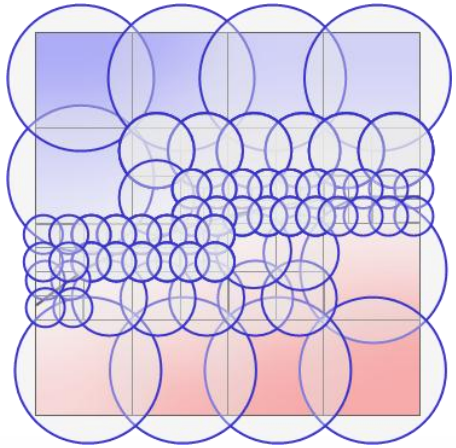
Implicit Surfaces

Implicit Surfaces:

- Mathematical background
- Differential properties
- Representations
- Data Fitting



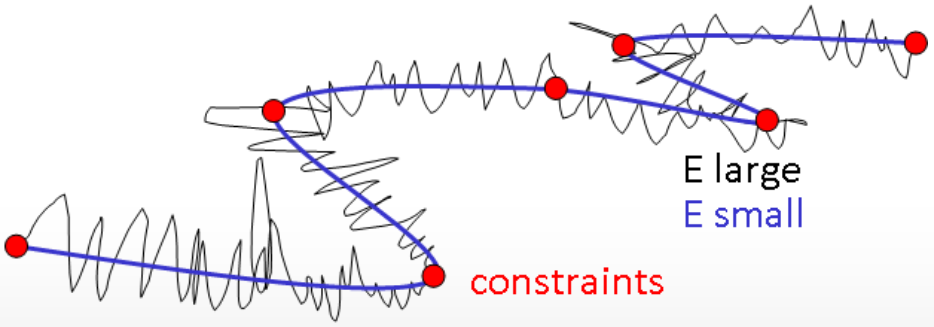


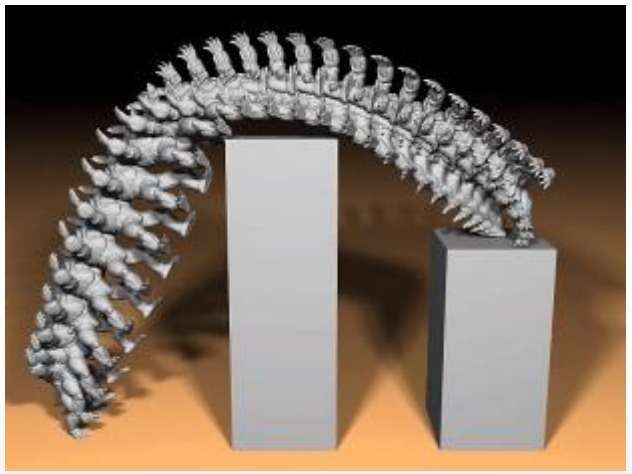


Variational Modeling

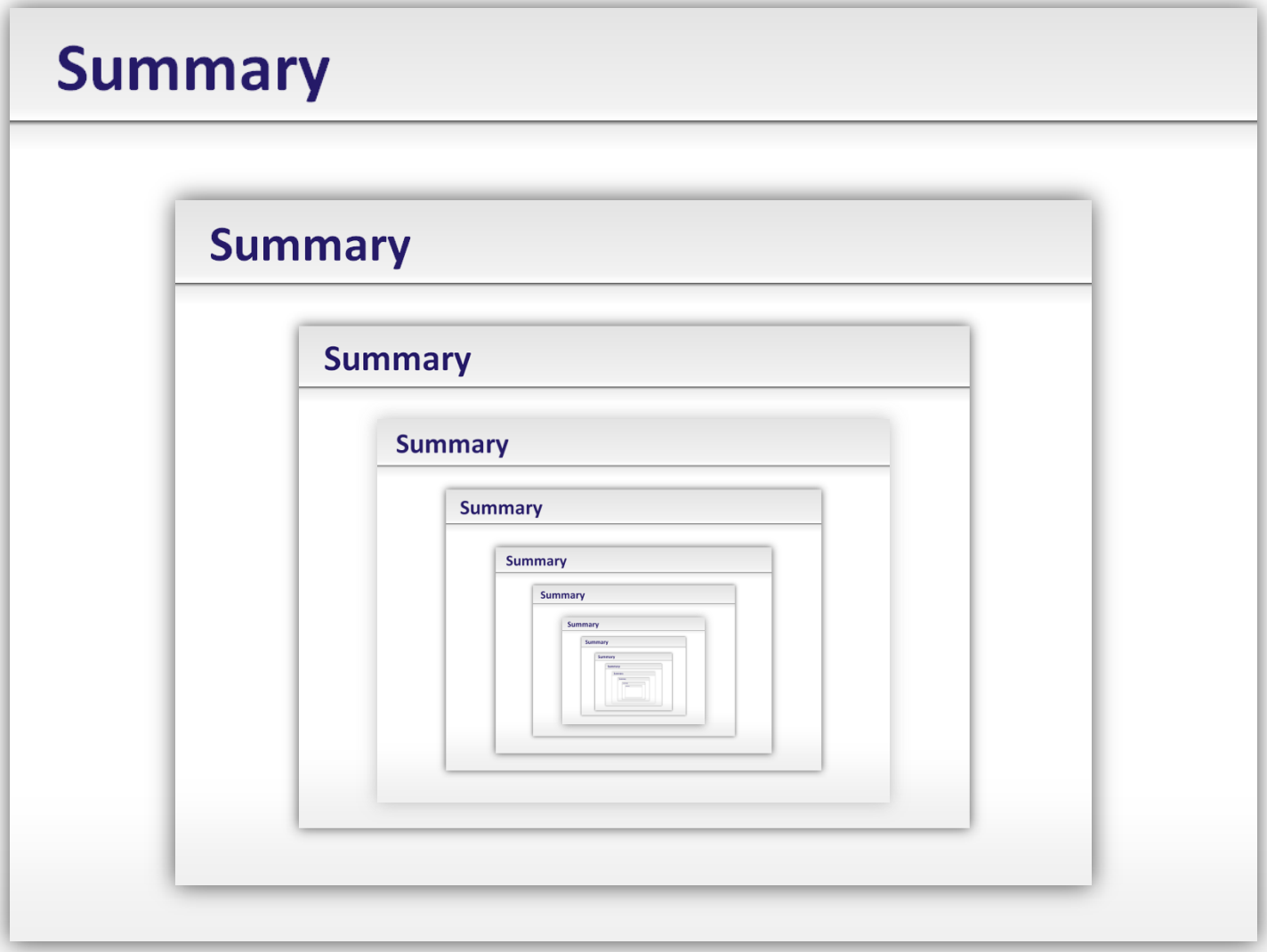
Variational Modeling

- Energy minimization on function spaces under constraints
- Numerical discretization, finite elements
- Variational Toolkit: Functionals, soft & hard constraints
- Euler-Lagrange equation
- Applications
  - Surface modeling
  - Dynamic modeling





Summary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Questions?