Computer Graphics

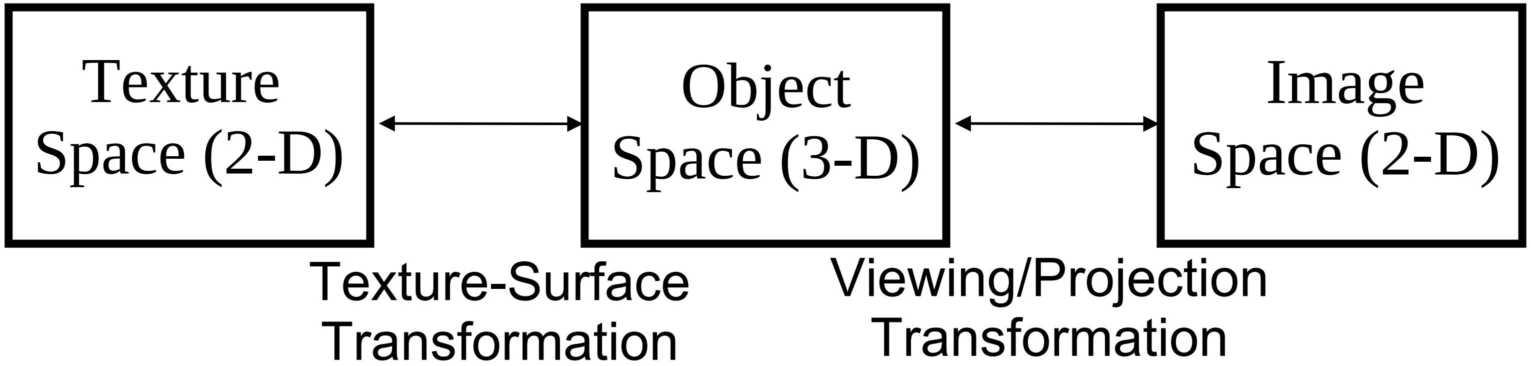
- Texturing & Procedural Methods -

Hendrik Lensch

Overview

- **Last time**
  - Shading
  - Texturing
- **Today**
  - Texturing (Cont.)
  - Procedural textures
  - Fractal landscapes
- **Next lecture**
  - Texture Filtering
  - Alias & signal processing

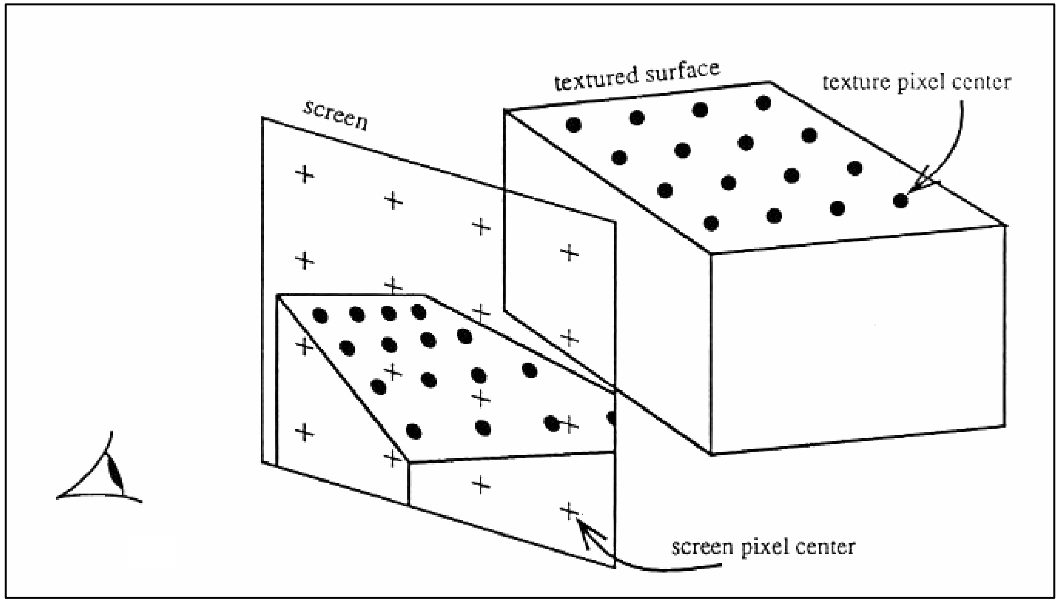
Texture Mapping Transformations

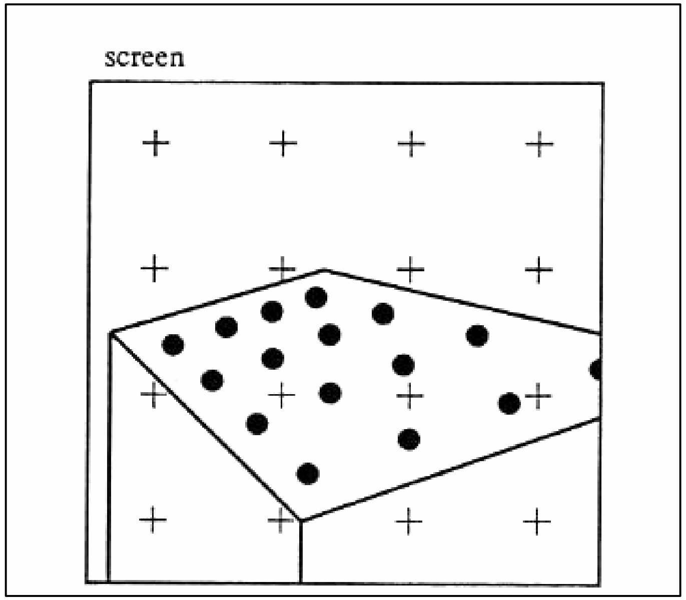


The texture is mapped onto a surface in 3-D object space, which is then mapped to the screen by the viewing projection. These two mappings are composed to find the overall 2-D texture space to 2-D image space mapping, and the intermediate 3-D space is often forgotten. This simplification suggests texture mapping's close ties with image warping and geometric distortion.

