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# Computer Graphics

- Ray Tracing I -

**Hendrik Lensch**

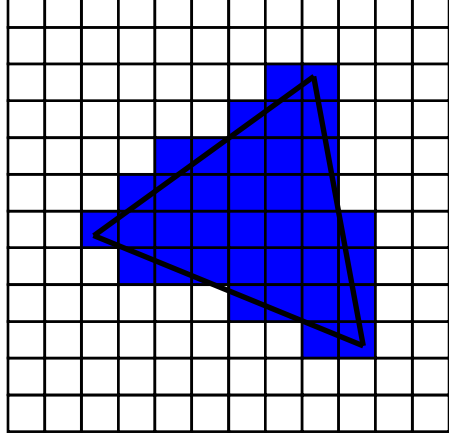
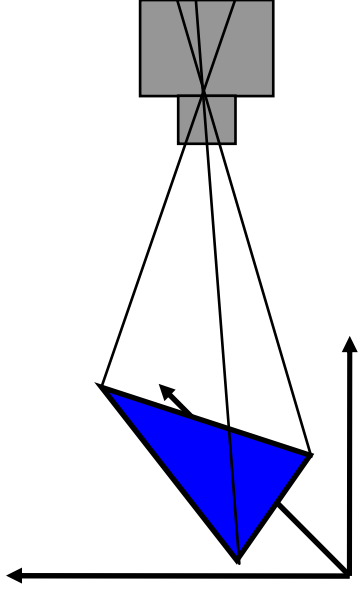
# Overview

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- **Last Lecture**
  - Introduction
- **Now**
  - Ray tracing I
    - Background
    - Basic ray tracing
    - What is possible?
    - Recursive ray tracing algorithm
- **Next lecture**
  - Ray tracing II: Spatial indices

# Current Graphics: Rasterization

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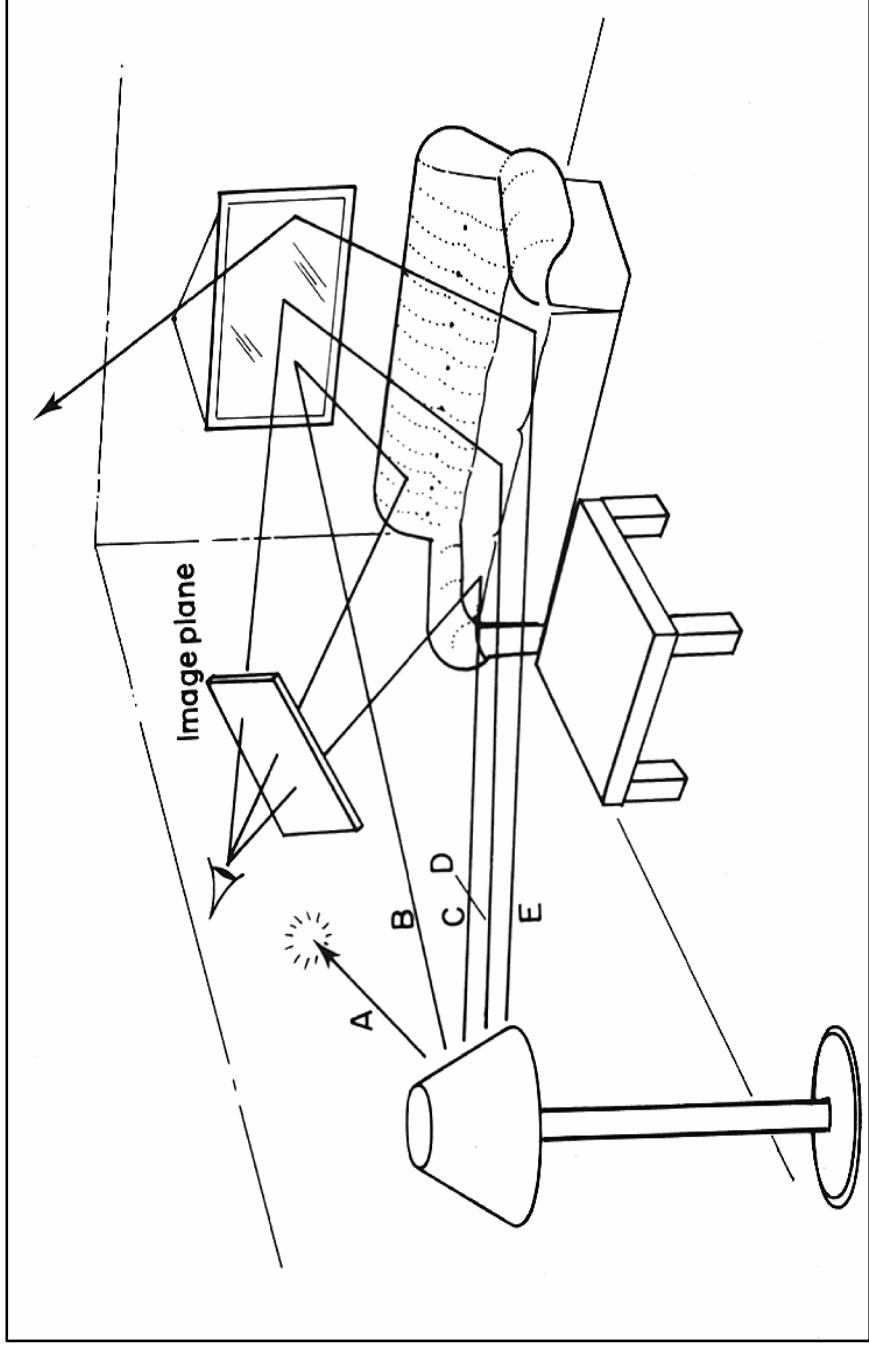


- **Primitive operation of all interactive graphics !!**
  - Scan convert a single triangle at a time
- **Sequentially processes every triangle individually**
  - Can never access more than one triangle
  - **But most effects need access to the world: shadows, reflection, global illumination**

# Tracing the Paths of Light

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- **Nature:**
  - Follow the path of *many* photons
  - Record those hitting the film in a camera



# Light Transport

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- **Light Distribution in a Scene**
  - Dynamic equilibrium
  - Newly created, scattered, and absorbed photons
- **Forward Light Transport:**
  - Start at the light sources
  - Shoot photons into scene
  - Reflect at surfaces (according to some reflection model)
  - Wait until they are absorbed or hit the camera (very seldom)  
→ Nature: massive parallel processing at the speed of light
- **Backward Light Transport:**
  - Start at the camera
  - Trace only paths that transport light towards the camera  
→ Ray tracing

# Ingredients

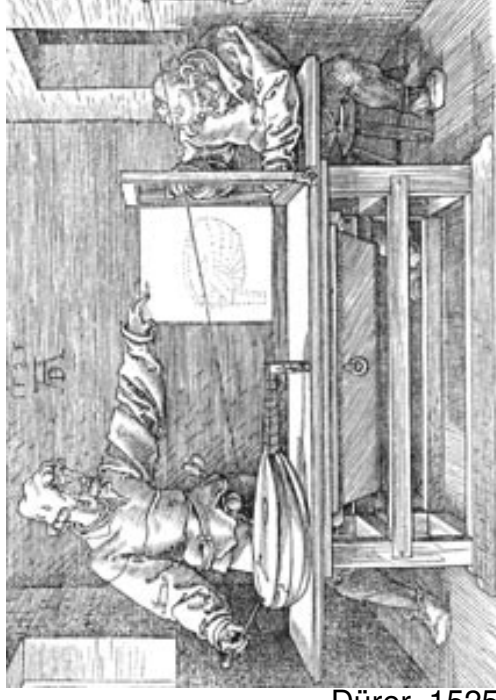
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- **Surfaces**
  - 3D geometry of objects in a scene
- **Surface reflectance characteristics**
  - Color, absorption, reflection, refraction, subsurface scattering
  - Local property, may vary over surface
  - Mirror, glass, glossy, diffuse, ...
- **Illumination**
  - Position, characteristics of light emitters
  - Repeatedly reflected light → indirect illumination
- **Assumption: air/empty space is totally transparent**
  - Excludes any scattering effects in participating media volumes
  - Would require solving a much more complex problem
  - Volume rendering, participating media

# Ray Tracing

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- **The Ray Tracing Algorithm**
  - One of the two fundamental rendering algorithms
- **Simple and intuitive**
  - Easy to understand and implement
- **Powerful and efficient**
  - Many optical global effects
    - shadows, reflection, refraction, and other
  - Efficient real-time implementation in SW and HW
- **Scalability**
  - Can work in parallel and distributed environments
  - Logarithmic scalability with scene size:  $O(\log n)$  vs.  $O(n)$
  - Output sensitive and demand driven
- **Not new**
  - Light rays: Empedocles (492-432 BC), Renaissance (Dürer, 1525)
  - Uses in lens design, geometric optics, ...



Dürer, 1525

# Ray Tracing

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- **Highly Realistic Images**
  - Ray tracing enables *correct* simulation of light transport

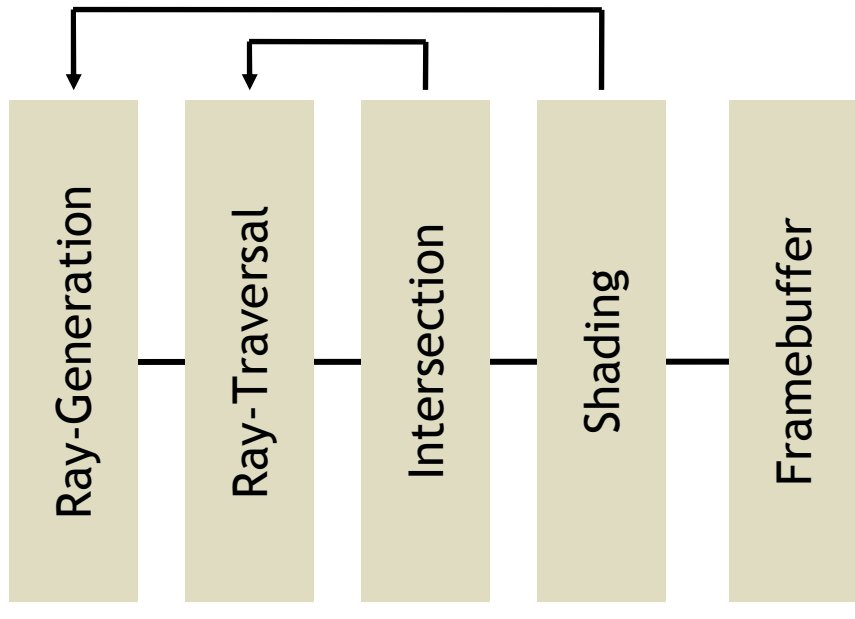
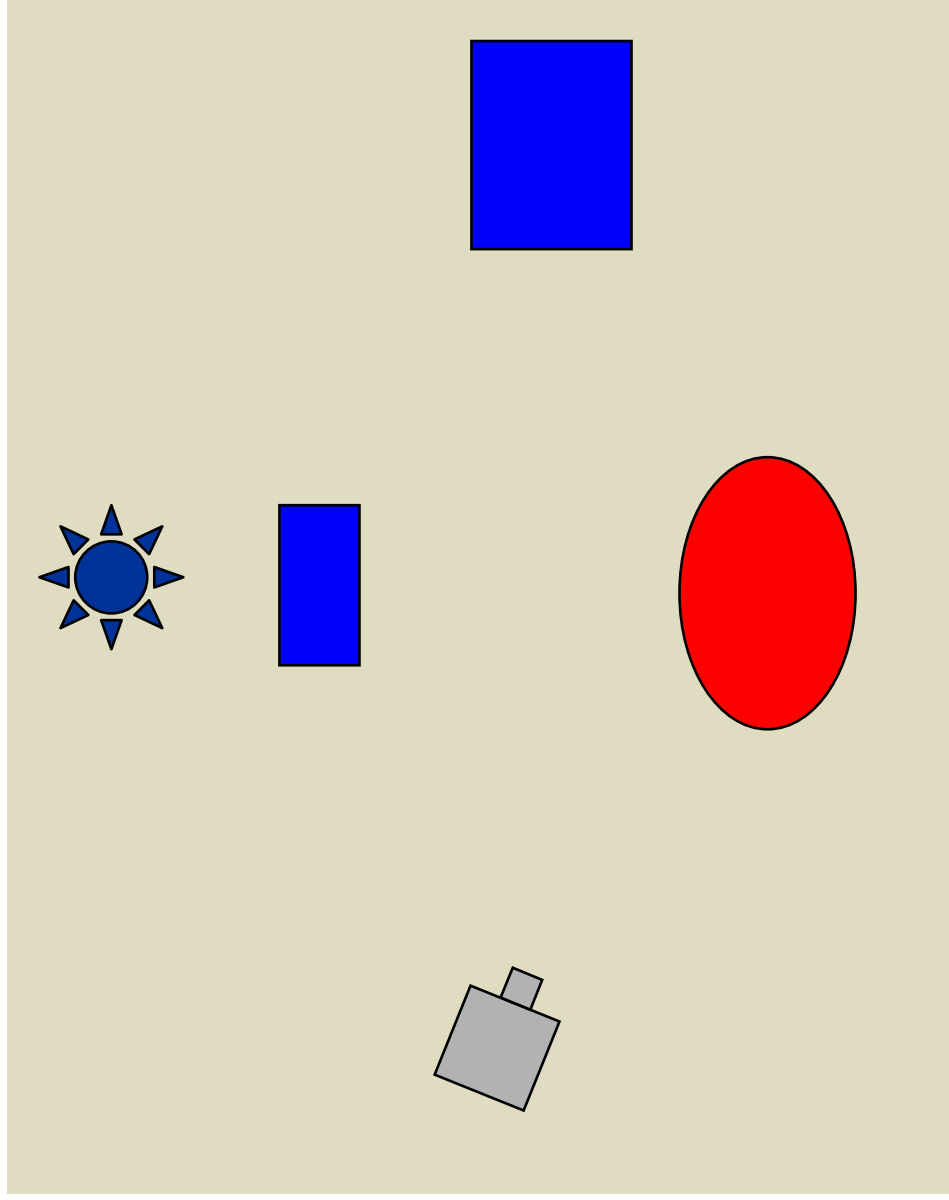


Internet Ray Tracing Competition, June 2002



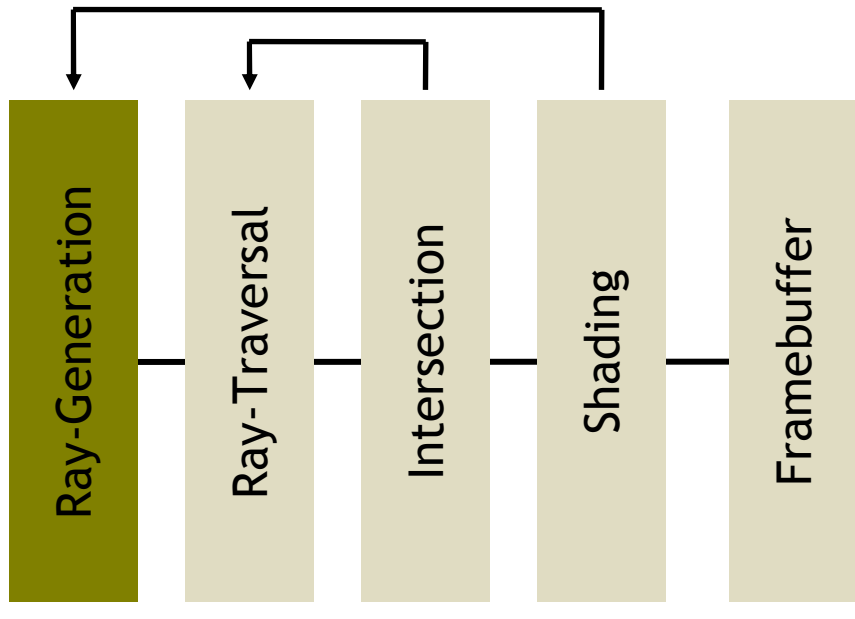
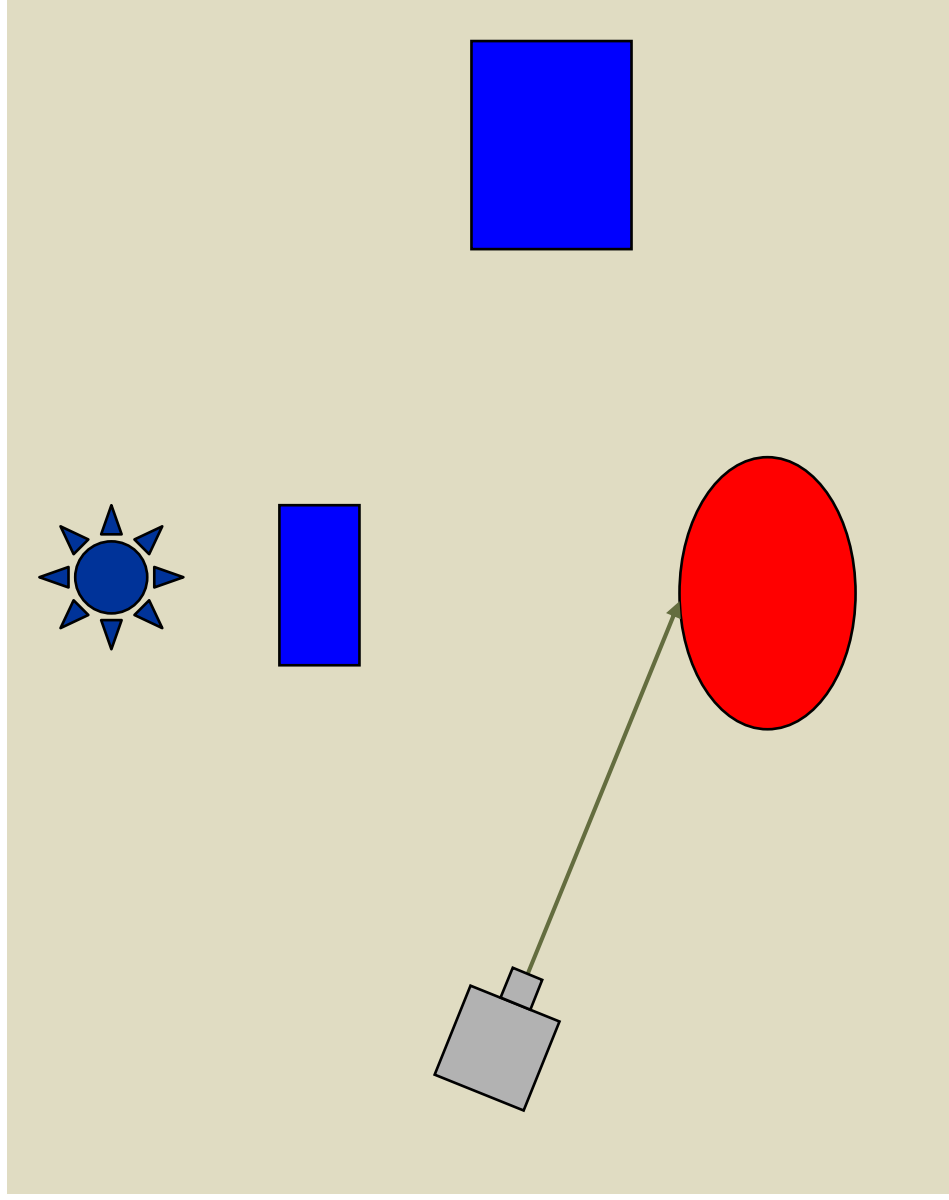
# Ray Tracing Pipeline

