Computer Graphics

- Light Transport -

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Overview

- **So far**
  - simple shading
- **Today**
  - Physics behind ray tracing
  - Physical light quantities
  - Perception of light
  - Light sources
  - Light transport simulation
- **Next lecture**
  - Light-matter interaction
  - Reflectance function
  - Reflection models

What is Light ?

- **Ray**
  - Linear propagation

  $\Rightarrow$ **Geometrical optics**
- **Vector**
  - Polarization

  $\Rightarrow$ **Jones Calculus**: matrix representation
- **Wave**
  - Diffraction, Interference

  $\Rightarrow$ **Maxwell equations**: propagation of light
- **Particle**
  - Light comes in discrete energy quanta: photons

  $\Rightarrow$ **Quantum theory**: interaction of light with matter
- **Field**
  - Electromagnetic force: exchange of virtual photons

  $\Rightarrow$ **Quantum Electrodynamics (QED)**: interaction between particles

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Light in Computer Graphics

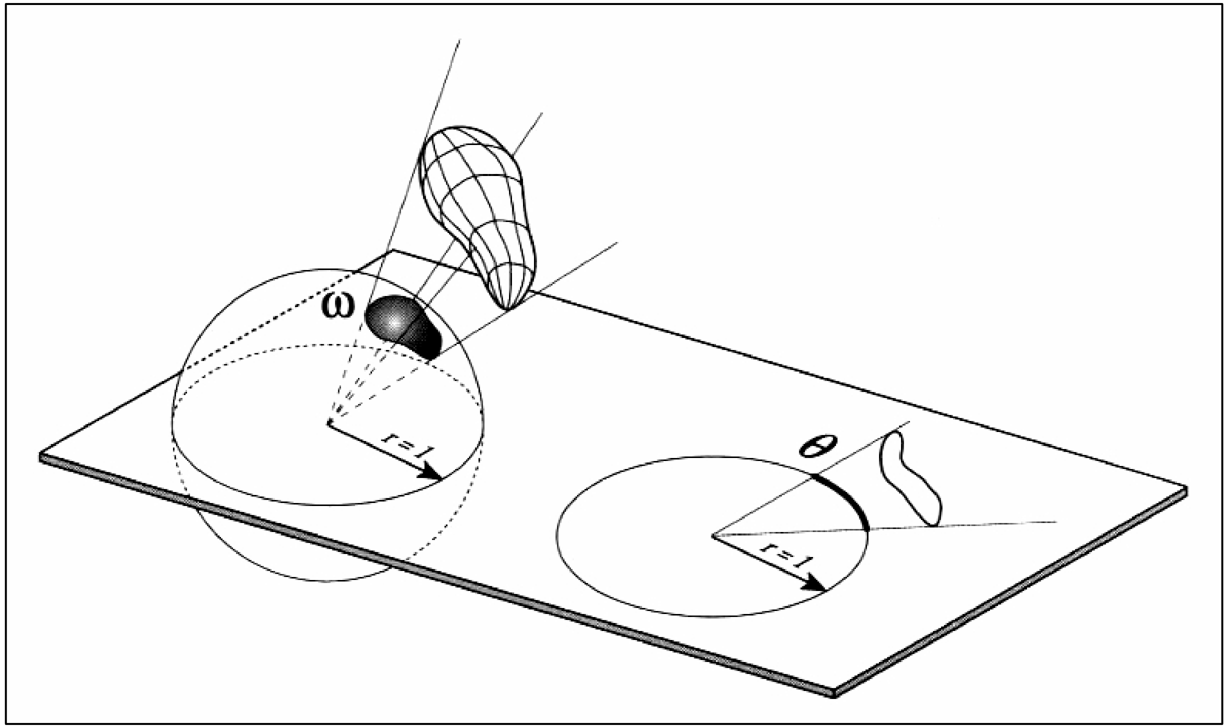
- **Based on human visual perception**
  - Macroscopic geometry
  - Tristimulus color model
  - Psycho-physics: tone mapping, compression, …
- **Ray optics**
  - Light: scalar, real-valued quantity
  - Linear propagation
  - Macroscopic objects
  - Incoherent light
  - Superposition principle: light contributions add up linearly
  - No attenuation in free space
- **Limitations**
  - Microscopic structures ($\approx\lambda$)
  - Diffraction, Interference
  - Polarization
  - Dispersion

Angle and Solid Angle

θ the angle subtended by a curve in the plane, is the length of the corresponding arc on the unit circle.

$\Omega, d\omega$ the solid angle subtended by an object, is the surface area of its projection onto the unit sphere,

Units for measuring solid angle: steradians [sr]



Solid Angle in Spherical Coordinates

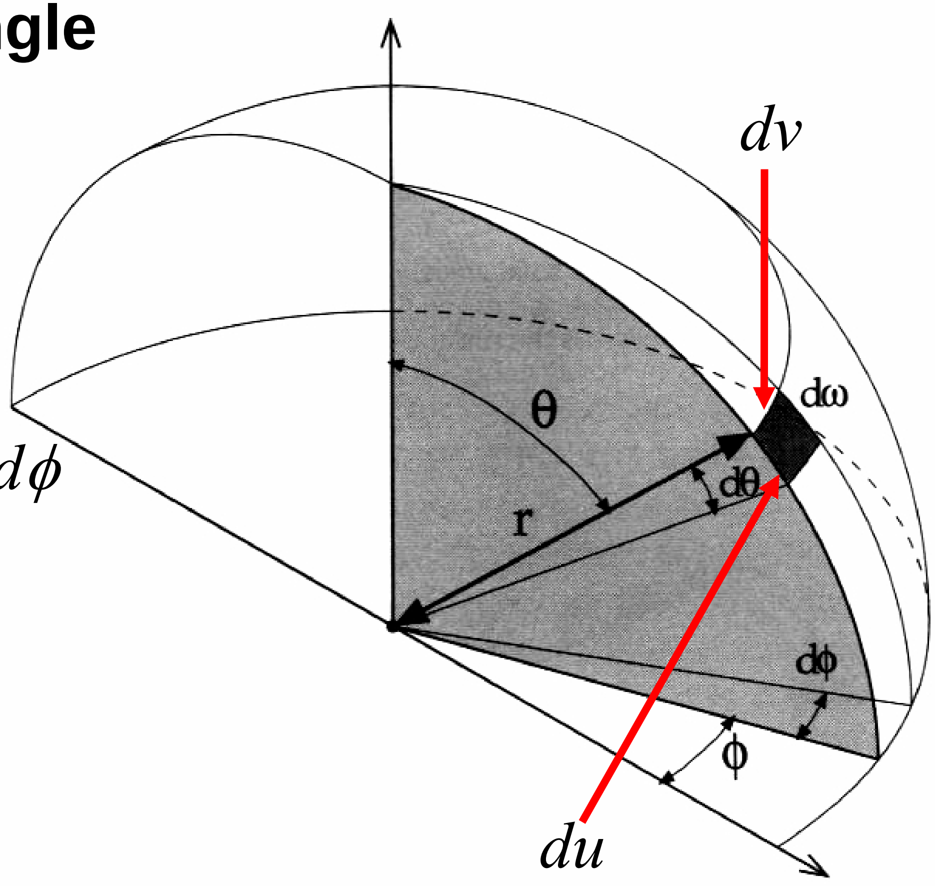
Infinitesimally small solid angle

$$du = r\,d\theta$$

$$dv = r\sin\theta\,d\phi$$

$$dA = du\,dv = r^2 \sin\theta\,d\theta\,d\phi$$

$$\Rightarrow \quad d\omega, d\Omega = \frac{dA}{r^2} = \sin\theta\,d\theta\,d\phi$$



