UNIVERSITÄT DES SAARLANDES
Dr.-Ing. Hendrik P.A. Lensch
Max Planck Institut Informatik
Art Tevs (tevs@mpi-inf.mpg.de)
Boris Ajdin (bajdin@mpi-inf.mpg.de)
Matthias Hullin (hullin@mpi-inf.mpg.de)

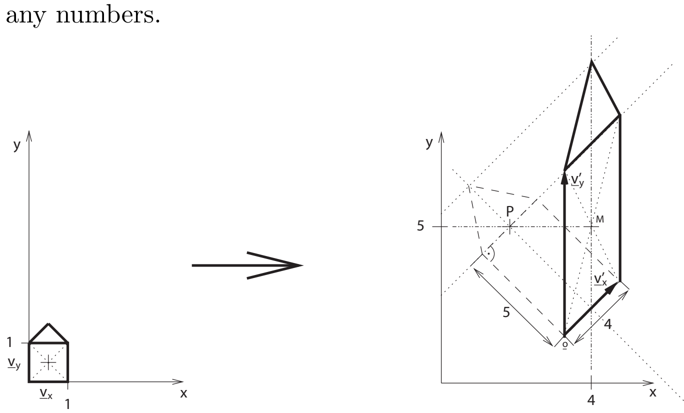
17. December 2007

Computer Graphics I
Assignment 8

Submission deadline for the exercises: Monday, 7th January 2008
Rule: Solutions have to be submitted in the lecture room before the lecture.

8.1 Transformations (50 Points)

In the picture below the left house should be transformed into the house on the right. The point M is at $(4, 5)$ and lines that look to be parallel are parallel. Please specify the complete transformation matrix as a sequence of primitive transformations (there's no need to calculate the final matrix). Do not guess any numbers.



8.2 Affine Spaces (20 Points)

Prove that the set of points $A = \{(x, y, z, w) \in R^4 \mid w = 1\}$ is an affine space. What is the associated vector space? You do *not* have to show that the associated vector space is a vector space. What is the difference between a point and a vector in that affine space?

Definition of an affine space: An affine space consists of a set of points P, an associated vector space V and an operation $+ \in P \times V \to P$ that fulfills the following axioms:

(1) `for` $p \in P$ `and` $v, w \in V : (p + v) + w = p + (v + w)$
(2) `for` $p, q \in P$ `there exists a unique` $v \in V$ `such that:` $p + v = q$

8.3 Rotations (30 Points)

Show that an arbitrary rotation around the origin in 2D can be represented by a combination of a shearing in y, a scaling in x and y and a shearing in x in this order. You have to derive the shearing and scaling matrices to an arbitrary rotation T.

$$T = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$$

8.4 Affine Spaces* (20 Points)

a) Prove that for any affine space $p + 0 = p$ is true. Use only the axioms of the definition 8.2 of an affine space.

b) A property of affine spaces is that a subtraction $- \in P \times P \to V$ exists that gives the connecting vector between two points. The definition of an affine space from Exercise 8.2 is missing such an axiom. How can you derive this operation from the axioms of an affine space?