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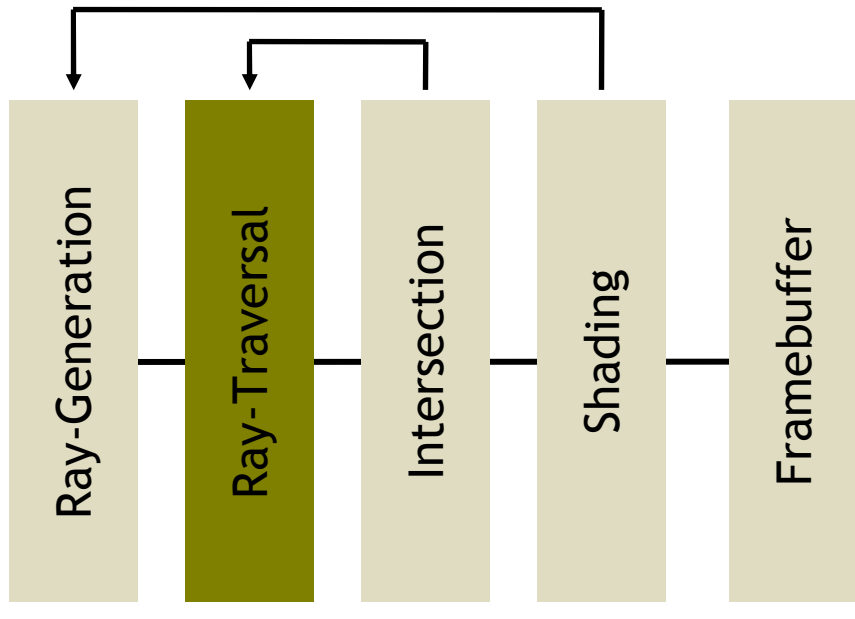
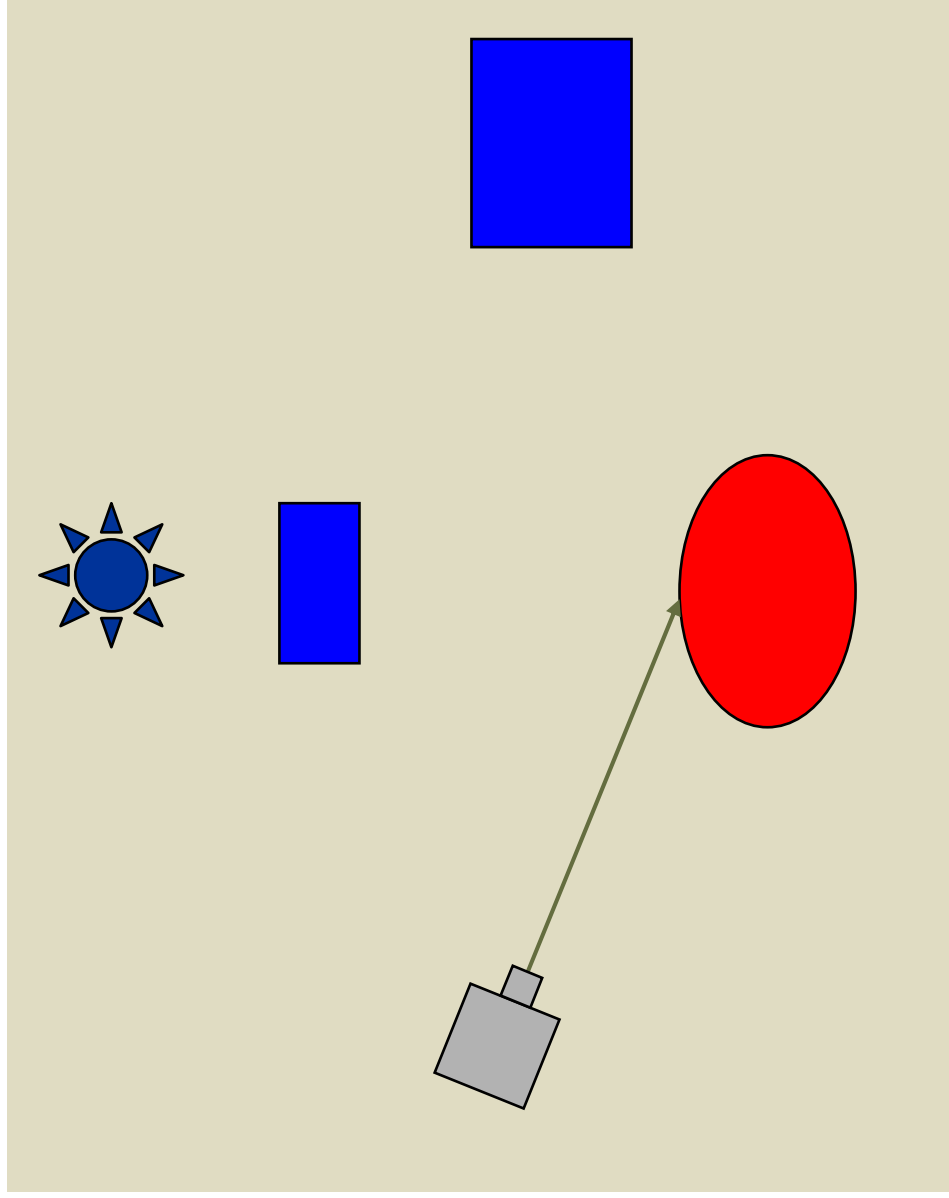
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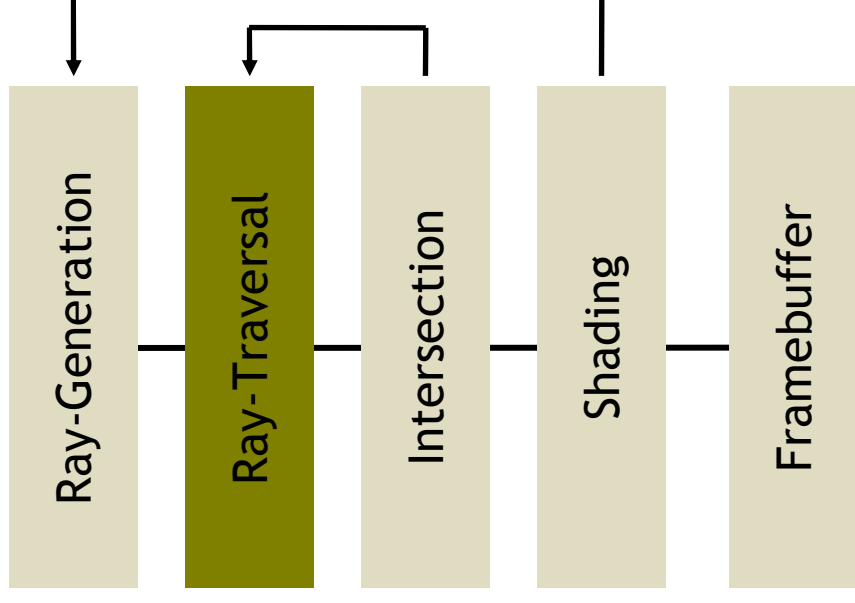
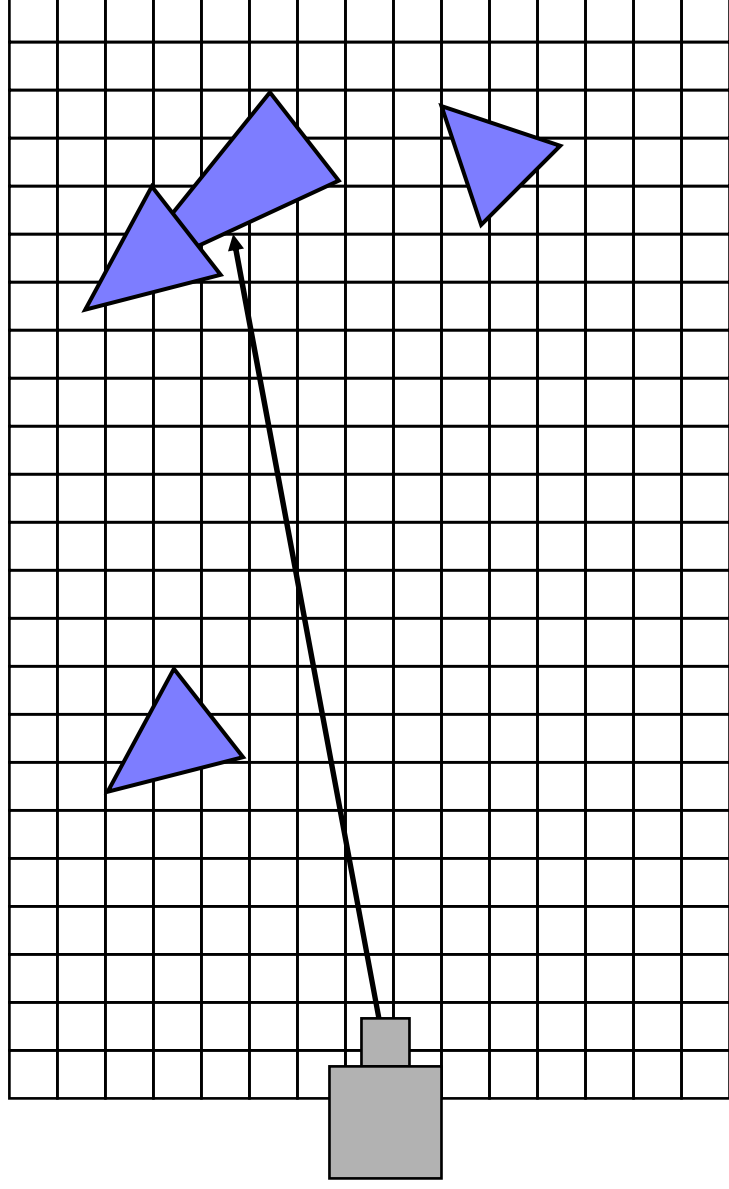
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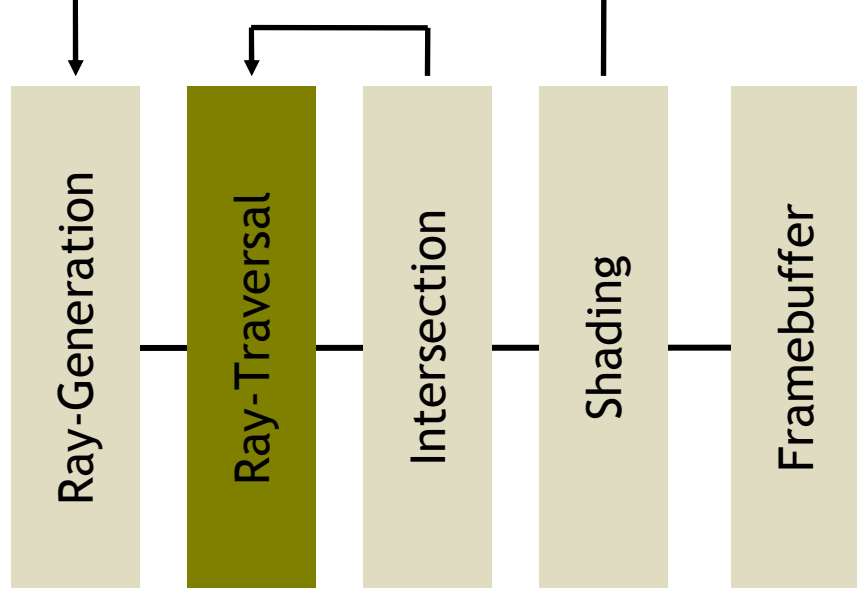
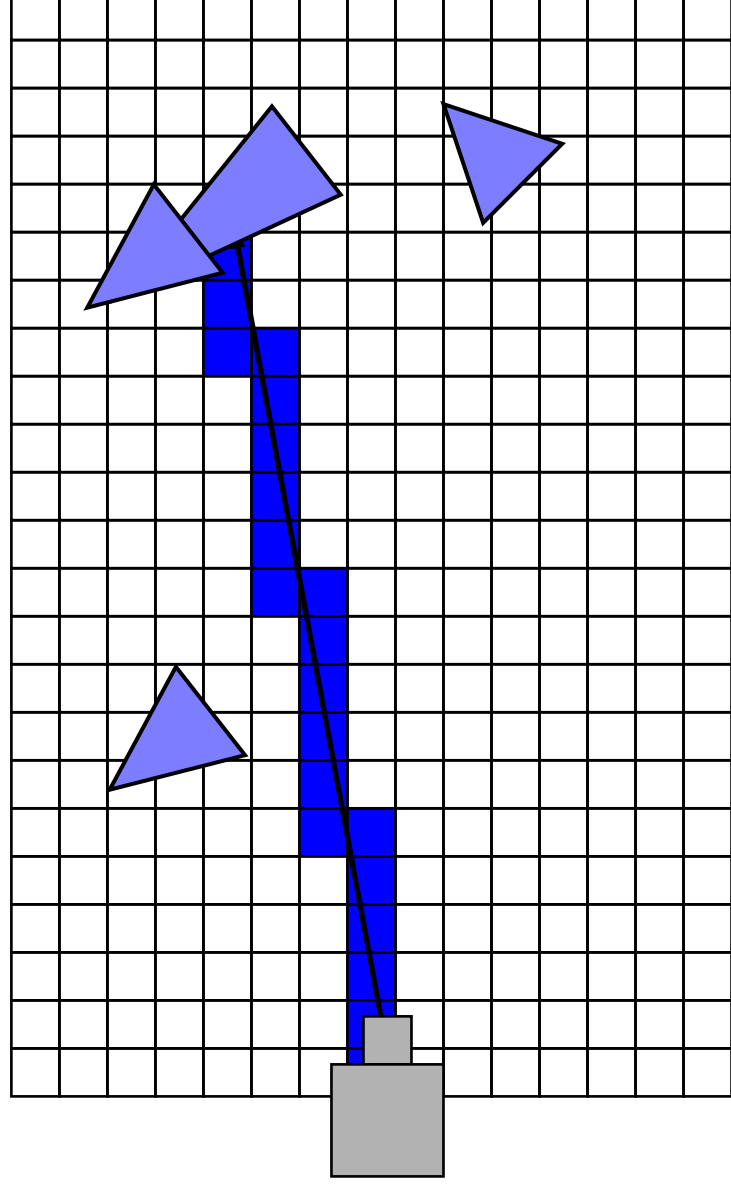