Finite solid angle

$$\Omega = \int_{\phi_0}^{\phi_1} d\phi \int_{\theta_0(\phi)}^{\theta_1(\phi)} \sin\theta\,d\theta$$

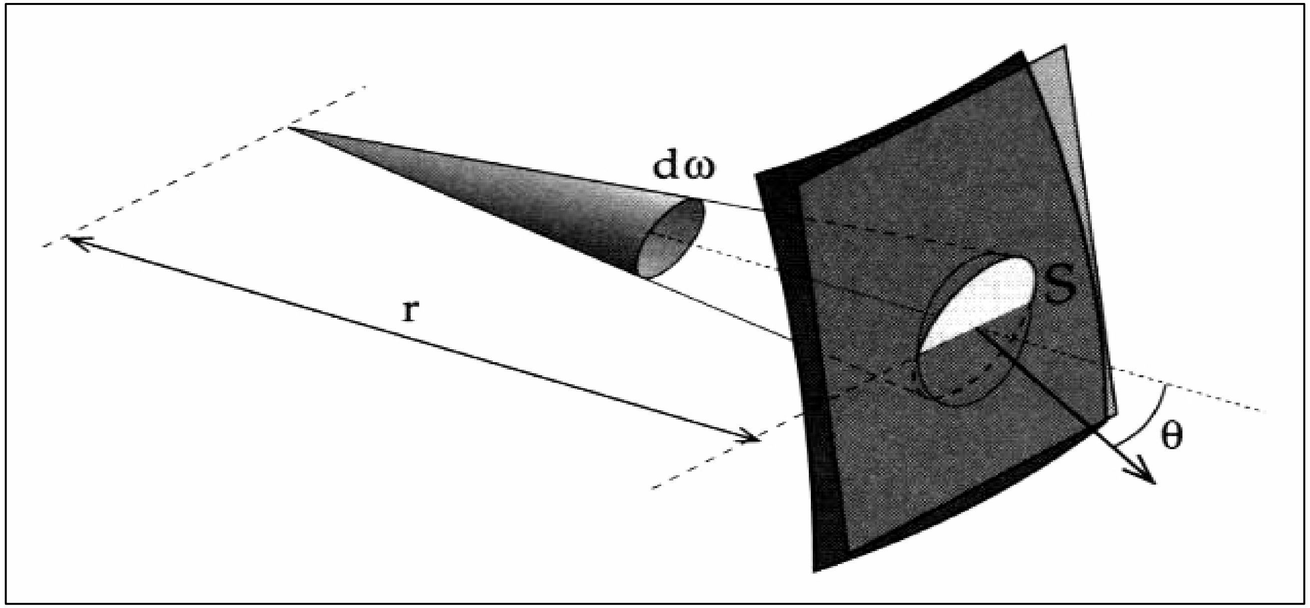
Projected Solid Geometry

The solid angle subtended by a small surface patch *S* with area Δ*A* is obtained (i) by projecting it orthogonal to the vector r to the origin

$$\Delta A \cos\theta$$

and (ii) dividing by the square of the distance to the origin:

$$\Delta\Omega \approx \frac{\Delta A \cos\theta}{r^2}$$



Radiometry

- **Definition:**
  - Radiometry is the science of measuring radiant energy transfers. Radiometric quantities have physical meaning and can be directly measured using proper equipment such as spectral photometers.

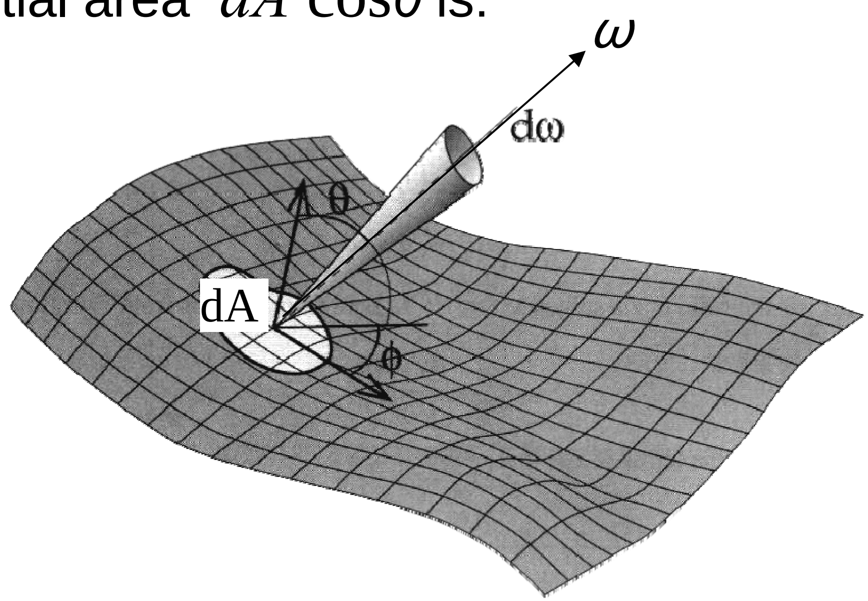
- **Radiometric Quantities**

– energy	[watt second]	$n \cdot h\nu$ (Photon Energy)
– radiant power (total flux)	[watt]	Φ
– radiance	[watt/(m^2 sr)]	L
– irradiance	[watt/m^2]	E
– radiosity	[watt/m^2]	B
– intensity	[watt/sr]	I

Radiometric Quantities: Radiance

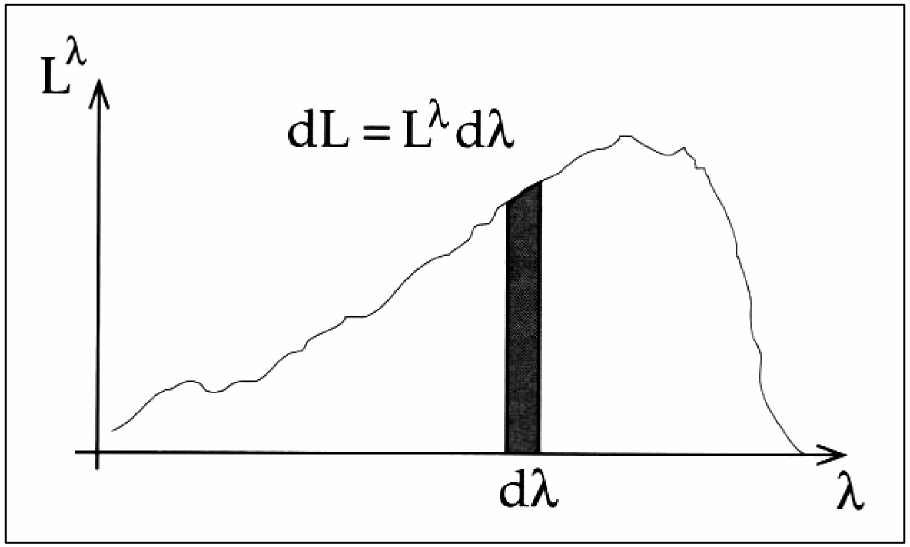
- Radiance is used to describe radiant energy transfer.
- Radiance L is defined as
  - **the power (flux) traveling at some point $\underline{x}$**
  - **in a specified direction $\underline{\omega} = (\theta,\varphi)$,**
  - **per unit area perpendicular to the direction of travel,**
  - **per unit solid angle.**
- Thus, the differential power $d^2\Phi$ radiated through the differential solid angle $d\omega$, from the projected differential area $dA\cos\theta$ is:

$$d^2\Phi = L(\underline{x},\underline{\omega})\, dA \cos\theta\, d\omega$$



Spectral Properties

- **Wavelength**
  - Since light is composed of electromagnetic waves of different frequencies and wavelengths, most of the energy transfer quantities are continuous functions of wavelength.
  - In graphics each measurement $L(\underline{x},\underline{\omega})$ is for a discrete band of wavelength only (often some abstract R, B, G)



Radiometric Quantities: Irradiance

Irradiance E is defined as the total power per unit area (flux density) incident onto a surface. To obtain the total flux incident to dA, the incoming radiance L_i is integrated over the upper hemisphere Ω_+ above the surface:

$$E \equiv \frac{d\Phi}{dA}$$

$$d\Phi = \left[\int_{\Omega_+} L_i(\underline{x}, \underline{\omega}) \cos\theta \, d\omega \right] dA$$

$$E = \int_{\Omega_+} L_i(\underline{x}, \underline{\omega}) \cos\theta \, d\omega = \int_0^{\pi/2} \int_0^{2\pi} L_i(\underline{x}, \underline{\omega}) \cos\theta \sin\theta \, d\theta \, d\phi$$

Radiometric Quantities: Radiosity

Radiosity B is defined as the total power per unit area (flux density) leaving a surface. To obtain the total flux radiated from dA, the outgoing radiance L_o is integrated over the upper hemisphere Ω_+ above the surface.

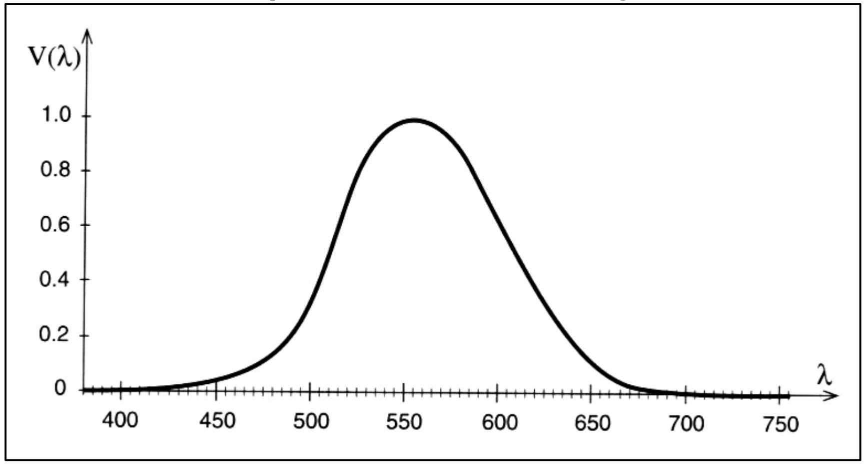
$$B \equiv \frac{d\Phi}{dA}$$

$$d\Phi = \left[\int_\Omega L_o(\underline{x}, \underline{\omega}) \cos\theta \, d\omega \right] dA$$

$$B = \int_\Omega L_o(\underline{x}, \underline{\omega}) \cos\theta \, d\omega = \int_0^{\pi/2} \int_0^{2\pi} L_o(\underline{x}, \underline{\omega}) \cos\theta \sin\theta \, d\theta \, d\phi$$

Photometry

- **Photometry:**
  - The human eye is sensitive to a limited range of radiation wavelengths (roughly from 380nm to 770nm).
  - The response of our visual system is not the same for all wavelengths, and can be characterized by the luminuous efficiency function V(λ), which represents the average human spectral response.
  - A set of photometric quantities can be derived from radiometric quantities by integrating them against the luminuous efficiency function V(λ).
  - Separate curves exist for light and dark adaptation of the eye.

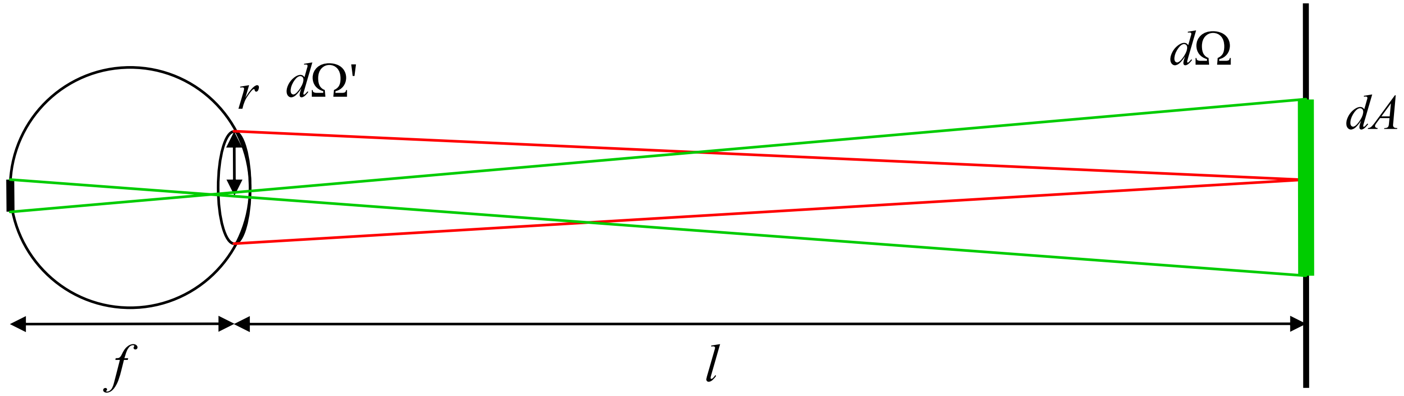


Radiometry vs. Photometry

Physics-based quantities **Perception-based quantities**

Radiometry		→	Photometry	
W	Radiant power	→	Luminous power	Lumens (lm)
W/m^2	Radiosity Irradiance	→	Luminosity Illuminance	Lux (lm/m^2)
$W/m^2/sr$	Radiance	→	Luminance	cd/m^2 ($lm/m^2/sr$)

Perception of Light



photons / second = **flux** = energy / time = power Φ

rod sensitive to flux

angular extend of rod = **resolution** (≈ 1 arc minute^2) $d\Omega$

projected rod size = **area** $dA \approx l^2 \cdot d\Omega$

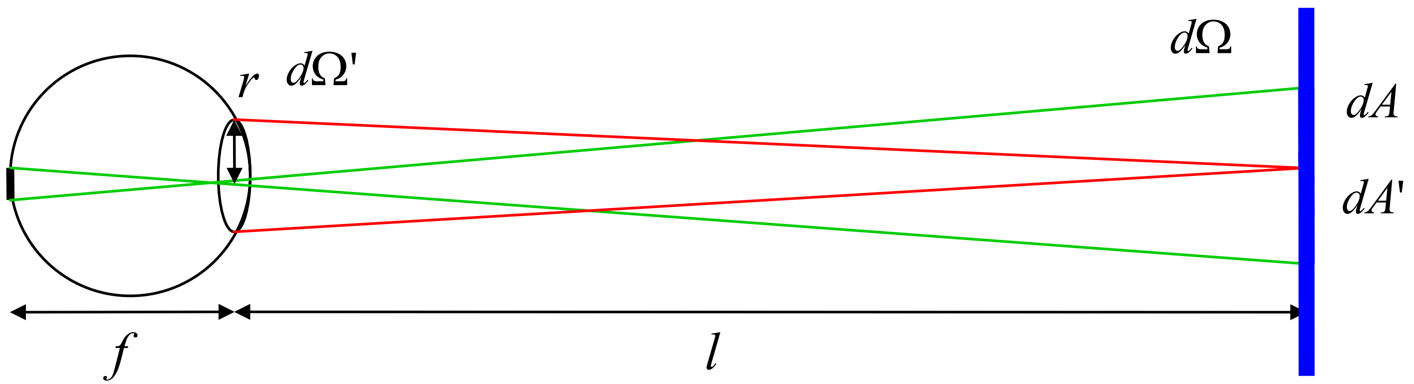
Angular extend of pupil aperture (r ≤ 4 mm) = **solid angle** $d\Omega' \approx \pi \cdot r^2 / l^2$

flux proportional to area and solid angle $\Phi \propto d\Omega' \cdot dA$

radiance = flux per unit area per unit solid angle $L = \frac{\Phi}{d\Omega' \cdot dA}$

The eye detects radiance

Brightness Perception



As l increases: $\Phi_0 \propto dA \cdot d\Omega' = l^2 d\Omega \cdot \pi \frac{r^2}{l^2} = \text{const}$

- $dA' > dA$: photon flux per rod stays constant
- $dA' < dA$: photon flux per rod decreases

Where does the Sun turn into a star ?