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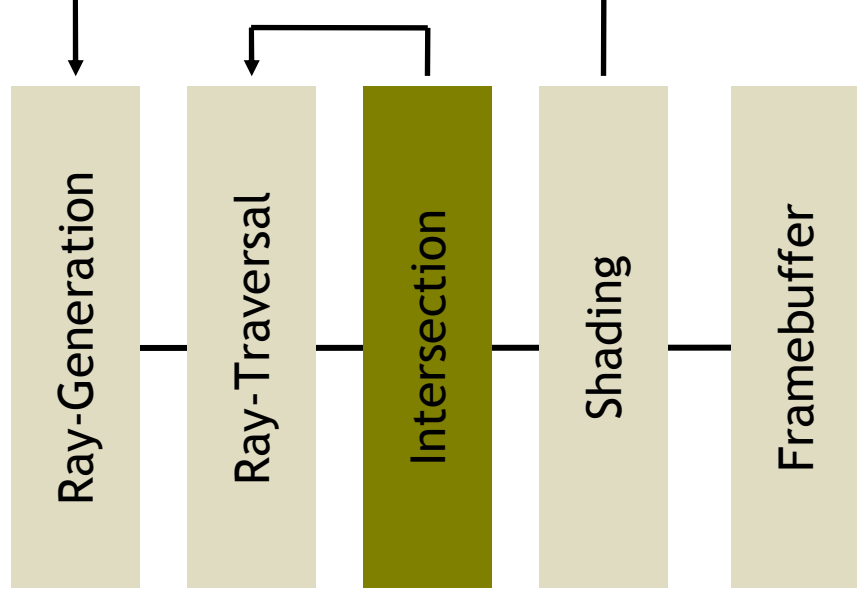
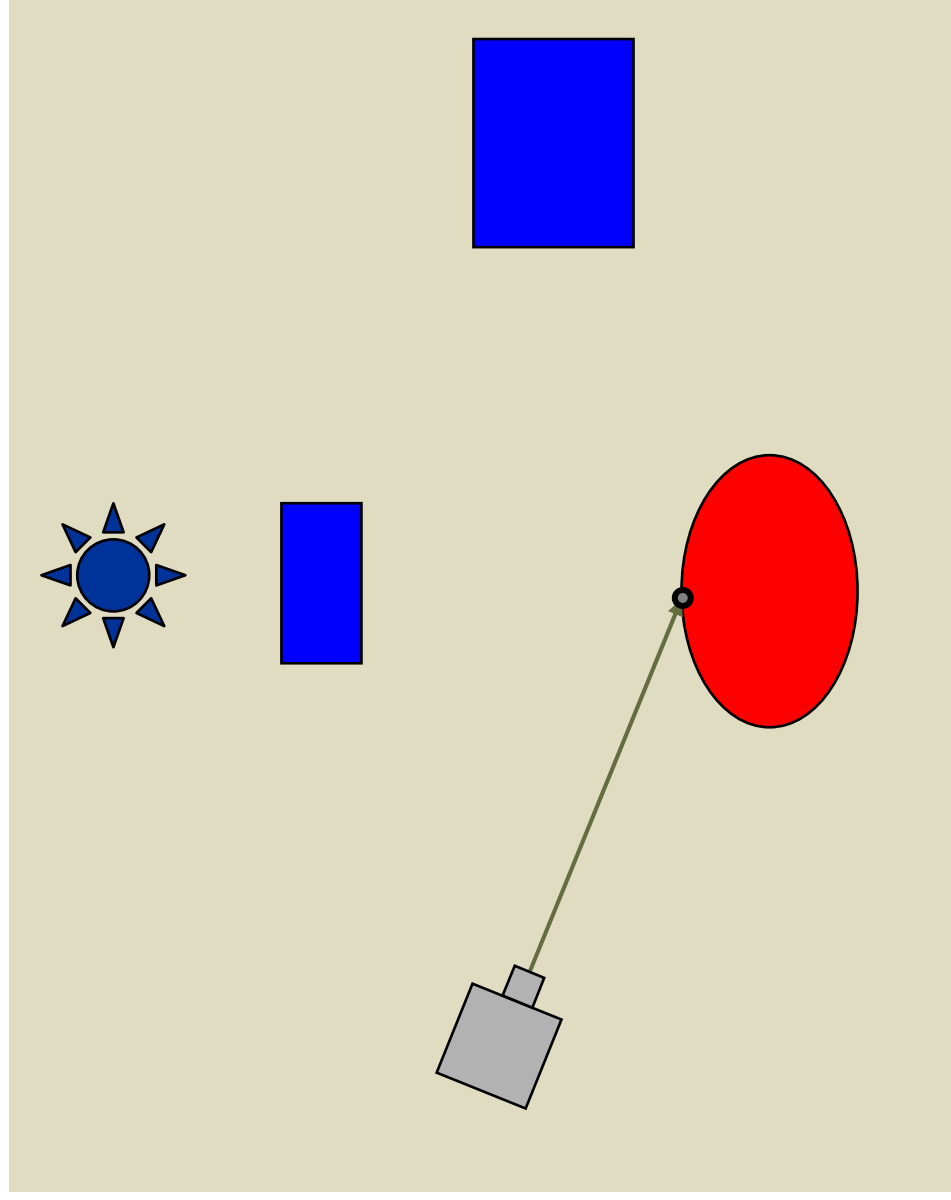
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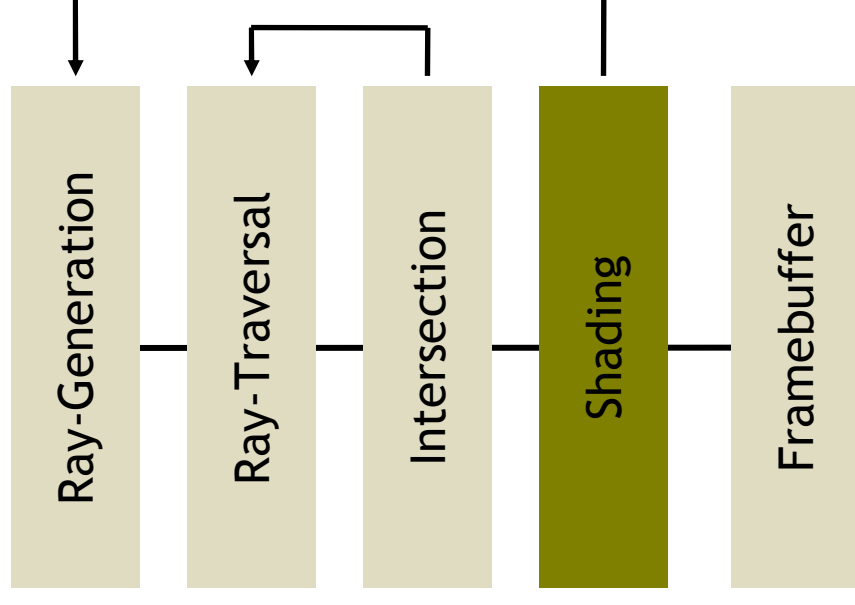
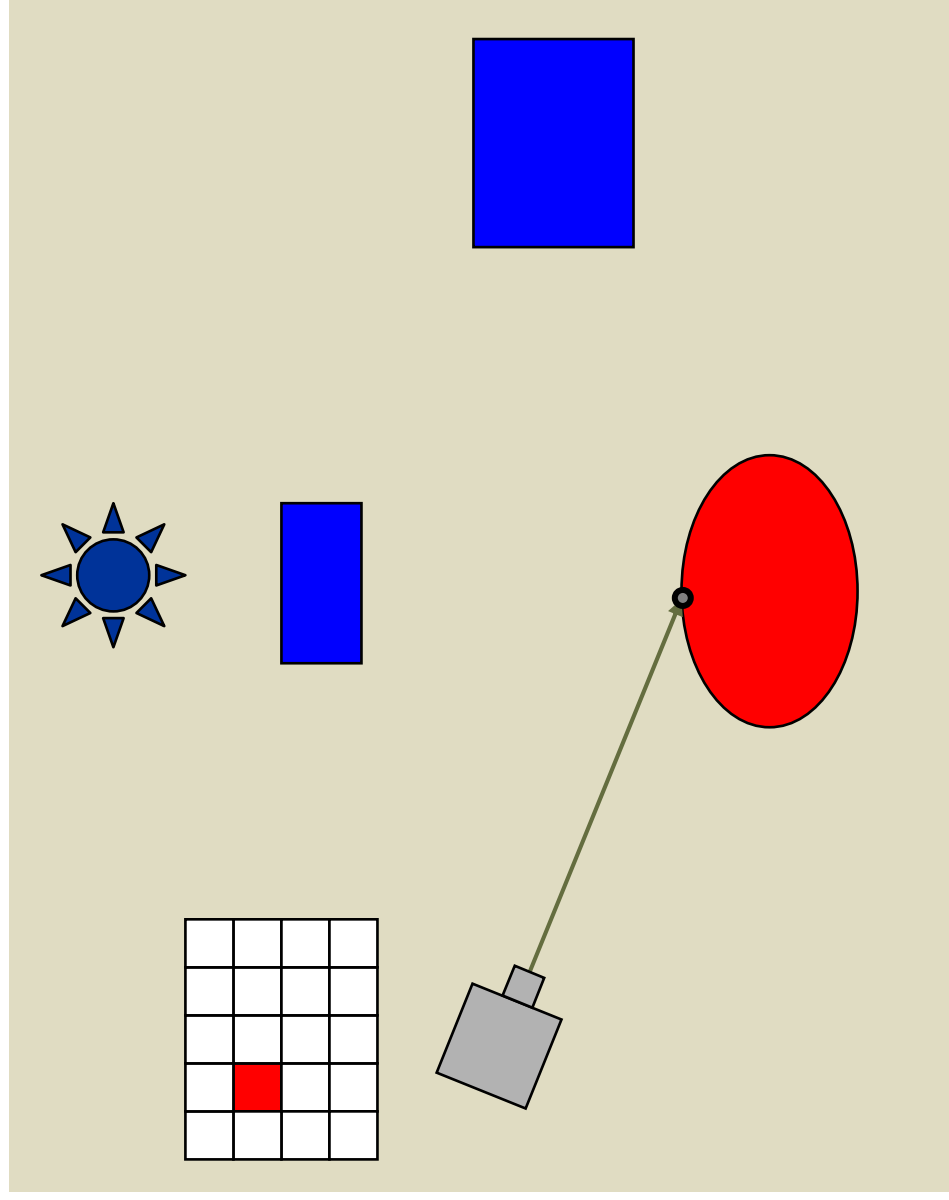
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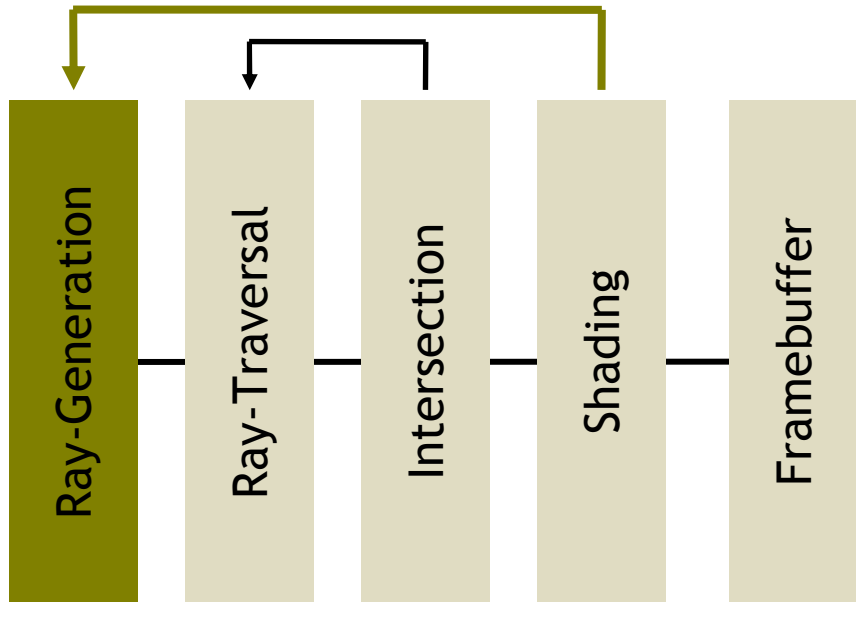
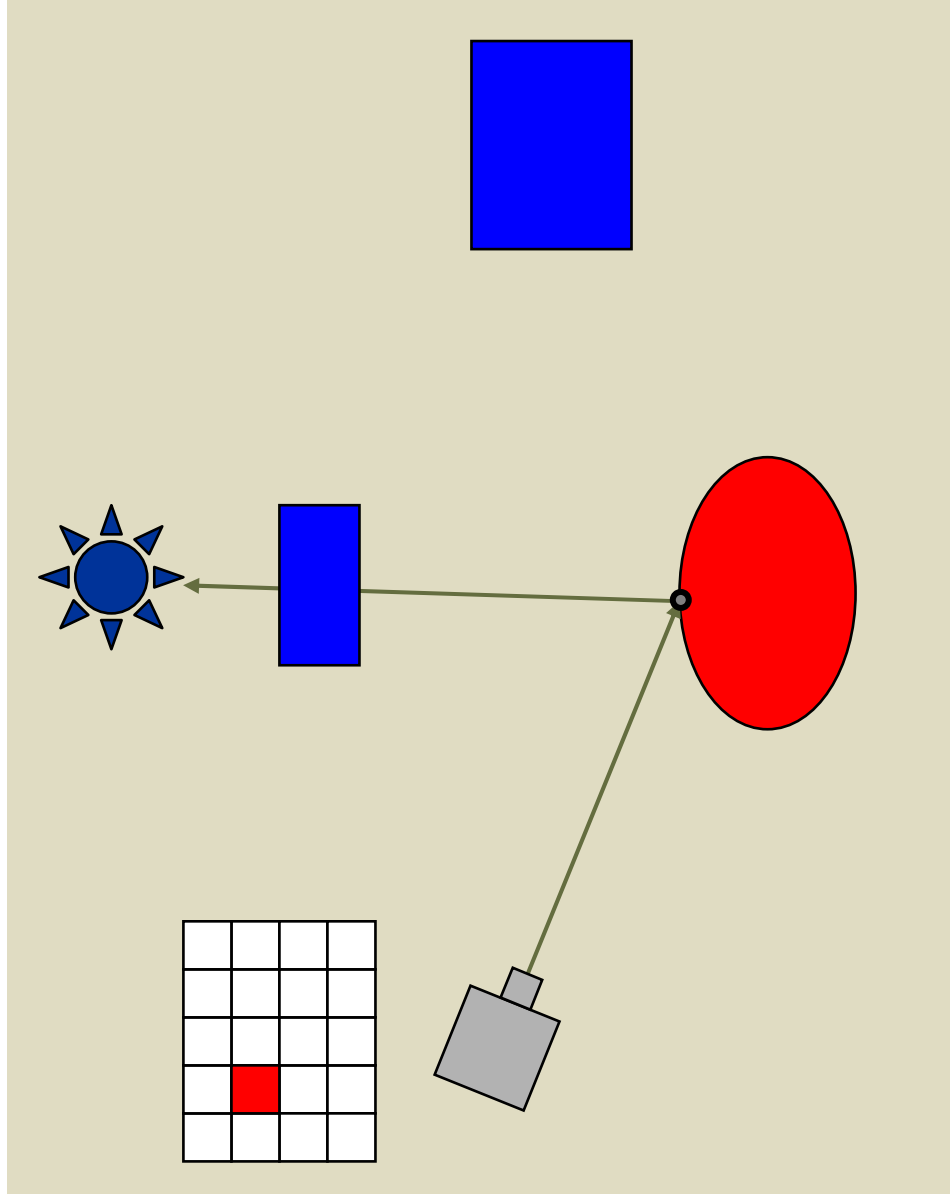
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# Ray Tracing Pipeline