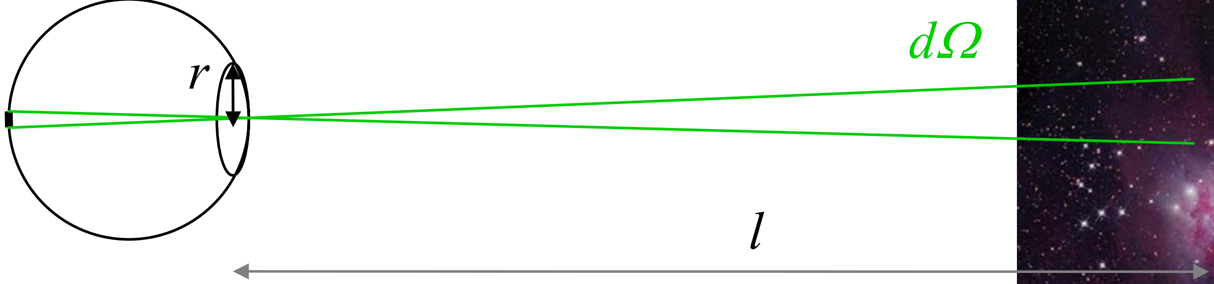
– Depends on apparent Sun disc size on retina

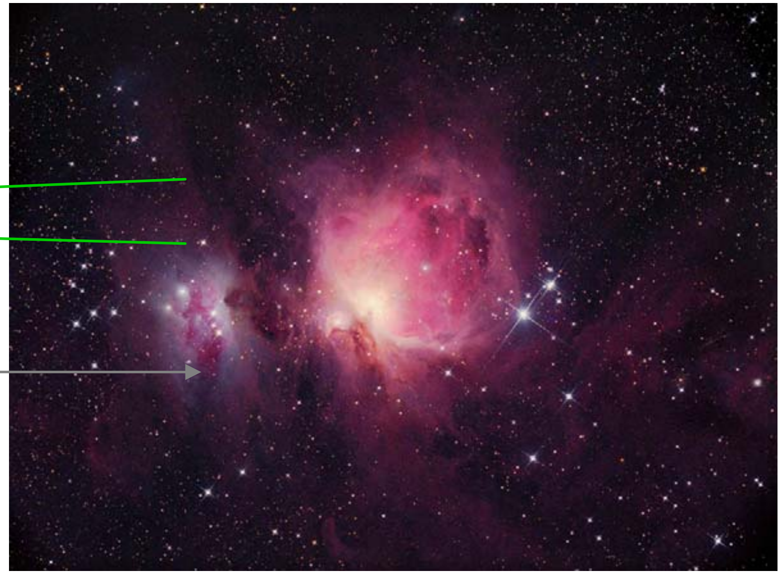
$\Rightarrow$ Photon flux per rod stays the same on Mercury, Earth or Neptune

$\Rightarrow$ Photon flux per rod decreases when dΩ' < 1 arc minute (beyond Neptune)

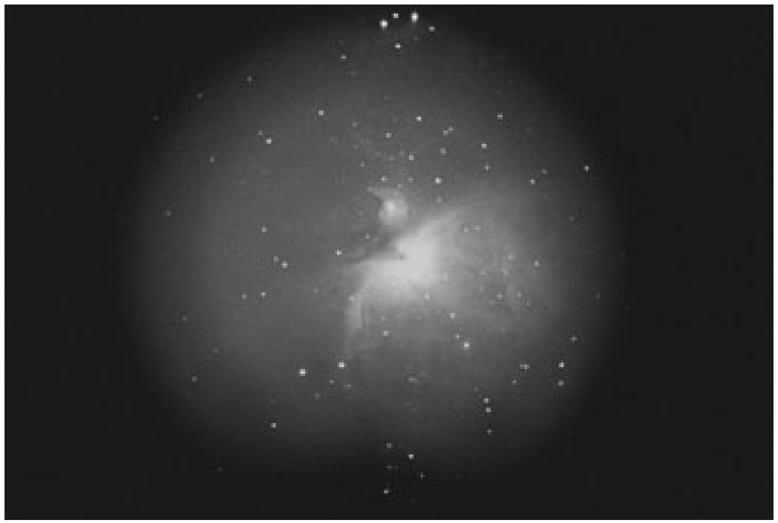
Brightness Perception II

Extended light source





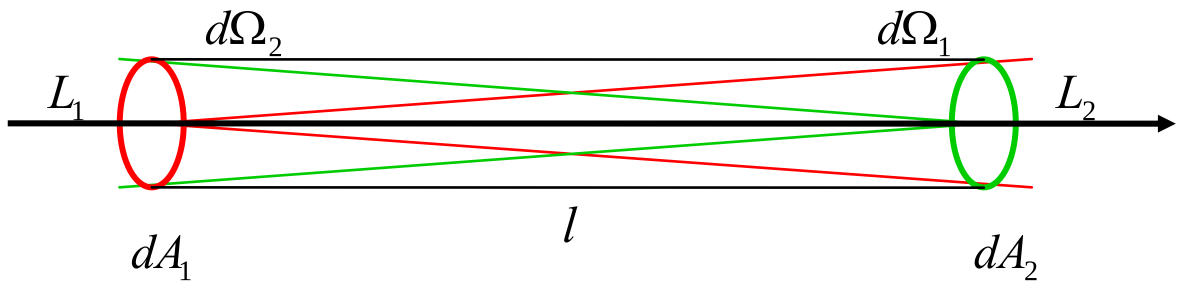
$$\Phi_0 = L_0 \cdot \pi \cdot r^2 \cdot d\Omega$$



⇒ Flux does not depend on distance *l*

⇒ Nebulae always appear b/w

Radiance in Space



Flux leaving surface 1 must be equal to flux arriving on surface 2

$$L_1 \cdot d\Omega_1 \cdot dA_1 = L_2 \cdot d\Omega_2 \cdot dA_2$$

From geometry follows $\quad d\Omega_1 = \frac{dA_2}{l^2} \qquad d\Omega_2 = \frac{dA_1}{l^2}$

Ray throughput $\quad T = d\Omega_1 \cdot dA_1 = d\Omega_2 \cdot dA_2 = \frac{dA_1 \cdot dA_2}{l^2}$

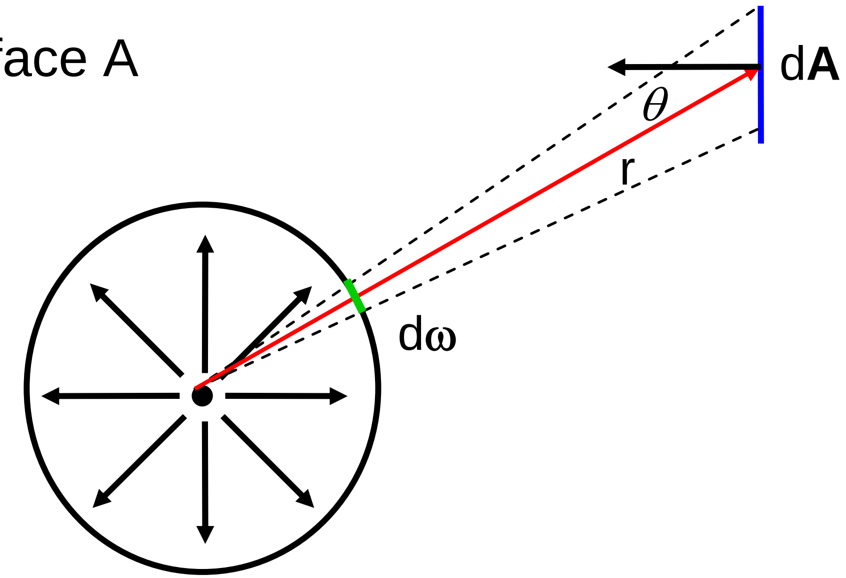
$$L_1 = L_2$$

The **radiance** in the direction of a light ray **remains constant** as it propagates along the ray

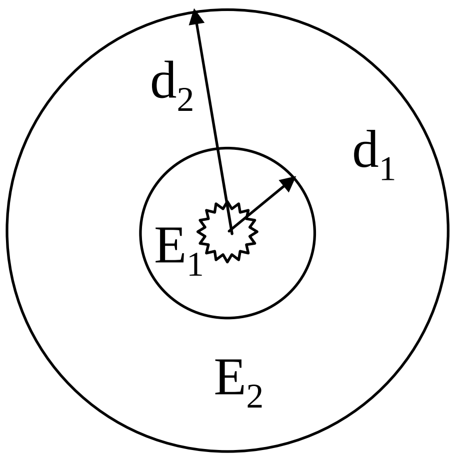
Point Light Source

- **Point light with isotropic radiance**
  - Power (total flux) of a point light source
    - ∀ Φ_g= Power of the light source [watt]
  - Intensity of a light source
    - $I = \Phi_g / (4\pi\ \text{sr})$ [watt/sr]
  - Irradiance on a sphere with radius r around light source:
    - $E_r = \Phi_g / (4\pi r^2)$ [watt/m²]
  - Irradiance on some other surface A

$$E(x) = \frac{d\Phi_g}{dA} = I\frac{d\omega}{dA} = \frac{\Phi_g}{4\pi}\cdot\frac{dA\cos\theta}{r^2 dA} = \frac{\Phi_g}{4\pi}\cdot\frac{\cos\theta}{r^2}$$



Inverse Square Law



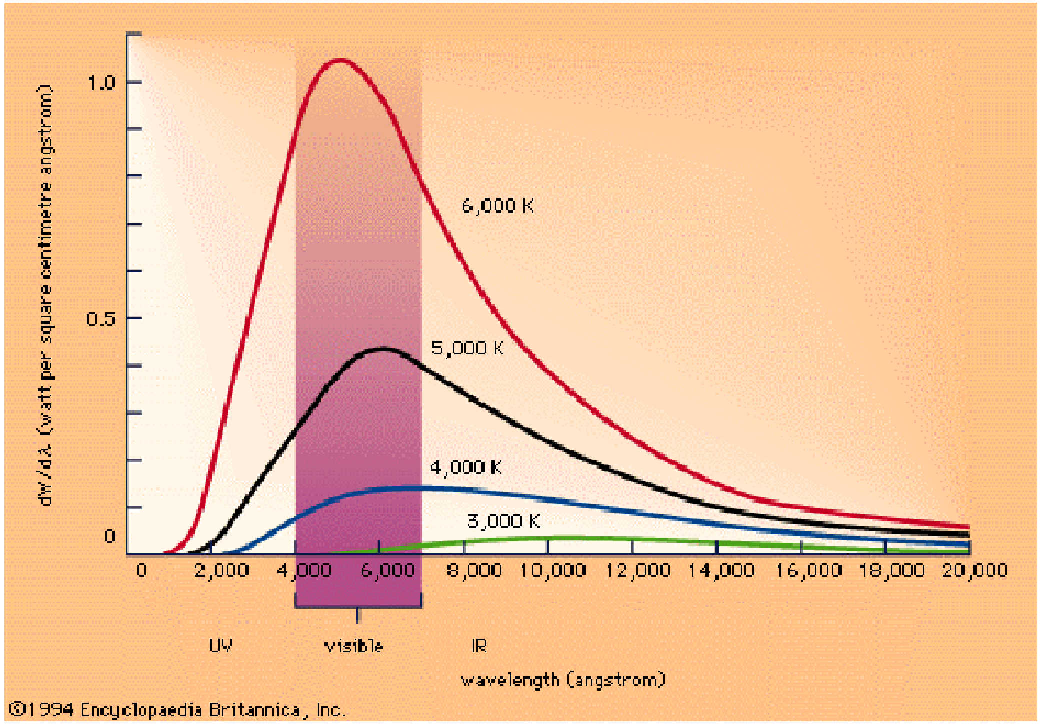
Irradiance E:

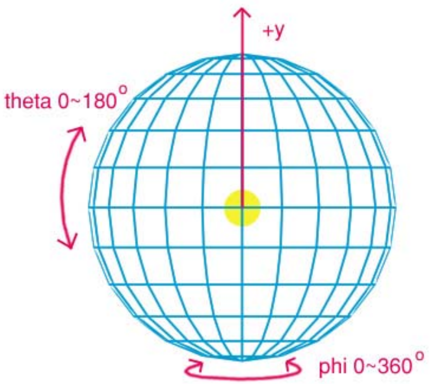
$$\frac{E_1}{E_2} = \frac{d_2^2}{d_1^2}$$

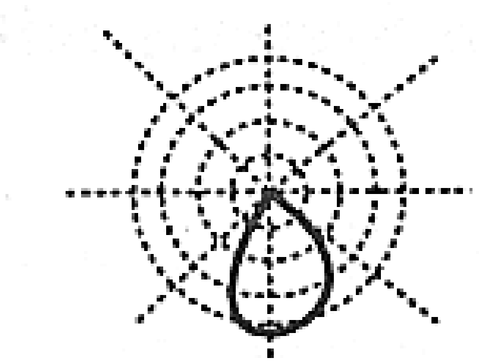
- **Irradiance *E*: power per m²**
  - Illuminating quantity
- **Distance-dependent**
  - Double distance from emitter: sphere area four times bigger
- **Irradiance falls off with inverse of squared distance**
  - For point light sources

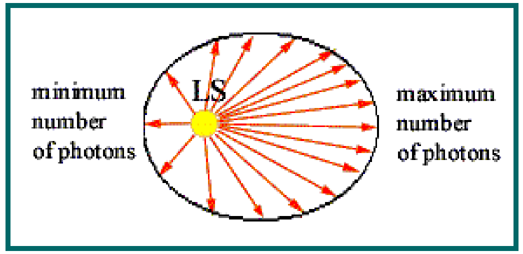
Light Source Specifications

- **Power (total flux)**
  - Emitted energy / time
- **Active emission size**
  - Point, area, volume
- **Spectral distribution**
  - Thermal, line spectrum
- **Directional distribution**
  - Goniometric diagram