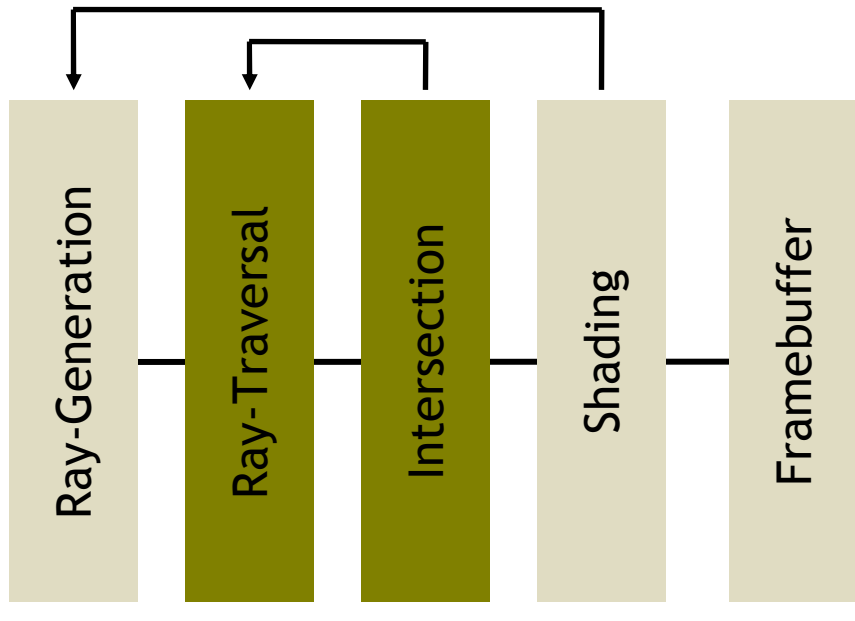
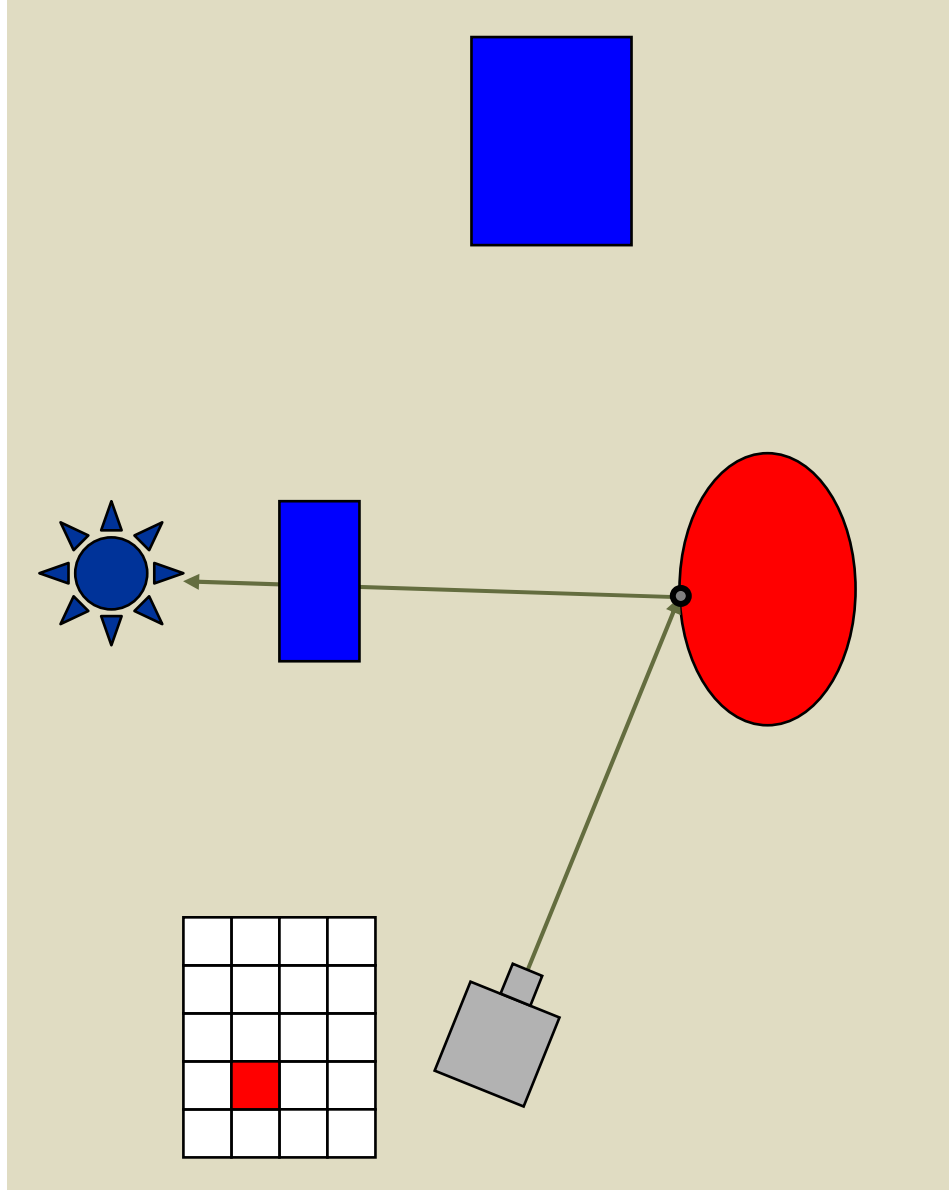
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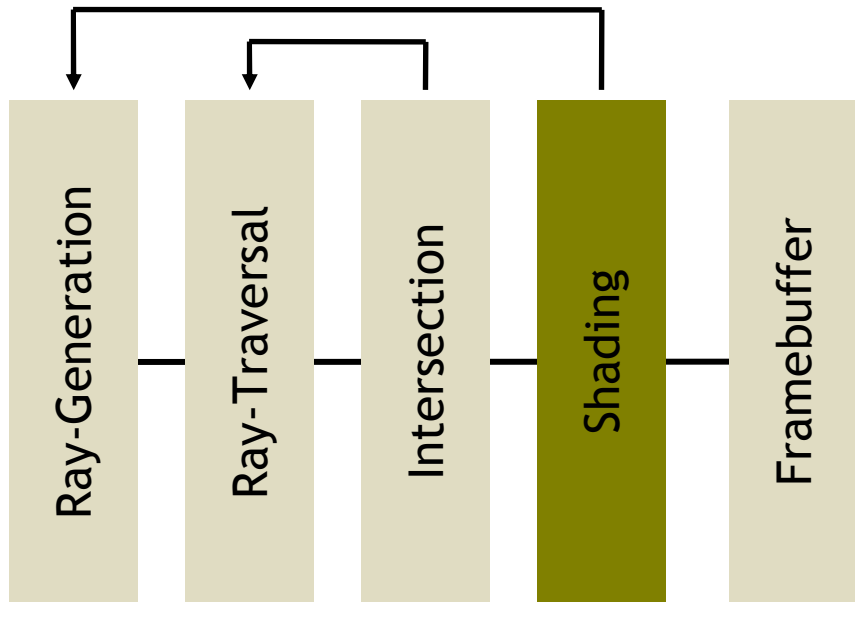
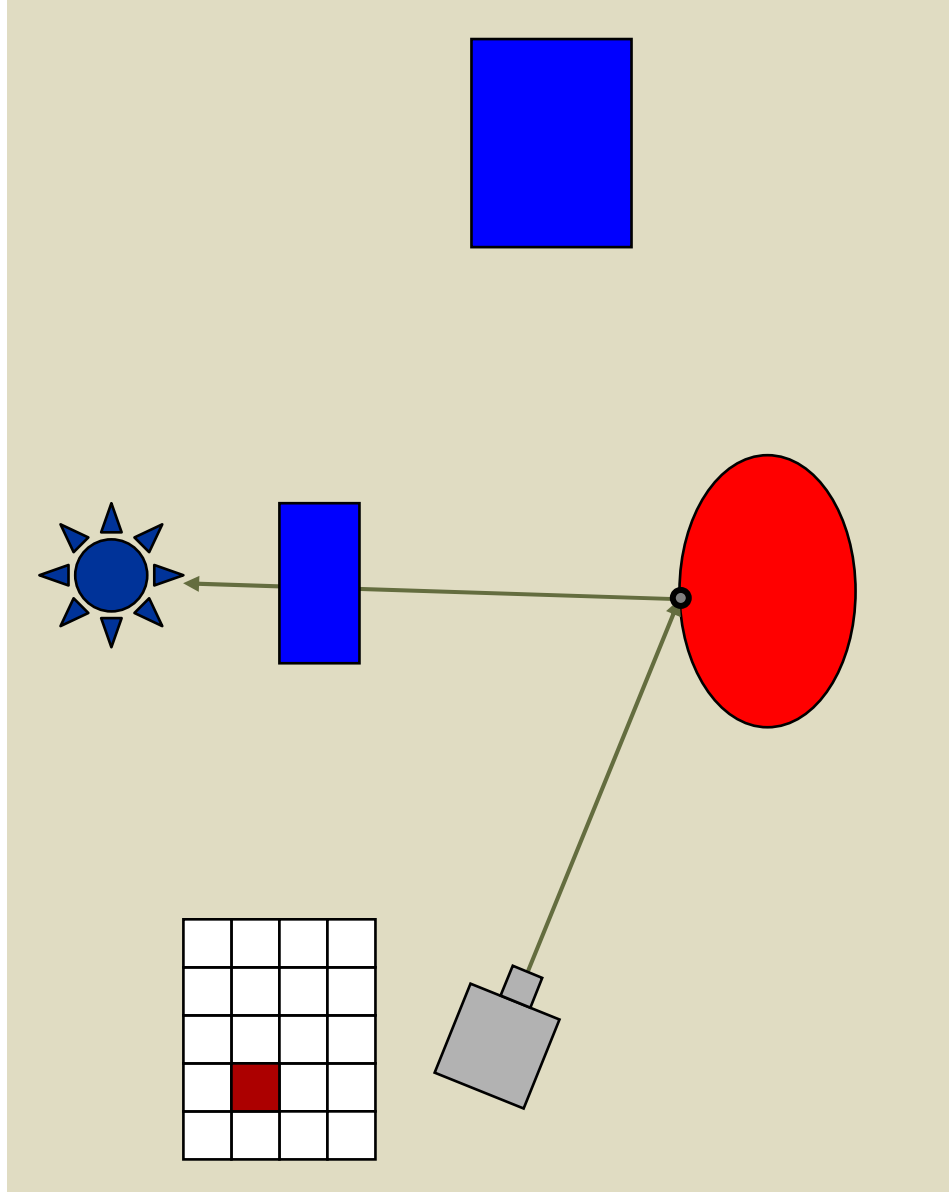
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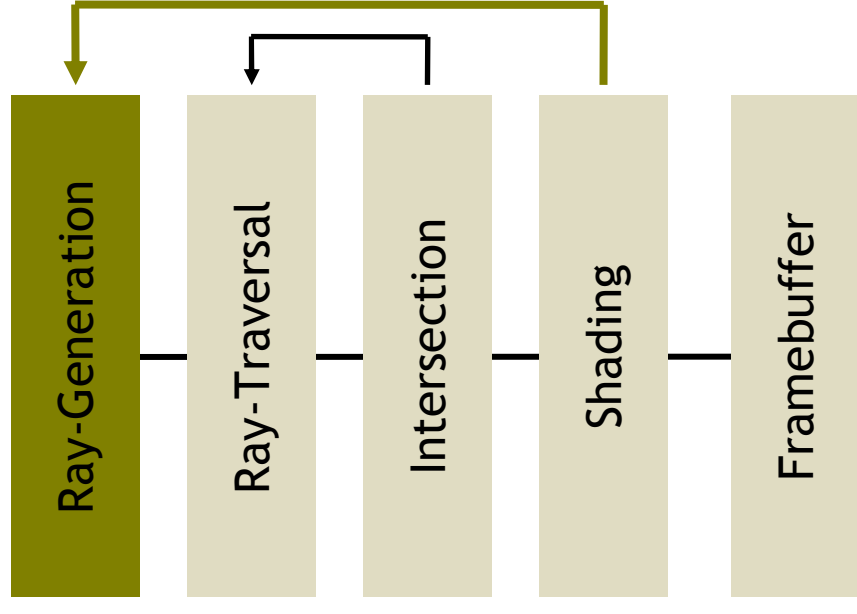
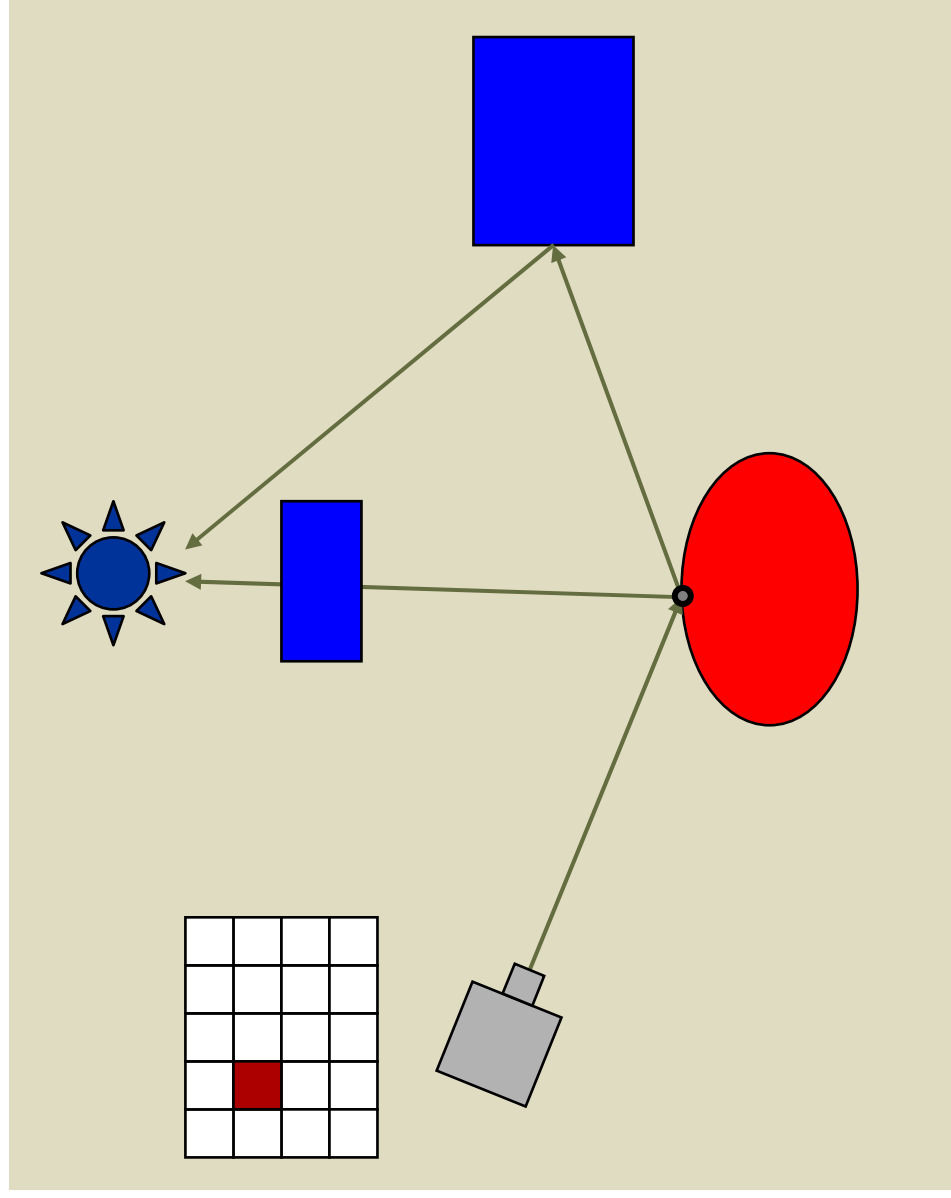
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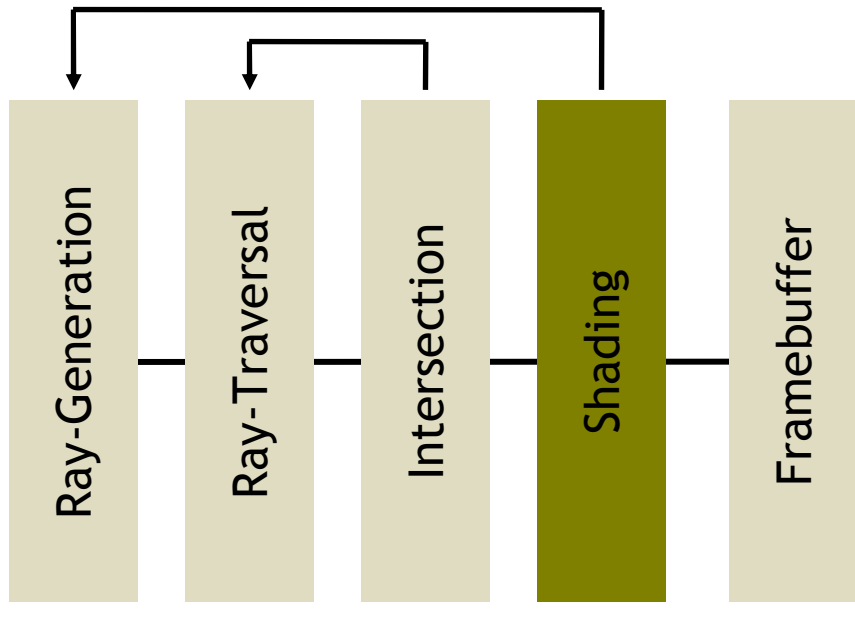
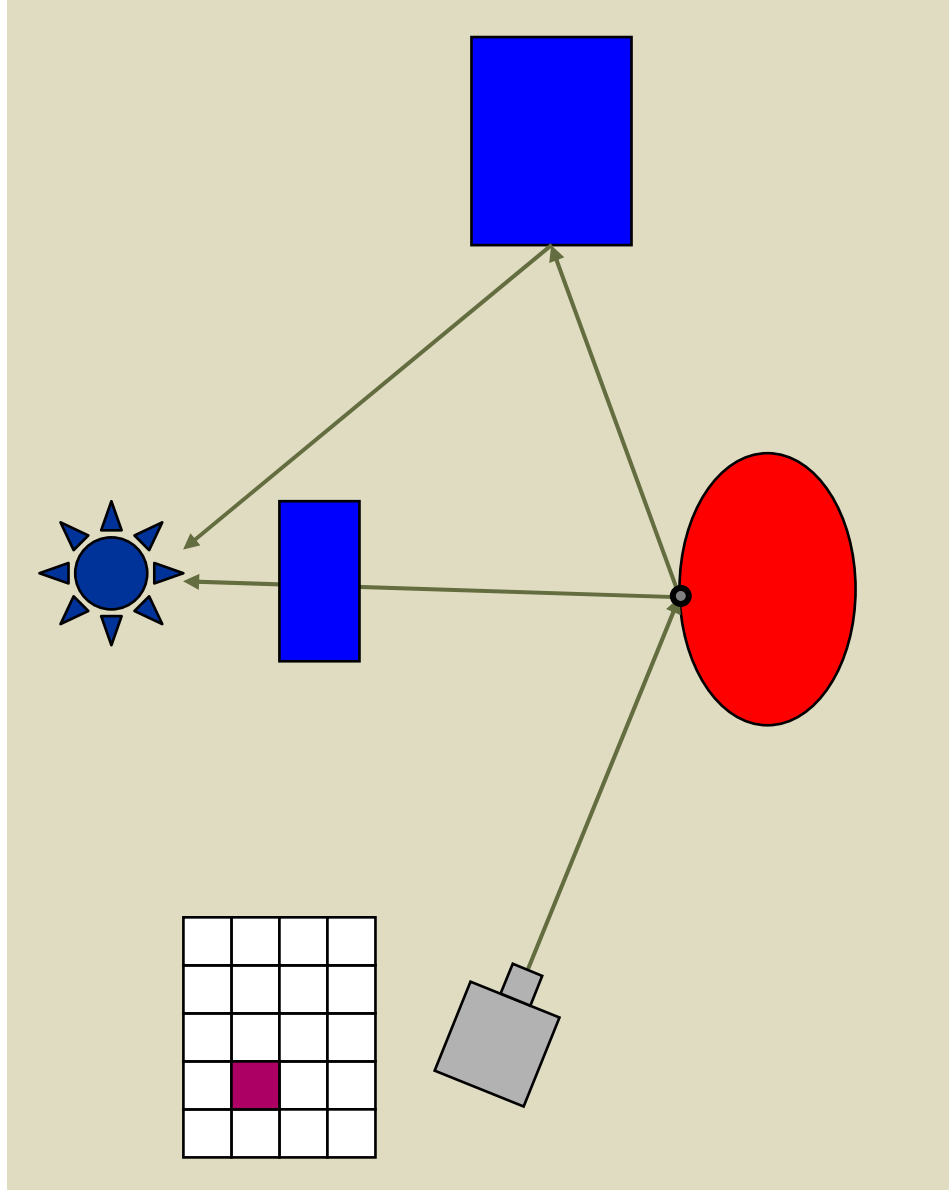
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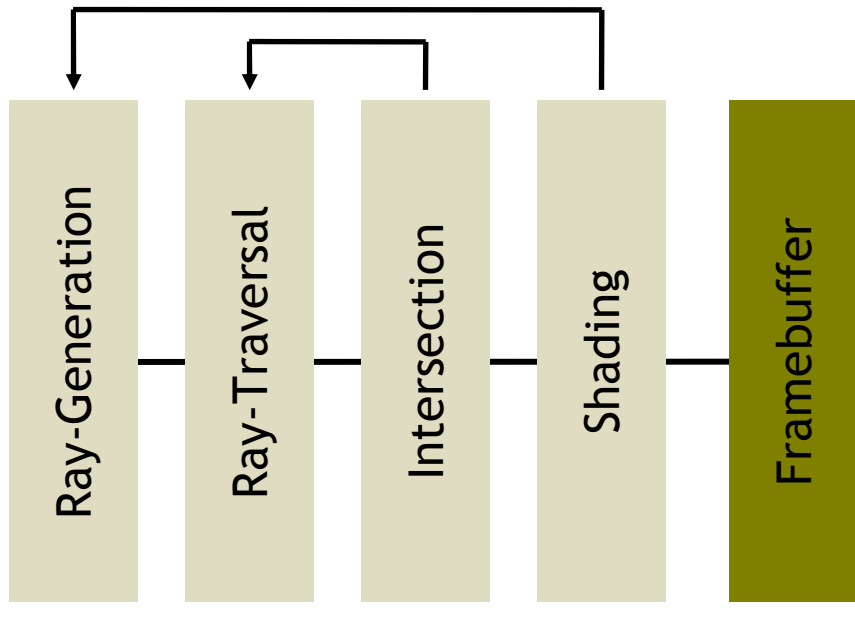
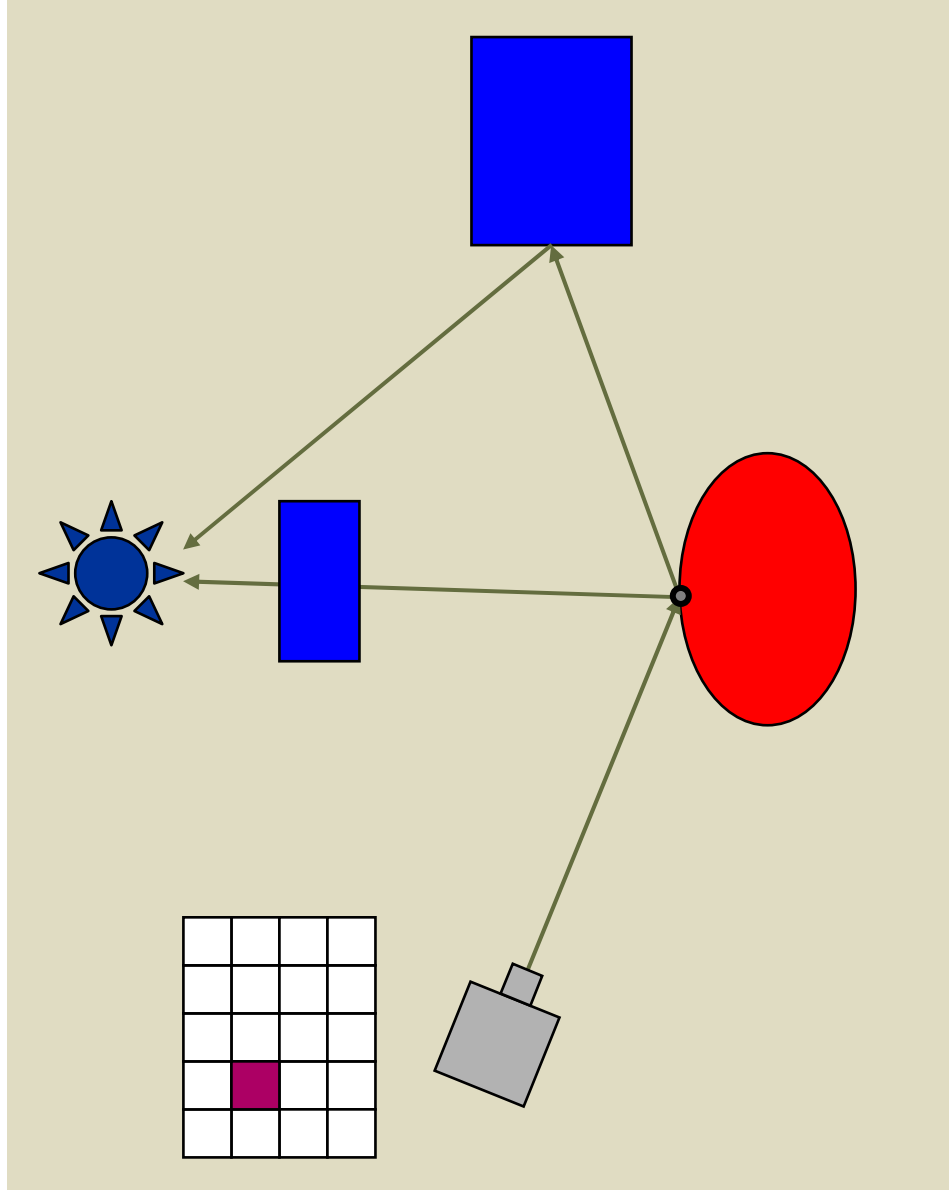
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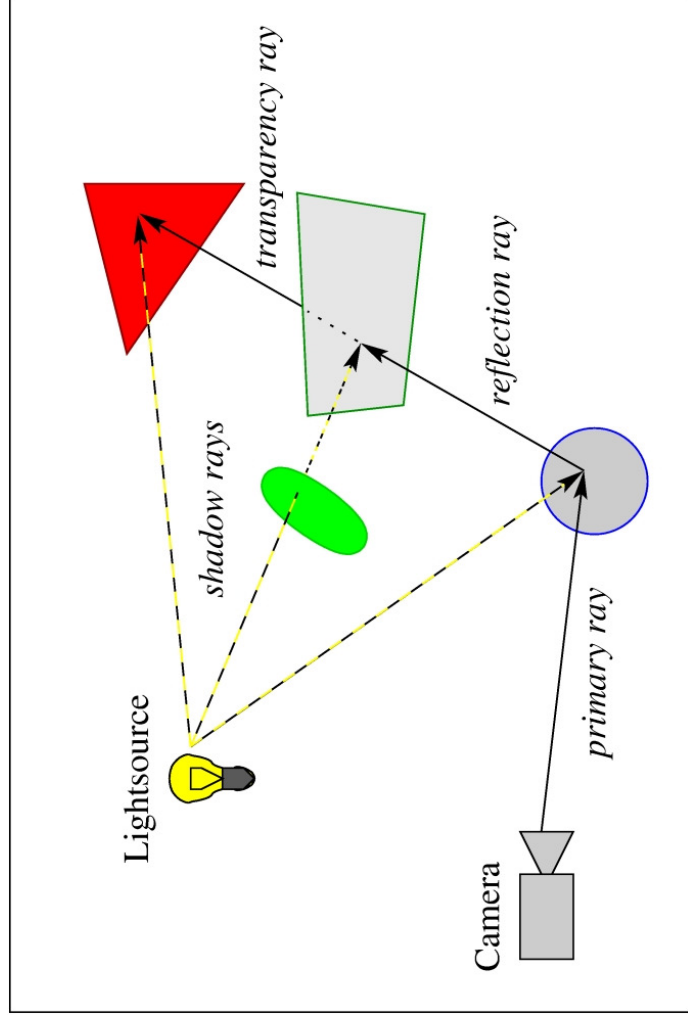
# Ray Tracing Pipeline

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# Ray Tracing

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- Global effects
- Parallel (as nature)
- Fully automatic
- Demand driven
- Per pixel operations
- Highly efficient

➔ Fundamental Technology for Next Generation Graphics

# Ray Tracing

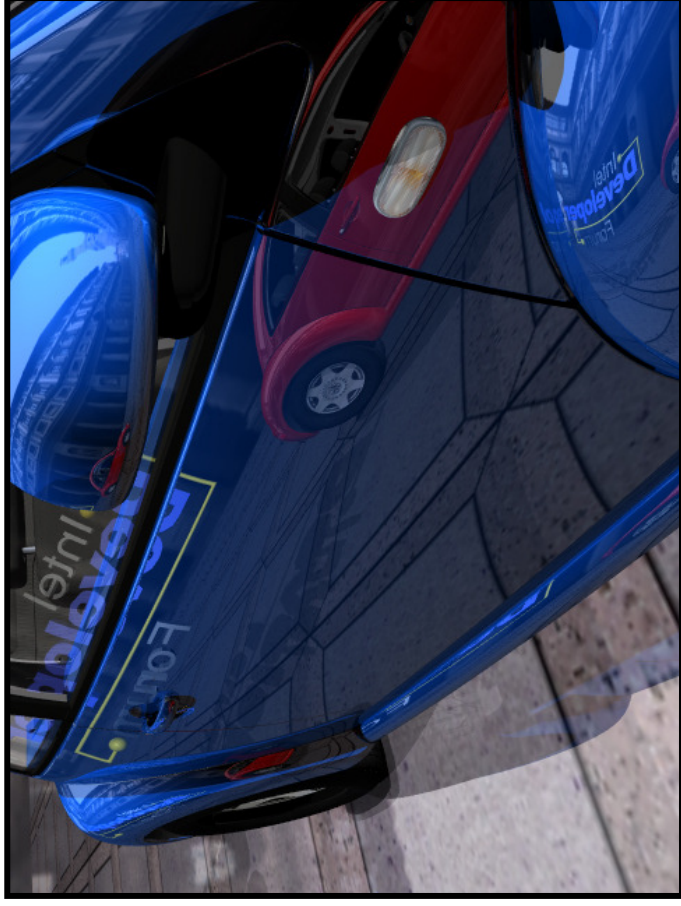
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- **In the Past**
  - Only used as an off-line technique
  - Was computationally far too demanding
  - Rendering times of minutes and hours
- **Recently**
  - Interactive ray tracing on supercomputers [Parker, U. Utah'98]
  - Interactive ray tracing on PCs [Wald'01]
  - Distributed ray tracing on PC clusters [Wald'01]
- **OpenRT-System** ([www.openrt.de](http://www.openrt.de))

# What is Possible?

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- **Models Physics of Global Light Transport**
  - Dependable, physically-correct visualization

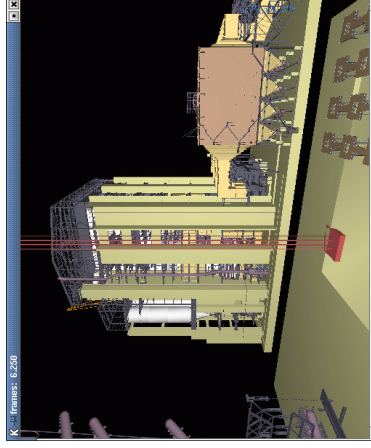




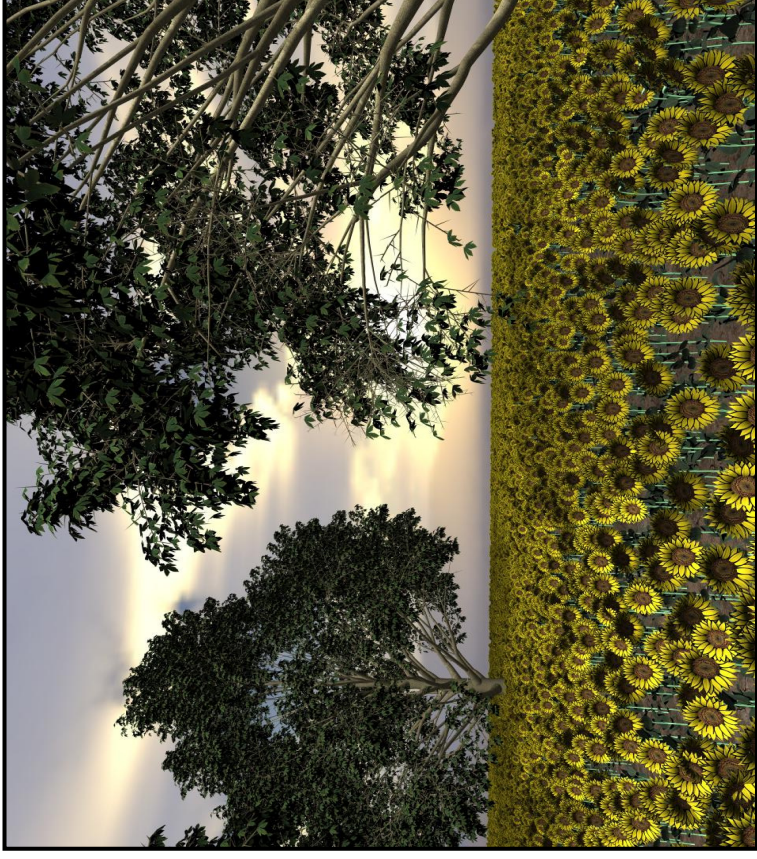
# What is Possible?

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- **Huge Models**
  - Logarithmic scaling in scene size



12.5 Million  
Triangles



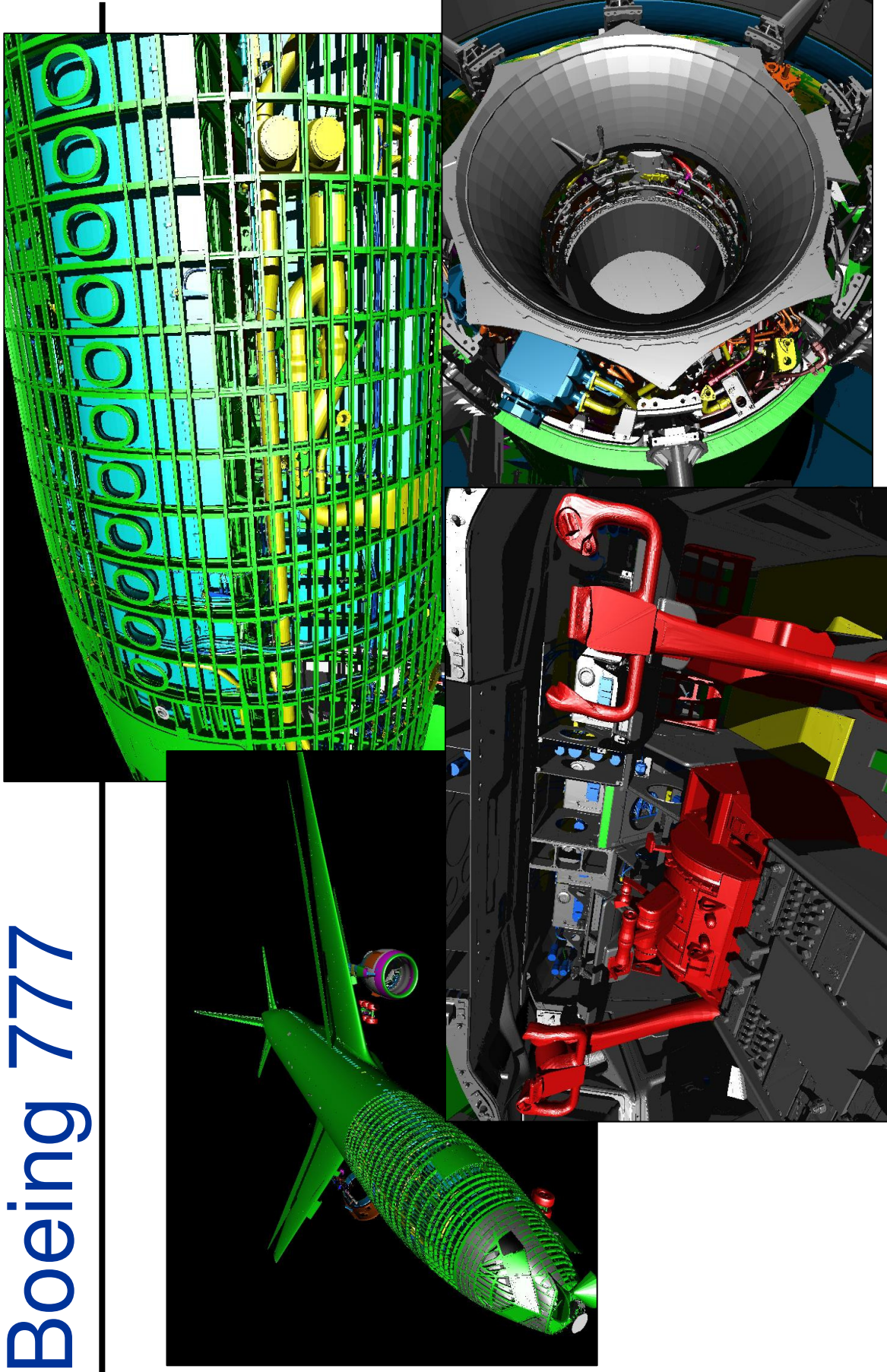
~1 Billion  
Triangles

# Huge & Realistic 3D Models

Outdoor environment: ~365,000 plants, ~1.5 billion triangles  
Rendered in realtime with skylight illumination on PC cluster