Sky Light

- **Sun**
  - Point source (approx.)
  - White light (by def.)
- **Sky**
  - Area source
  - Scattering: blue
- **Horizon**
  - Brighter
  - Haze: whitish
- **Overcast sky**
  - Multiple scattering in clouds
  - Uniform grey

Courtesy Lynch & Livingston

Light Source Classification

Radiation characteristics

- **Directional light**
  - Spot-lights
  - Beamers
  - Distant sources
- **Diffuse emitters**
  - Torchieres
  - Frosted glass lamps
- **Ambient light**
  - “Photons everywhere”

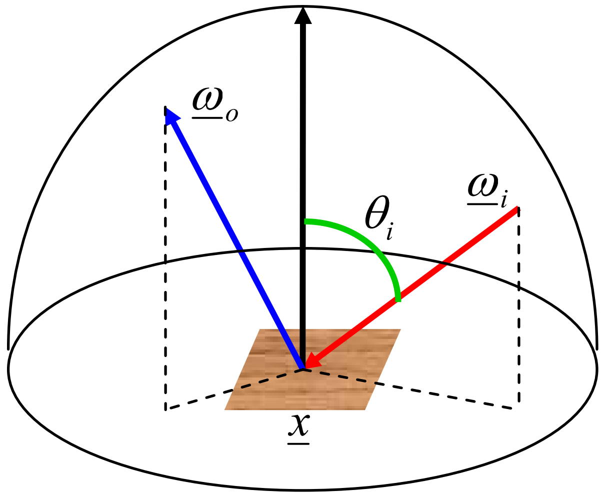
Emitting area

- **Volume**
  - neon advertisements
  - sodium vapor lamps
- **Area**
  - CRT, LCD display
  - (Overcast) sky
- **Line**
  - Clear light bulb, filament
- **Point**
  - Xenon lamp
  - Arc lamp
  - Laser diode

Surface Radiance

$$L(\underline{x},\underline{\omega}_o) = L_e(\underline{x},\underline{\omega}_o) + \int_{\Omega} f_r(\underline{\omega}_i,\underline{x},\underline{\omega}_o)\, L_i(\underline{x},\underline{\omega}_i) \cos\theta_i \, d\underline{\omega}_i$$

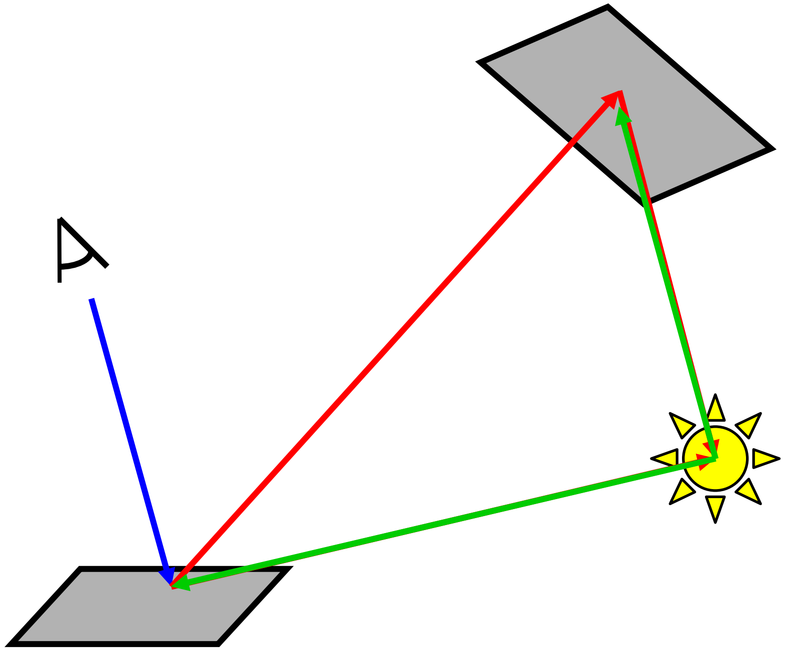
- **Visible surface radiance** $L(\underline{x},\underline{\omega}_o)$
  - Surface position $\underline{x}$
  - Outgoing direction $\underline{\omega}_o$
  - Incoming illumination direction $\underline{\omega}_i$
- **Self-emission** $L_e(\underline{x},\underline{\omega}_o)$
- **Reflected light**
  - Incoming radiance from all directions $L_i(\underline{x},\underline{\omega}_i)$
  - Direction-dependent reflectance (BRDF: bidirectional reflectance distribution function) $f_r(\underline{\omega}_i,\underline{x},\underline{\omega}_o)$



Ray Tracing

$$L(\underline{x},\underline{\omega}_o) = L_e(\underline{x},\underline{\omega}_o) + \int_\Omega f_r(\underline{\omega}_i,\underline{x},\underline{\omega}_o)\, L_i(\underline{x},\underline{\omega}_i)\cos\theta_i \, d\underline{\omega}_i$$

- **Simple ray tracing**
  - Illumination from light sources only - **local illumination (integral ➔ sum)**
  - Evaluates angle-dependent reflectance function - **shading**
- **Advanced ray tracing techniques**
  - Recursive ray tracing
    - Multiple reflections/refractions (for specular surfaces)
  - Forward ray tracing
    - Stochastic sampling (Monte Carlo methods)
    - Photon mapping
  - Combination of both



Light Transport in a Scene

- **Scene**
  - Lights (emitters)
  - Object surfaces (partially absorbing)
- **Illuminated object surfaces become emitters, too !**
  - Radiosity = Irradiance – absorbed photons flux density
    - Radiosity: photons per second per m^2 leaving surface
    - Irradiance: photons per second per m^2 incident on surface
- **Light bounces between all mutually visible surfaces**
- **Invariance of radiance in free space**
  - No absorption in-between objects
- **Dynamic Energy Equilibrium**
  - emitted photons = absorbed photons (+ escaping photons)

➔ **Global Illumination**

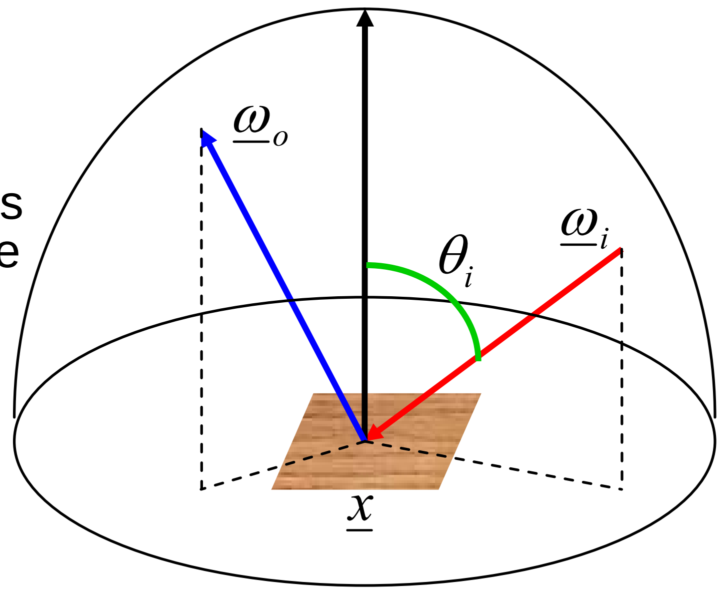
(Surface) Rendering Equation

- **In Physics: *Radiative Transport Equation***
- **Expresses energy equilibrium in scene**

$$L(\underline{x},\underline{\omega}_o) = L_e(\underline{x},\underline{\omega}_o) + \int_{\Omega} f_r(\underline{\omega}_i,\underline{x},\underline{\omega}_o)\, L_i(\underline{x},\underline{\omega}_i) \cos\theta_i \, d\underline{\omega}_i$$

total radiance = emitted radiance + reflected radiance

- **First term: emissivity of the surface**
  - non-zero only for light sources
- **Second term: reflected radiance**
  - integral over all possible incoming directions of irradiance times angle-dependent surface reflection function
- **Fredholm integral equation of 2nd kind**
  - unknown radiance appears on lhs and inside the integral
  - Numerical methods necessary to compute approximate solution