# Boeing 777

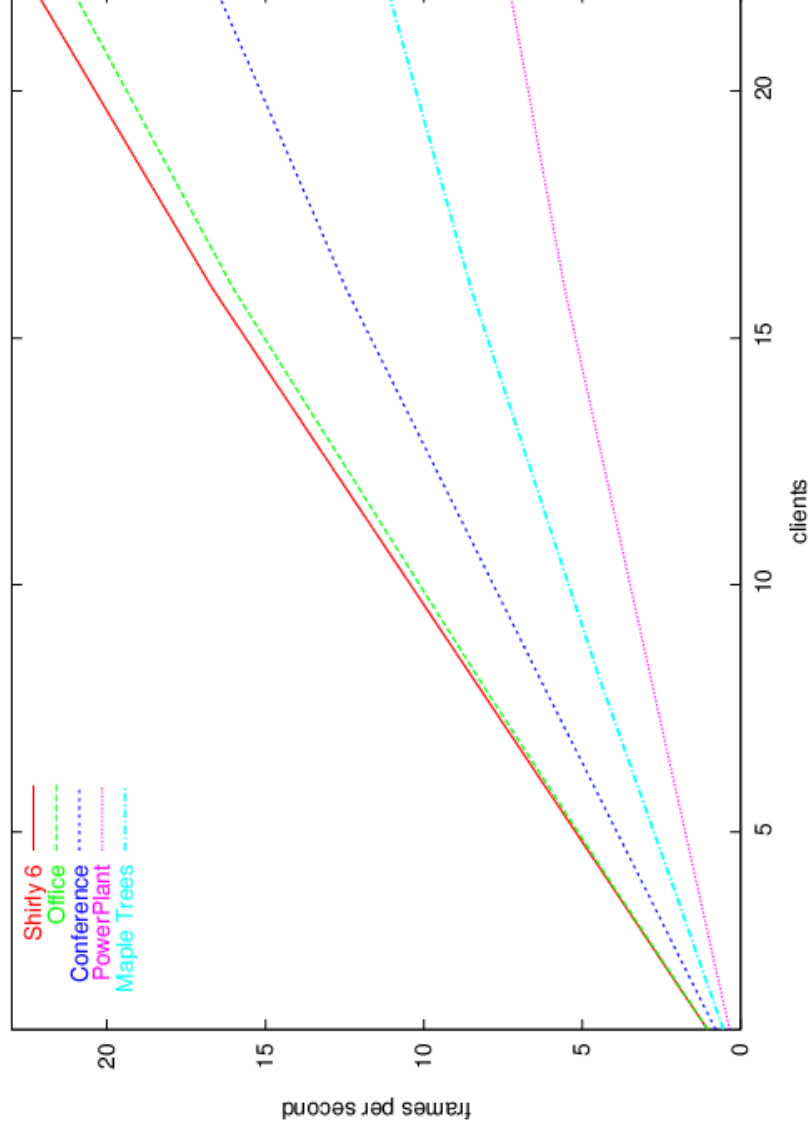


Boeing 777: ~350 million individual polygons, ~30 GB on disk

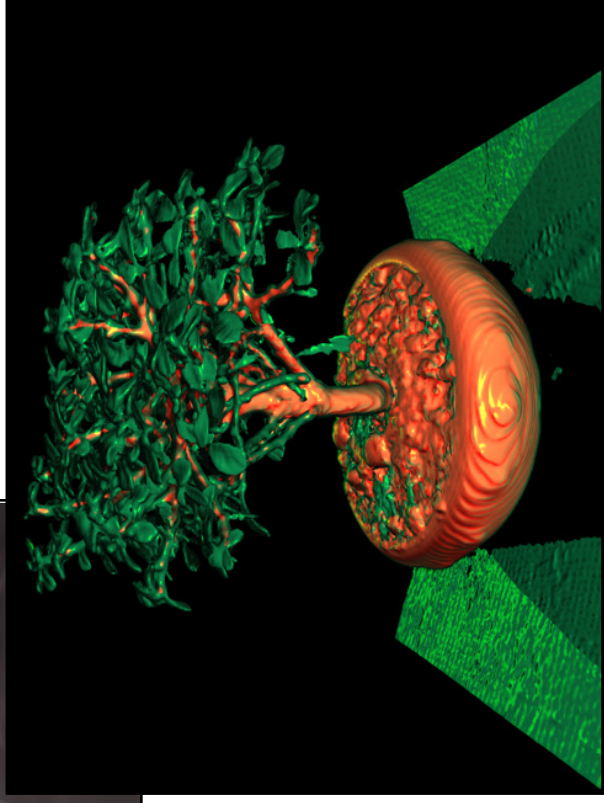
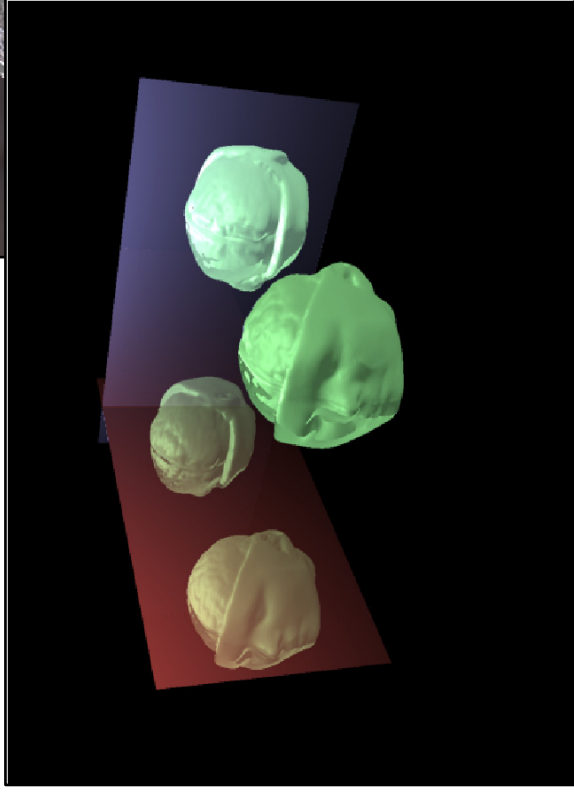
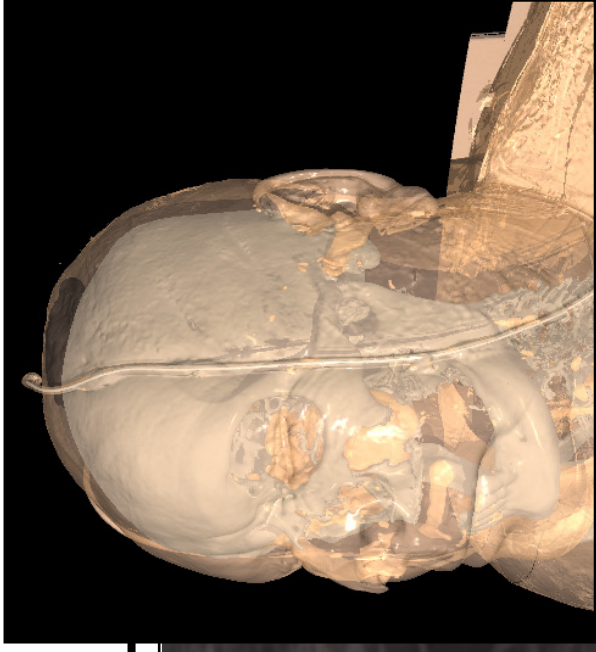
# What is Possible?

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- **Highly Scalable**
  - Output sensitivity with build-in occlusion culling
  - Linear in number of pixels, rays, and processors



# Volume Visualization



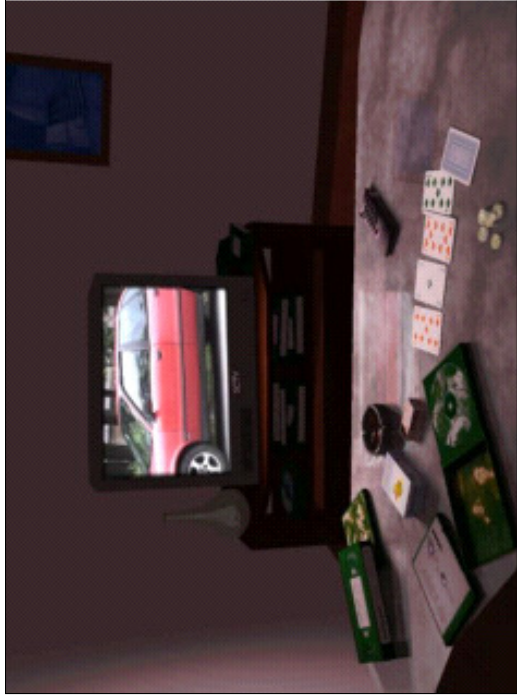
## Iso-Surface Volume Rendering

# Measured Materials

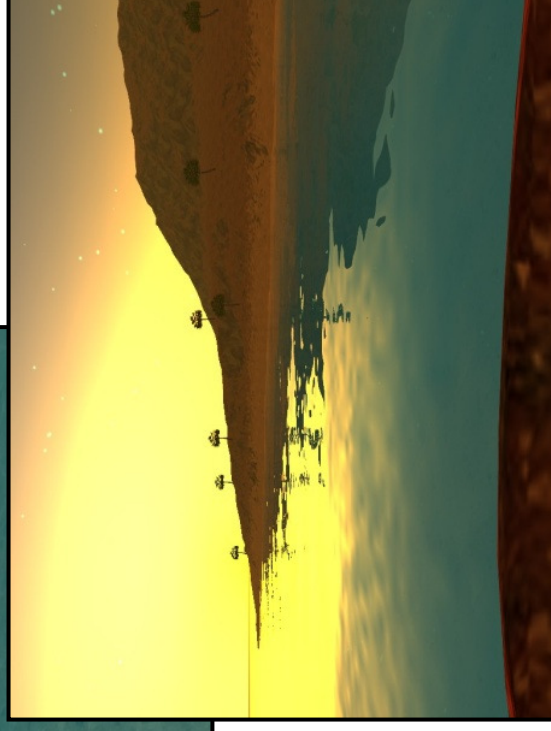
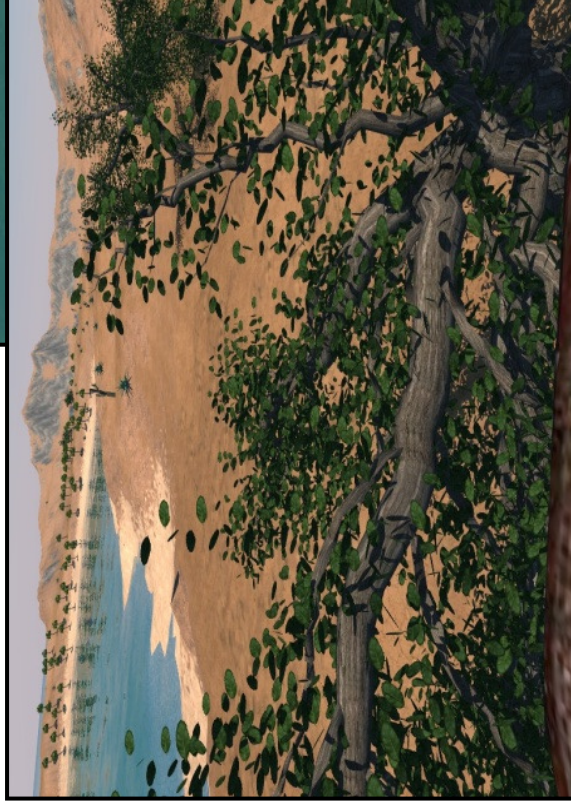
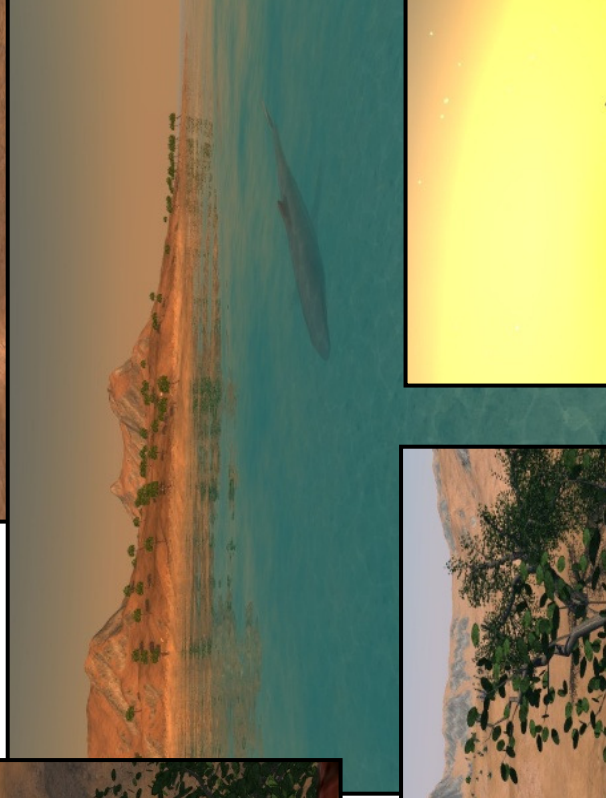
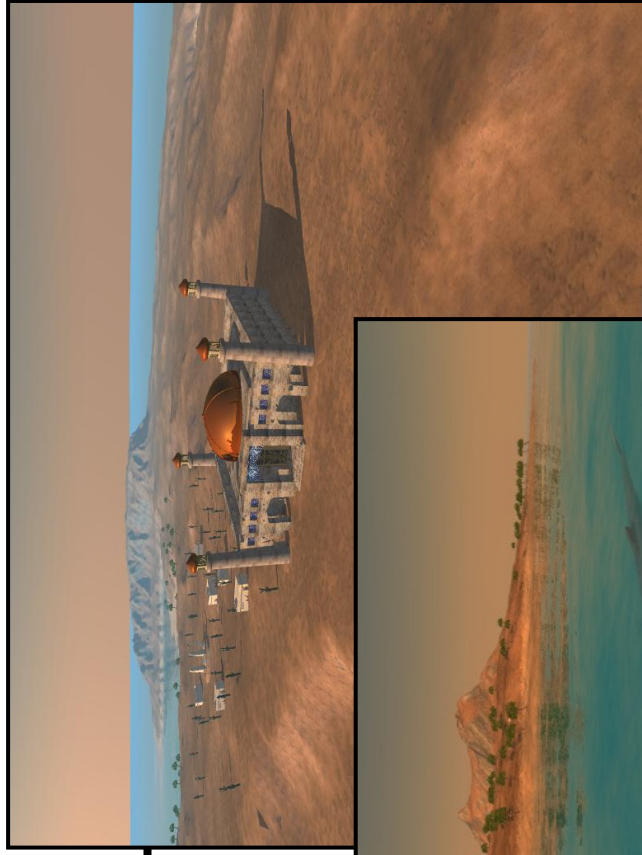


# Realistic Visualization: VR/AR

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# Games?





# Realtime Lighting Simulation

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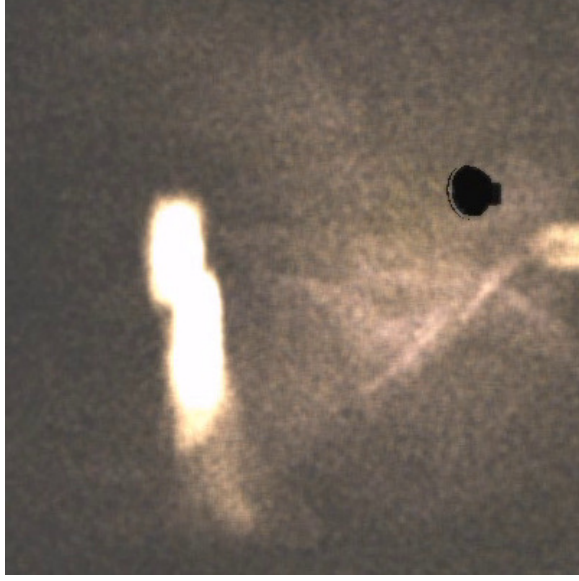
# Lighting Simulation

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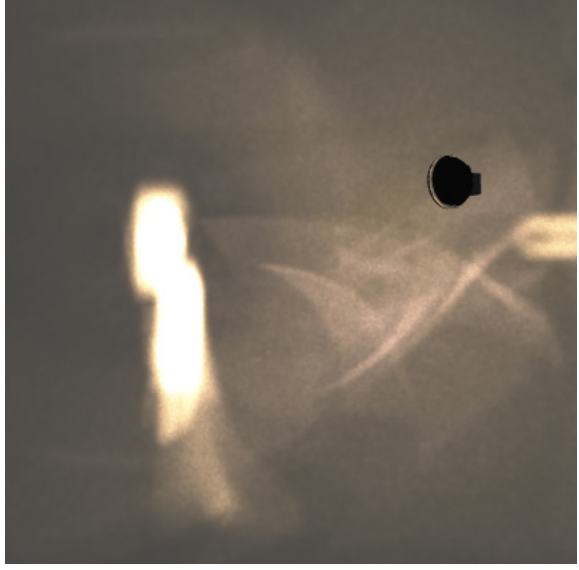
- Complex Scattering
- Highly accurate Results



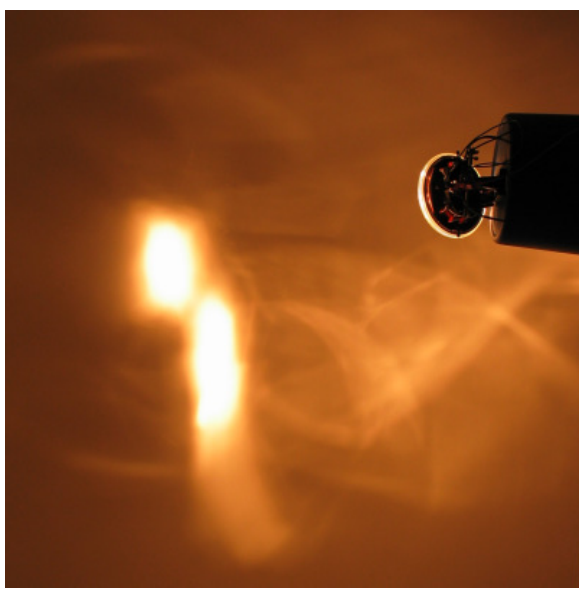
250k / 3 fps



25M / 11 fps



Photograph



# Fundamental Ray Tracing Steps

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- **Generation of primary rays**
  - Rays from viewpoint along viewing directions into 3D scene
  - (At least) one ray per picture element (pixel)
- **Ray tracing**
  - Traversal of spatial index structures
  - Intersection of ray with scene geometry
- **Shading**
  - From intersection, determine “light color” sent along primary ray
  - Determines “pixel color”
  - Needed
    - Local material color and reflection properties
      - Object texture
    - Local illumination of intersection point
      - Can be hard to determine correctly

# Ray and Object Representations

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- **Ray in space:**  $\underline{r}(t) = \underline{o} + t \underline{d}$ 
  - $\underline{o} = (o_x, o_y, o_z)$
  - $\underline{d} = (d_x, d_y, d_z)$
- **Scene geometry**
  - Sphere:  $(\underline{p}-\underline{c}) \cdot (\underline{p}-\underline{c}) - r^2 = 0$ 
    - $\underline{c}$  : sphere center
    - $r$  : sphere radius
    - $\underline{p}$  : any surface point
  - Plane:  $(\underline{p}-\underline{a}) \cdot \underline{n} = 0$ 
    - Implicit definition
    - $\underline{n}$  : surface normal
    - $\underline{a}$  : one given surface point
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  - Triangles: Plane intersection plus barycentric coordinates



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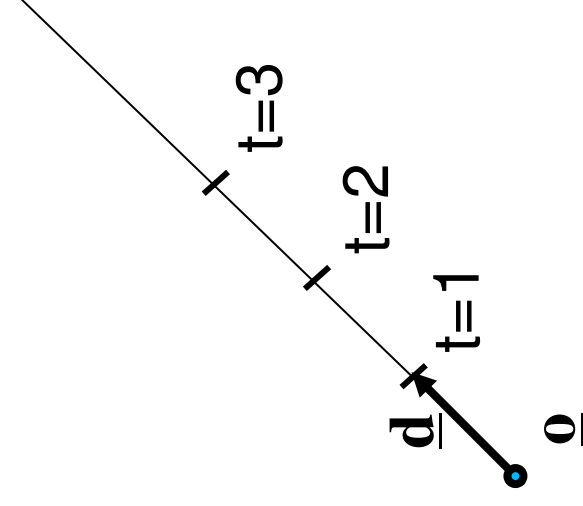
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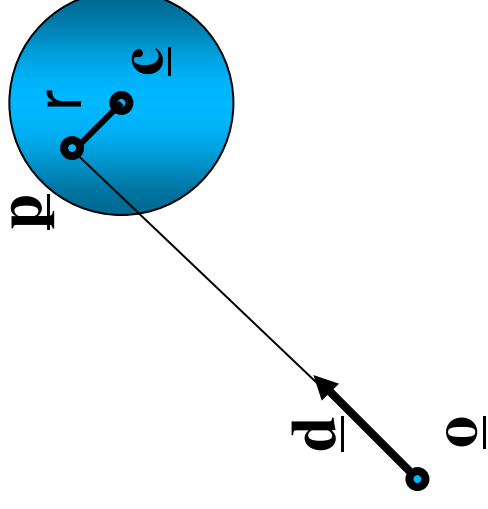


ray parameterization

# Ray and Object Representations

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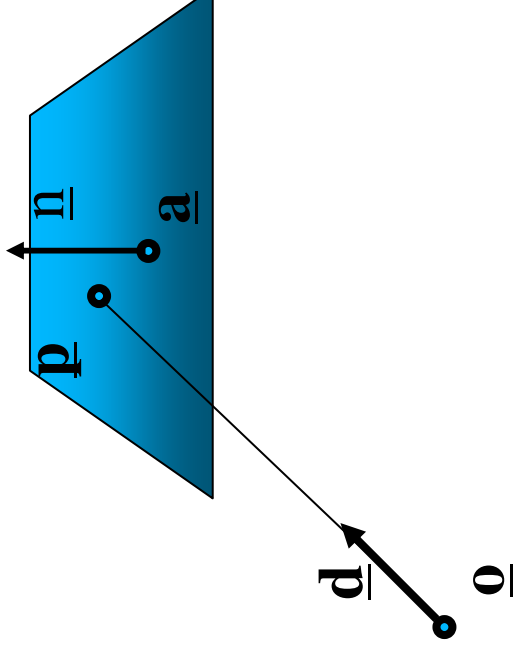
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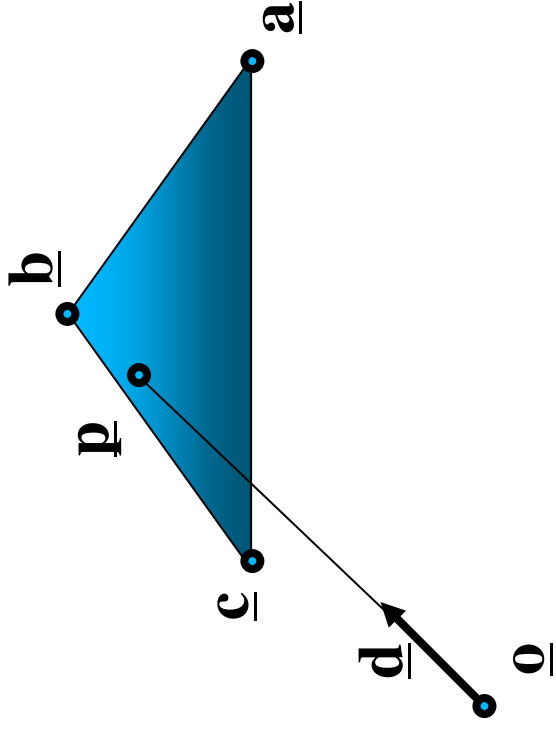
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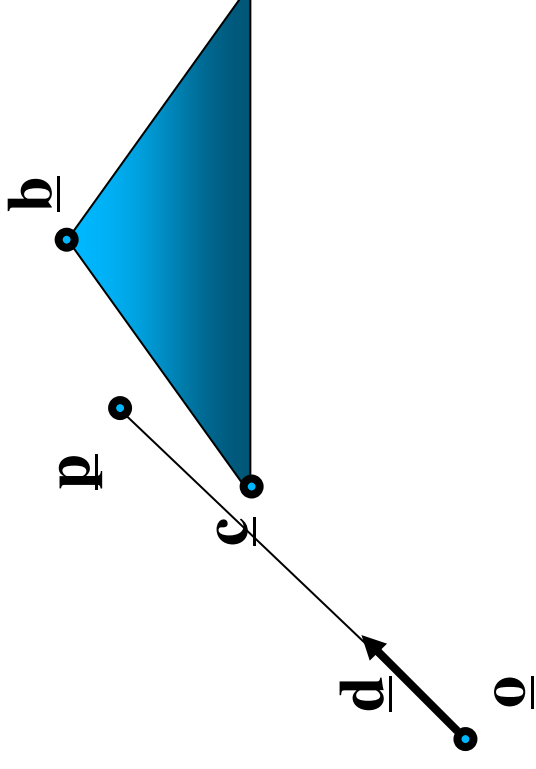




# Ray and Object Representations

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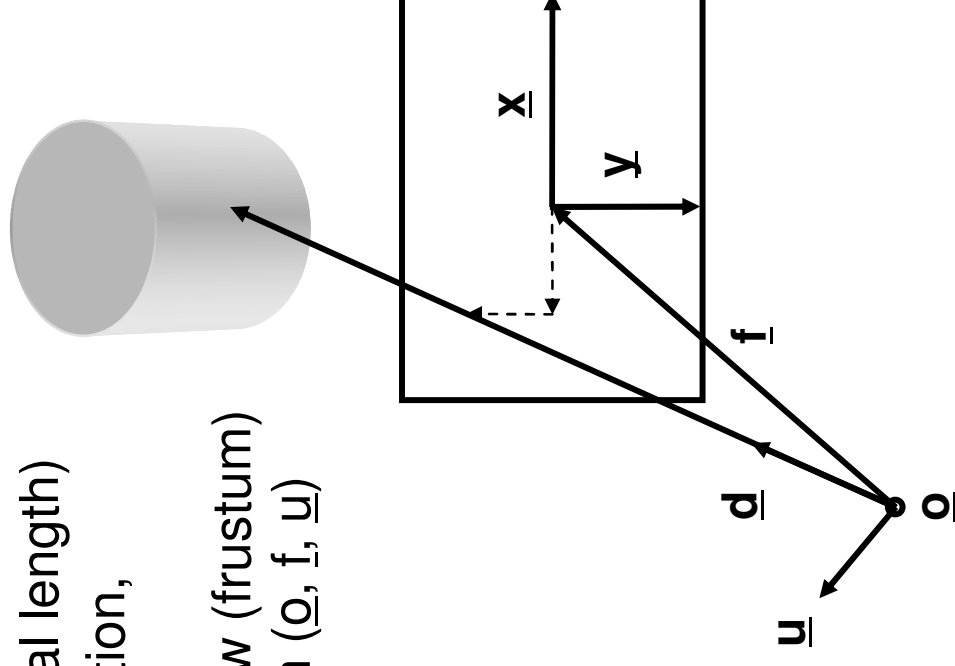
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# Perspective Camera Model

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- **Definition of the pinhole camera**
  - $\underline{o}$ : Origin (point of view)
  - $\underline{f}$ : Vector to center of view (focal length)
  - $\underline{u}$ : Up-vector of camera orientation, in one plane with  $\underline{y}$  vector
  - $\underline{x}, \underline{y}$ : Span half the viewing window (frustum) relative to coordinate system ( $\underline{o}, \underline{f}, \underline{u}$ )
  - $xres, yres$ : Image resolution

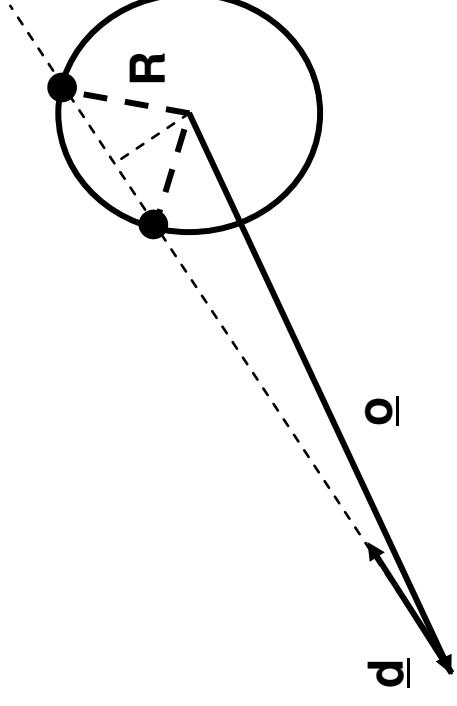


```
for (x= 0; x < xres; x++)
  for (y= 0; y < yres; y++)
  {
     $\underline{d} = \underline{f} + 2(x/xres - 0.5) \cdot \underline{x}$ 
      + 2(y/yres - 0.5)  $\cdot \underline{y}$ ;
     $\underline{d} = \underline{d} / |\underline{d}|$ ; // Normalize
    col= trace( $\underline{o}, \underline{d}$ );
    write_pixel(x, y, col);
  }
```

# Intersection Ray – Sphere

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- **Sphere**
  - Given a sphere at the origin  
 $x^2 + y^2 + z^2 - 1 = 0$
  - Given a ray  
 $\underline{r} = \underline{o} + t\underline{d}$  ( $r_x = o_x + td_x$  and so on)  
 $\underline{o}$ : origin,  $\underline{d}$ : direction
  - Substituting the ray into the equation for the sphere gives  
 $t^2(d_x^2 + d_y^2 + d_z^2) + 2t(d_x o_x + d_y o_y + d_z o_z) + (o_x^2 + o_y^2 + o_z^2) - 1 = 0$ 
    - Easily solvable with standard techniques
    - **But beware of numerical imprecision**
  - Alternative: Geometric construction
    - Ray and center span a plane
    - Simple 2D construction



# Intersection Ray – Plane

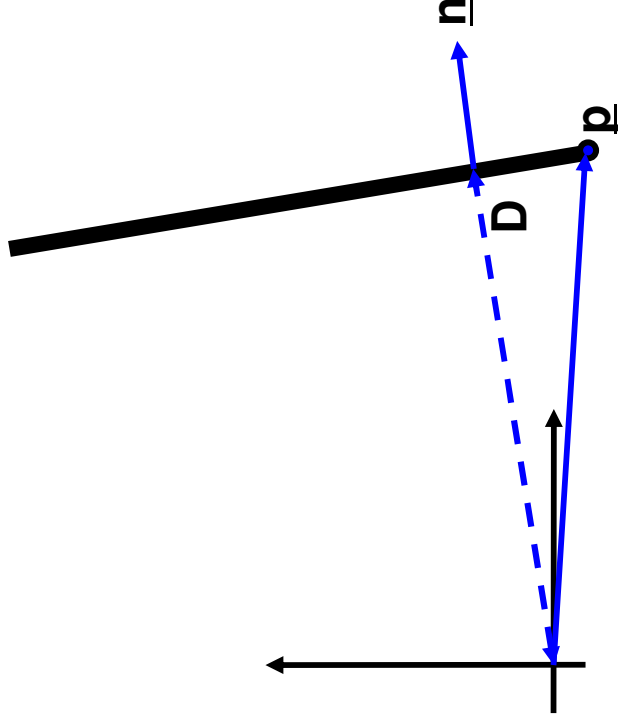
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- **Plane: Implicit representation (Hesse form)**
  - Plane equation:  $\underline{p} \cdot \underline{n} - D = 0$ ,  $|\underline{n}| = 1$ 
    - $\underline{n}$ : Normal vector:
    - $D$ : Normal distance of plane from  $(0, 0, 0)$ :

- **Two possible approaches**

- Geometric
- Mathematic
  - Substitute  $\underline{o} + t\underline{d}$  for  $\underline{p}$
  - $(\underline{o} + t\underline{d}) \cdot \underline{n} - D = 0$
  - Solving for  $t$  gives

$$t = \frac{D - \underline{o} \cdot \underline{n}}{\underline{d} \cdot \underline{n}}$$



# Intersection Ray – Plane

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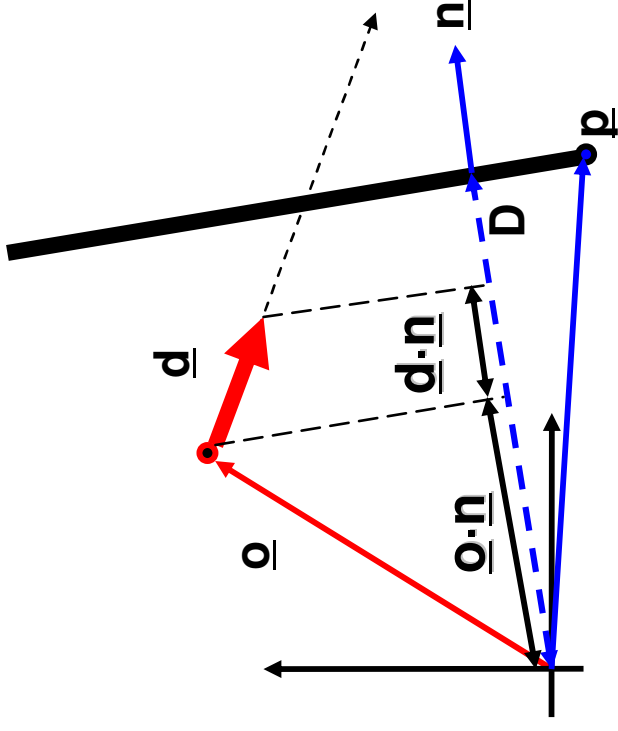
- **Plane: Implicit representation (Hesse form)**