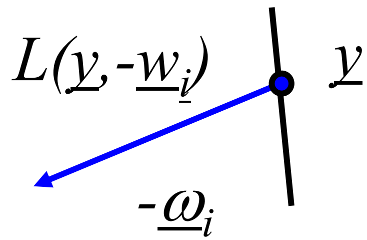
Rendering Equation II

- **Outgoing illumination at a point**

$$L(\underline{x},\underline{\omega}_o) = L_e(\underline{x},\underline{\omega}_o) + L_r(\underline{x},\underline{\omega}_o)$$
$$= L_e(\underline{x},\underline{\omega}_o) + \int_{\Omega_+} f_r(\underline{\omega}_i,\underline{x},\underline{\omega}_o) L_i(\underline{x},\underline{\omega}_i)\cos\theta_i \, d\underline{\omega}_i$$

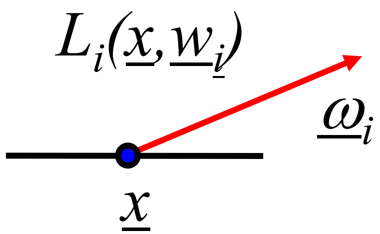
- **Linking with other surface points**
  - Incoming radiance at $\underline{x}$ is outgoing radiance at $\underline{y}$

$$L_i(\underline{x},\underline{\omega}_i) = L(\underline{y},-\underline{\omega}_i) = L(RT(\underline{x},\underline{\omega}_i),-\underline{\omega}_i)$$



  - Ray-Tracing operator

$$\underline{y} = RT(\underline{x},\underline{\omega}_i)$$



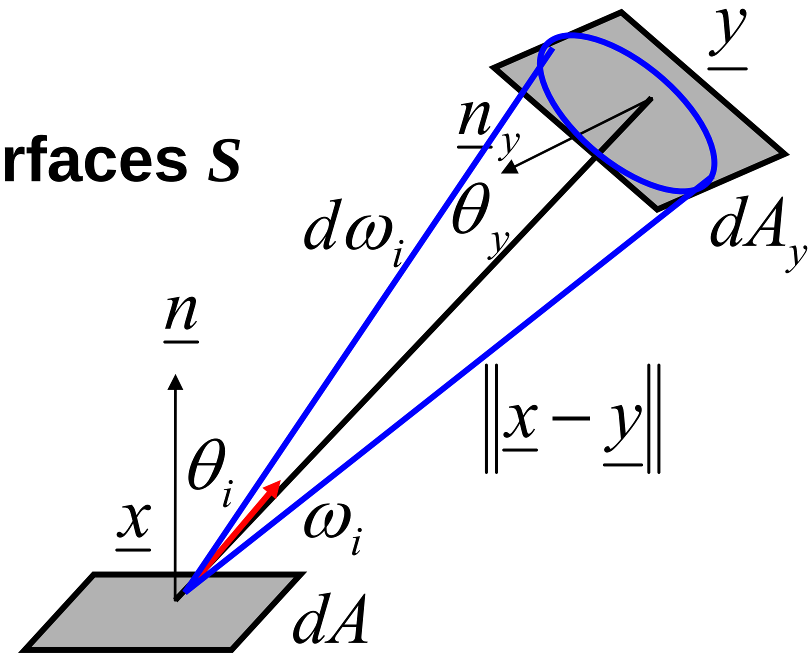
Rendering Equation III

- **Directional parameterization**

$$L(\underline{x},\underline{\omega}_o) = L_e(\underline{x},\underline{\omega}_o) + \int_{\Omega_+} f_r(\underline{\omega}_i,\underline{x},\underline{\omega}_o) L(\underline{y}(\underline{x},\underline{\omega}_i),-\underline{\omega}_i)\cos\theta_i \, d\omega_i$$

- **Re-parameterization over surfaces *S***

$$d\omega_i = \frac{\cos\theta_y}{\|\underline{x}-\underline{y}\|^2} \, dA_y$$



$$L(\underline{x},\underline{\omega}_o) = L_e(\underline{x},\underline{\omega}_o) + \int_{\underline{y}\in S} f_r(\underline{\omega}_i,\underline{x},\underline{\omega}_o) \, L(\underline{y},\underline{\omega}_i(\underline{x},\underline{y})) V(\underline{x},\underline{y}) \frac{\cos\theta_i \cos\theta_y}{\|\underline{x}-\underline{y}\|^2} \, dA_y$$

Rendering Equation IV

$$L(\underline{x},\underline{\omega}_o) = L_e(\underline{x},\underline{\omega}_o) + \int_{\underline{y}\in S} f_r(\underline{\omega}_i,\underline{x},\underline{\omega}_o)\, L(\underline{y},\underline{\omega}_i(\underline{x},\underline{y}))\, \mathrm{V}(\underline{\mathrm{x}},\underline{\mathrm{y}}) \frac{\cos\theta_i \cos\theta_y}{\left\|\underline{\mathrm{x}}-\underline{y}\right\|^2}\, dA_y$$

- **Geometry term**

$$G(\underline{x},\underline{y}) = V(\underline{x},\underline{y}) \frac{\cos\theta_i \cos\theta_y}{\|\underline{x}-\underline{y}\|^2}$$

- **Visibility term**

$$V(\underline{x},\underline{y}) = \begin{cases} 1 & \text{if visible} \\ 0 & \text{if not visible} \end{cases}$$

- **Integration over all surfaces**

$$L(\underline{x},\underline{\omega}_o) = L_e(\underline{x},\underline{\omega}_o) + \int_{\underline{y}\in S} f_r(\underline{\omega}_i,\underline{x},\underline{\omega}_o)\, L(\underline{y},\underline{\omega}_i(\underline{x},\underline{y}))\, G(\underline{x},\underline{y})\, dA_y$$

Rendering Equation: Approximations

- **Using RGB instead of full spectrum**
  - follows roughly the eye's sensitivity
- **Dividing scene surfaces into small patches**
  - Assumes locally constant reflection, visibility, geometry terms
- **Sampling hemisphere along finite, discrete directions**
  - simplifies integration to summation
- **Reflection function model**
  - Parameterized function
    - ambient: constant, non-directional, background light
    - diffuse: light reflected uniformly in all directions
    - specular: light of higher intensity in mirror-reflection direction
  - Lambertian surface (only diffuse reflection) - **Radiosity**
- *Approximations based on empirical foundations*
  - ➢ ***An example: polygon rendering in OpenGL***

Radiosity Equation

$$L(\underline{x},\underline{\omega}_o) = L_e(\underline{x},\underline{\omega}_o) + \int_{\underline{y}\in S} f_r(\underline{\omega}_i,\underline{x},\underline{\omega}_o)\, L(\underline{y},\underline{\omega}_i(x,y))\, G(\underline{x},\underline{y})\, dA_y$$