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- **Two possible approaches**

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  - Substitute  $\underline{o} + t\underline{d}$  for  $\underline{p}$
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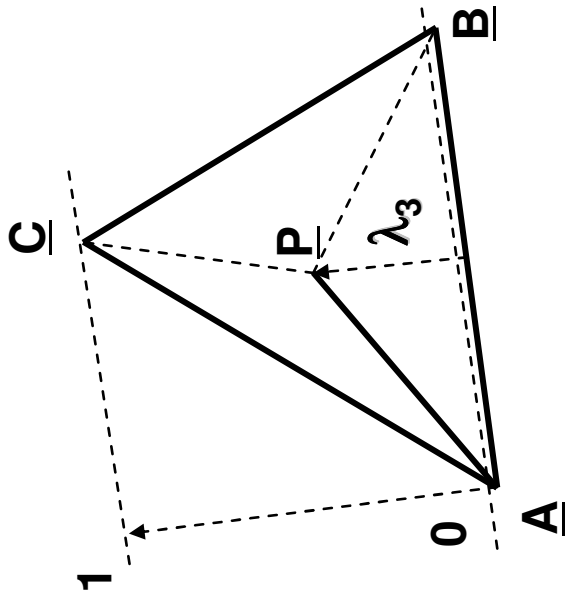
$$t = \frac{D - \underline{o} \cdot \underline{n}}{\underline{d} \cdot \underline{n}}$$



# Intersection Ray – Triangle

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- **Barycentric coordinates**
  - Non-degenerate triangle ABC
  - Every point P in the plane can be described using
$$\underline{P} = \lambda_1 \underline{A} + \lambda_2 \underline{B} + \lambda_3 \underline{C}$$
    - $\lambda_1 + \lambda_2 + \lambda_3 = 1$ 
      - barycentric coordinates = ratio of signed areas
$$\lambda_3 = \angle(APB) / \angle(ACB)$$
 etc
  - For fixed  $\lambda_3$ ,  $\underline{P}$  may move parallel to AB
  - For  $\lambda_1 + \lambda_2 = 1$ 
$$\underline{P} = (1 - \lambda_3) (\lambda_1 \underline{A} + \lambda_2 \underline{B}) + \lambda_3 \underline{C} \quad (0 < \lambda_3 < 1)$$
    - $\underline{P}$  moves between  $\underline{C}$  and AB

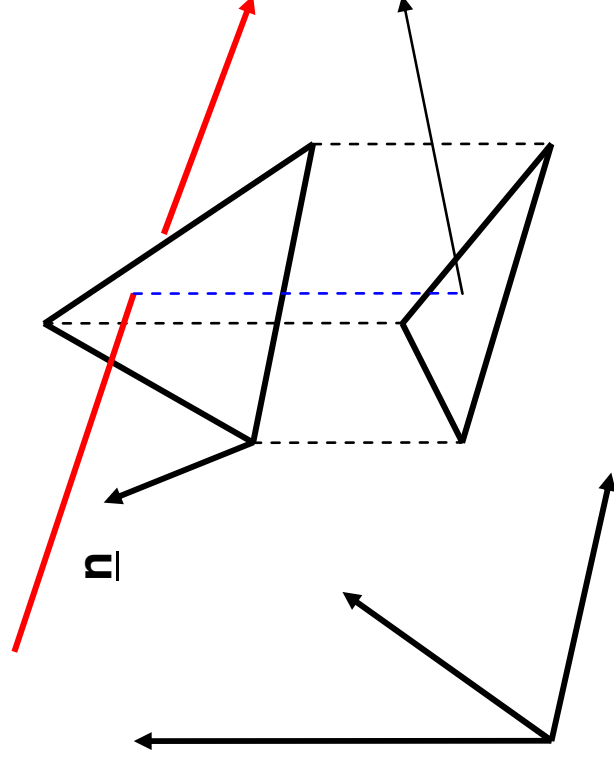


- **Point is in triangle, iff all  $\lambda_i$  greater or equal than zero**

# Intersection Ray – Triangle (2)

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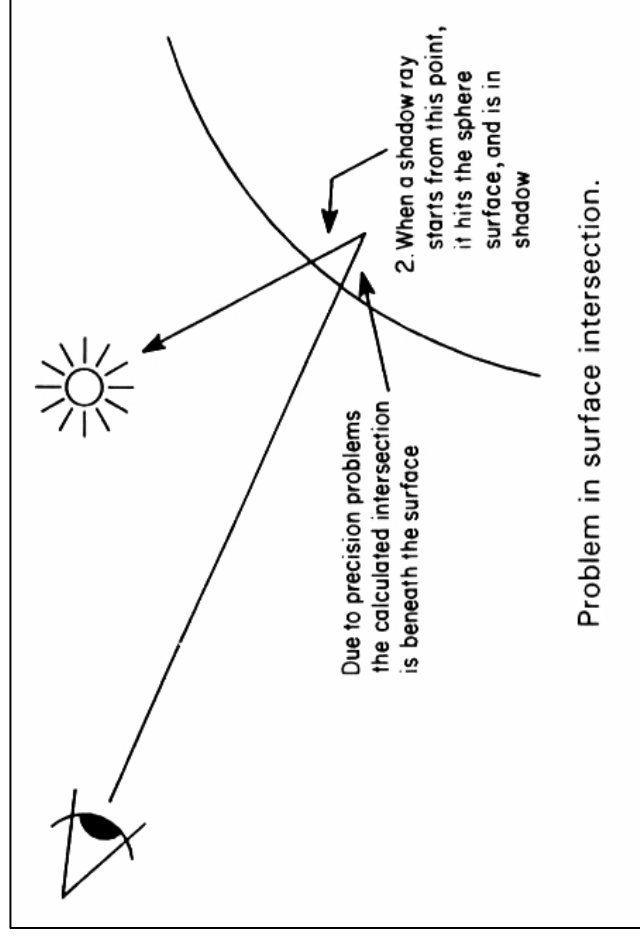
- **Compute intersection with triangle plane**
- **Given the 3D intersection point**
  - Project point into xy, xz, yz coordinate plane
  - Use coordinate plane that is most aligned
    - xy: if  $n_z$  is maximal, etc.
  - Coordinate plane and 2D vertices can be pre-computed
- **Compute barycentric coordinates**
- **Test for positive BCs**



# Precision Problems

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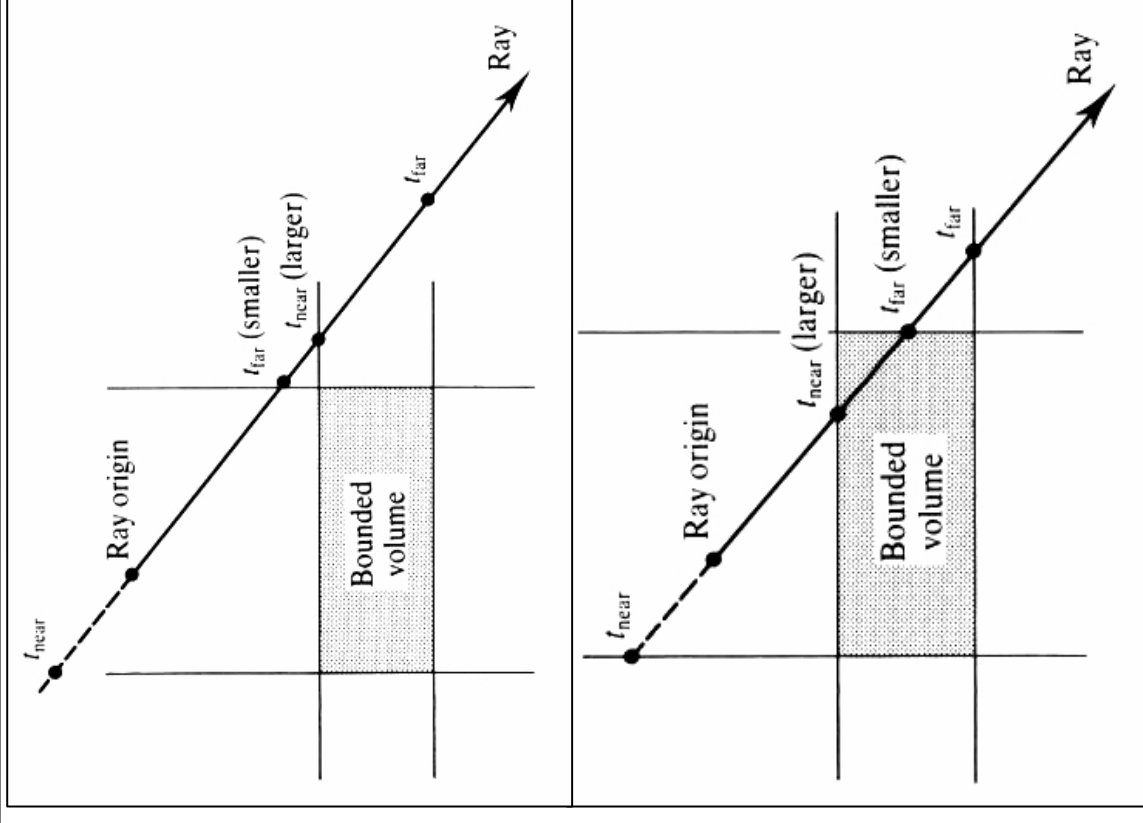
Inaccuracies of the intersection points computations due to floating-point arithmetic can result in incorrect shadow rays (self-shadowing) or infinite loops for secondary rays which have origins at a previously found intersection point. A simple solution is to check if the value of parameter  $t$  (used for intersection point calculations) is within some tolerance. For example, if  $\text{abs}(t) < 0.00001$ , then that  $t$  describes the origin of some ray as being on the object. The tolerance should be scaled to the size of the environment.





# Intersection Ray-Box

- Boxes are important for
  - bounding volumes
  - hierarchical structures
- Intersection test
  - test pairs of parallel planes in turn
  - calculate intersection distances
  - $t_{near}$  (first plane) and  $t_{far}$  (second plane)
  - If the value of  $t_{near}$  for one pair of planes is greater than  $t_{far}$  for another pair of planes, the ray cannot intersect the box.



# History of Intersection Algorithms

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- **Ray-geometry intersection algorithms**
  - Polygons: [Appel '68]
  - Quadrics, CSG: [Goldstein & Nagel '71]
  - Recursive Ray Tracing: [Whitted '79]
  - Tori: [Roth '82]
  - Bicubic patches: [Whitted '80, Kajiya '82]
  - Algebraic surfaces: [Hanrahan '82]
  - Swept surfaces: [Kajiya '83, van Wijk '84]
  - Fractals: [Kajiya '83]
  - Deformations: [Barr '86]
  - NURBS: [Stürzlinger '98]
  - Subdivision surfaces: [Kobbelt et al '98]

# Shading

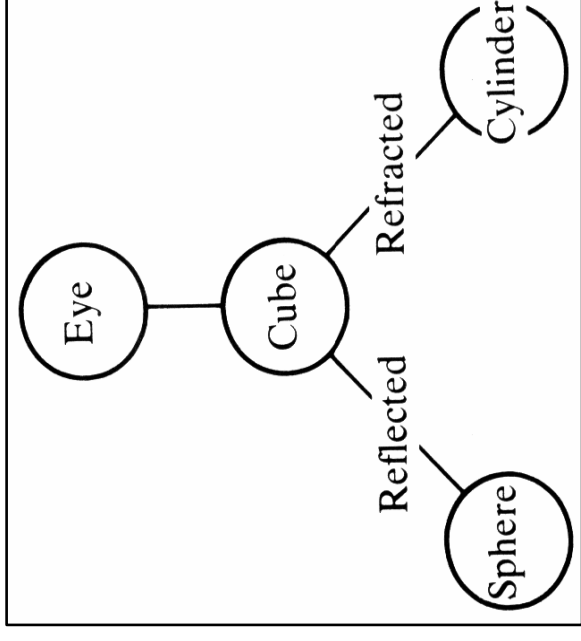
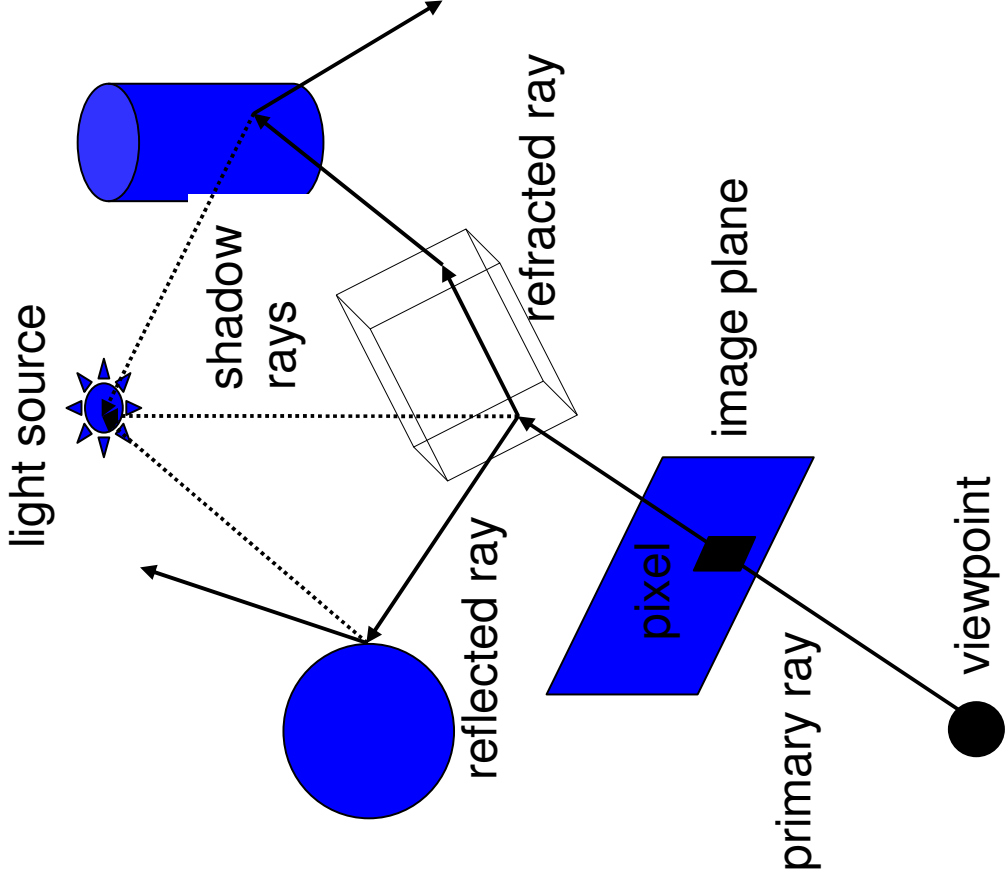
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- **Intersection point determines primary ray's "color"**
- **Diffuse object: color at intersection point**
  - No variation with viewing angle: diffuse (Lambertian)
  - Must still be illuminated
    - Point light source: shadow ray
    - Scales linearly with received light (Irradiance)
    - No illumination: in shadow = black
- **Non-Lambertian Reflectance**
  - Appearance depends on illumination *and* viewing direction
    - Local Bi-directional Reflectance Distribution Function (BRDF)
  - Simple cases
    - Mirror, glass: secondary rays
- **Area light sources, indirect illumination can be difficult**

# Recursive Ray Tracing

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- **Searching recursively for paths to light sources**
  - Interaction of light & material at intersection points
  - Recursively trace new rays in reflection, refraction, and light direction



# Ray Tracing Algorithm

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- **Trace(ray)**
  - Search the next intersection point → (hit, material)
  - Return Shade(ray, hit, material)
- **Shade(ray, hit, material)**
  - For each light source
    - if ShadowTrace(ray to light source, distance to light)
      - Calculate reflected radiance (i.e. Phong)
      - Adding to the reflected radiance
  - If mirroring material
    - Calculate radiance in reflected direction: Trace(R(ray, hit))
    - Adding mirroring part to the reflected radiance
  - Same for transmission
  - Return reflected radiance
- **ShadowTrace(ray, dist)**
  - Return false, if intersection point with distance < dist has been found

# Ray Tracing

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- **Incorporates into a single framework**
  - Hidden surface removal
    - Front to back traversal
    - Early termination once first hit point is found
  - Shadow computation
    - Shadow rays/ shadow feelers are traced between a point on a surface and a light sources
  - Exact simulation of some light paths
    - Reflection (reflected rays at a mirror surface)
    - Refraction (refracted rays at a transparent surface, Snell's law)
- **Limitations**
  - Easily gets inefficient for full global illumination computations
    - Many reflections (exponential increase in number of rays)
    - Indirect illumination requires many rays to sample all incoming directions

# Ray Tracing: Approximations

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- **Usually RGB color model instead of full spectrum**
- **Finite number of point lights instead of full indirect light**
- **Approximate material reflectance properties**
  - Ambient: constant, non-directional background light
  - Diffuse: light reflected uniformly in all directions,
  - Specular: perfect reflection, refraction
- **All are based on purely empirical foundation**

# Wrap-Up

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- **Background**
  - Forward light transport vs. backward search in RT
- **Ray tracer**
  - Ray generation, ray-object intersection, shading
- **Ray-geometry intersection calculation**
  - Sphere, plane, triangle, box
- **Recursive ray tracing algorithm**
  - Primary, secondary, shadow rays
- **Next lecture**
  - Acceleration techniques