- **Diffuse reflection only** $\rho(\underline{x}) = \int_0^{\pi/2}\int_0^{2\pi} f_r(\underline{x})\cos\theta\, d\omega = \int_0^{\pi/2}\int_0^{2\pi} f_r(\underline{x})\cos\theta\sin\theta\, d\phi\, d\theta$

$$f_r(\underline{\omega}_i,\underline{x},\underline{\omega}_o) = f_r(\underline{x}) \;\Rightarrow\; \rho(\underline{x}) = \pi\, f_r(\underline{x})$$

diffuse reflectance

- **Radiance $\Rightarrow$ Radiosity**

$$L(\underline{x},\underline{\omega}_o) \rightarrow B(\underline{x})/\pi$$

$$B(\underline{x}) = B_e(\underline{x}) + \rho(\underline{x})\, E(\underline{x})$$

$$= B_e(\underline{x}) + \rho(\underline{x}) \int_{\underline{y}\in S} F(\underline{x},\underline{y})\, B(\underline{y})\, dA_y$$

- **Form factor**

$$F(\underline{x},\underline{y}) = \frac{G(\underline{x},\underline{y})}{\pi}$$

percentage of light leaving dA_y that arrives at dA

Linear Operators

- **Properties**
  - Fredholm integral of 2nd kind
  - Global linking
    - Potentially each point with each other
    - Often sparse systems (occlusions)
  - No consideration of volume effects!!
- **Linear operator**
  - acts on functions like matrices act on vectors
  - Superposition principle
  - Scaling and addition

$$B(\underline{x}) = B_e(\underline{x}) + \rho(\underline{x}) \int_{\underline{y} \in S} F(\underline{x}, \underline{y})\, B(\underline{y})\, dA_y$$

$$f(x) = g(x) + (K \circ f)(x)$$

$$(K \circ f)(x) \equiv \int k(x, y) f(y)\, dy$$

$$K \circ (af + bg) = a(K \circ f) + b(K \circ g)$$

Formal Solution of Integral Equations

$$B(\underline{x}) = B_e(\underline{x}) + \rho(\underline{x}) \int_{\underline{y} \in S} F(\underline{x}, \underline{y})\, B(\underline{y})\, dA_y$$

- **Integral equation**

$$B = B_e + K \circ B$$

$$\Rightarrow (I - K) \circ B = B_e$$

- **Formal solution**

$$B = (I - K)^{-1} \circ B_e$$

- **Neumann series**

$$\frac{1}{1-x} = 1 + x + x^2 + \ldots$$

$$\frac{1}{I-K} = I + K + K^2 + \ldots$$

$$(I-K)\frac{1}{I-K} = (I-K)(I + K + K^2 + \ldots)$$
$$= (I + K + K^2 + \ldots) - (K + K^2 + \ldots) = I$$

Formal Solutions II

- **Successive approximation**

$$\frac{1}{I-K}B_e = B_e + K \circ B_e + K^2 \circ B_e + \ldots$$
$$= (B_e + K \circ (B_e + K \circ (B_e + \ldots$$

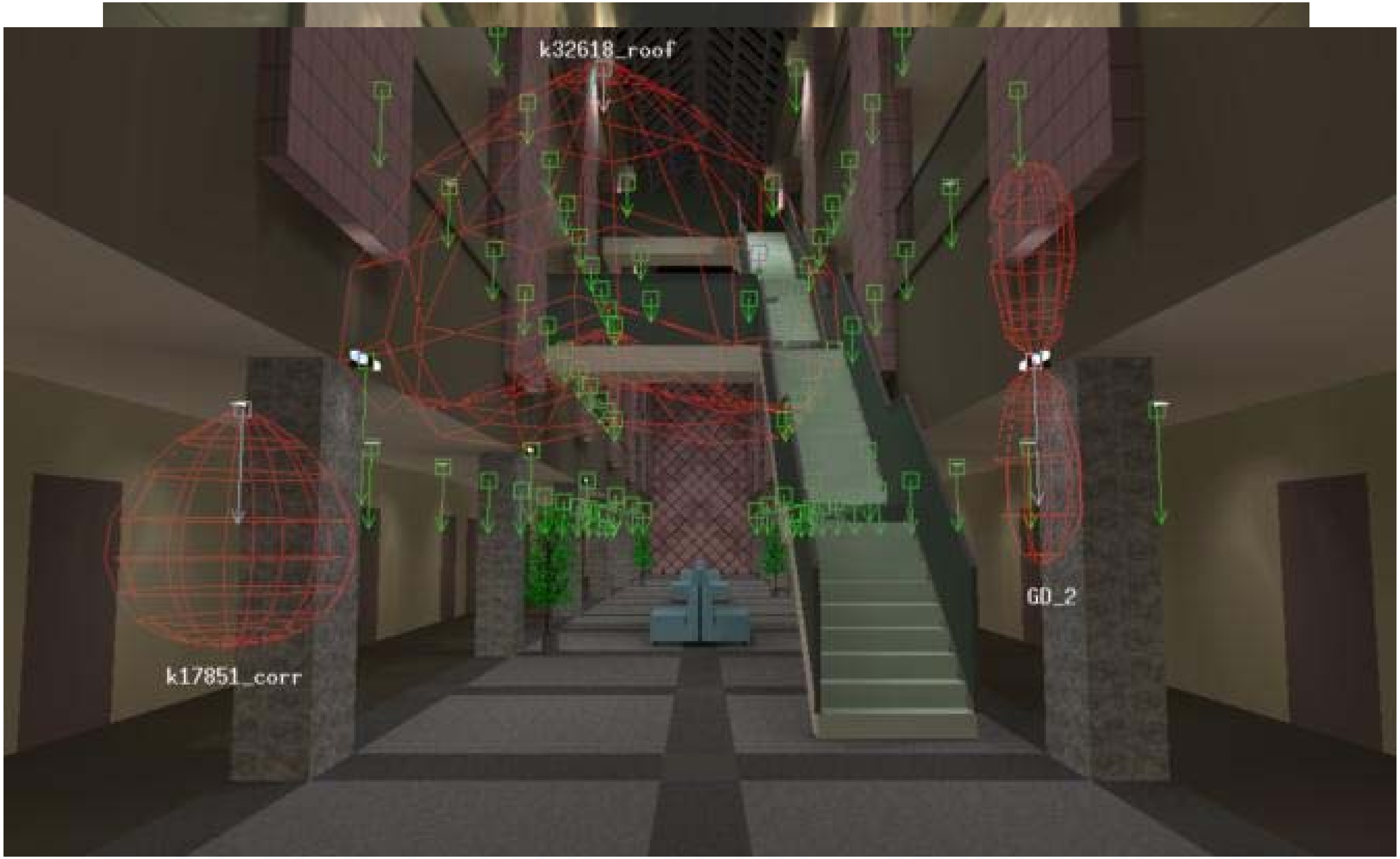
  - Direct light from the light source
  - Light which is reflected and transported one time
  - Light which is reflected and transported n-times

$$B_1 = B_e$$
$$B_2 = B_e + K \circ B_1$$
$$\ldots$$
$$B_n = B_e + K \circ B_{n-1}$$

Lighting Simulation

Courtesy Karol Myszkowski, MPII

Lighting Simulation



Courtesy Karol Myszkowski, MPII

Lighting Simulation

Courtesy Karol Myszkowski, MPII

Wrap-up

- **Physical Quantities in Rendering**
  - Radiance
  - Radiosity
  - Irradiance
  - Intensity
- **Light Perception**
- **Light Sources**
- **Rendering Equation**
  - Integral equation
  - Balance of radiance
- **Radiosity**
  - Diffuse reflectance function
  - Radiative equilibrium between emission and absorption, escape
  - System of linear equations
  - Iterative solution