Texture space (u,v)
Object space (x_o,y_o,z_o)
Screen space (x,y)

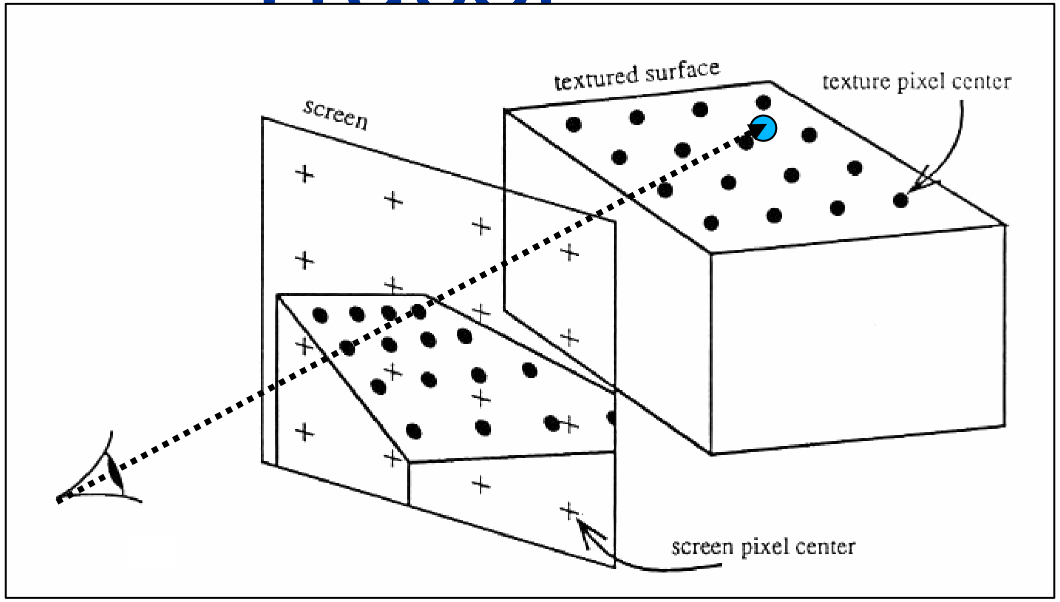
2D Texturing

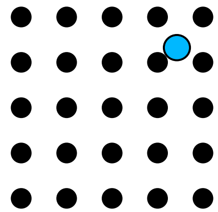




- **2D texture mapped onto object**
- **Object projected onto 2D screen**
- **2D→2D: warping operation**
- **Uniform sampling ?**
- **Hole-filling/blending ?**

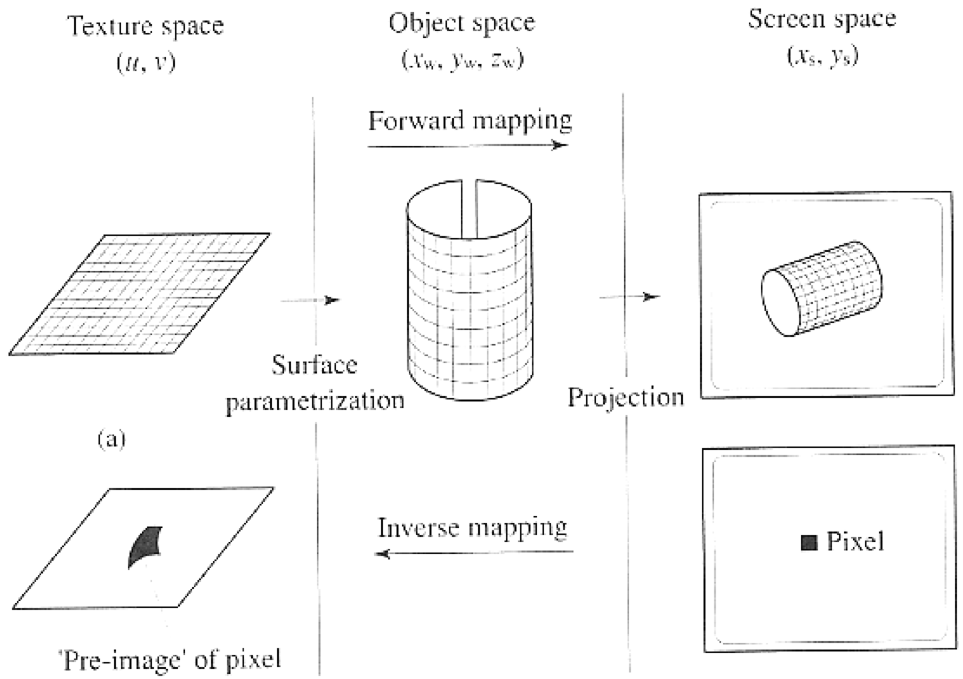
Texture Mapping in a Ray Tracer





- **approximation:**
  - ray hits surface
  - surface location corresponds to coordinate inside a texture

2D Texture Mapping



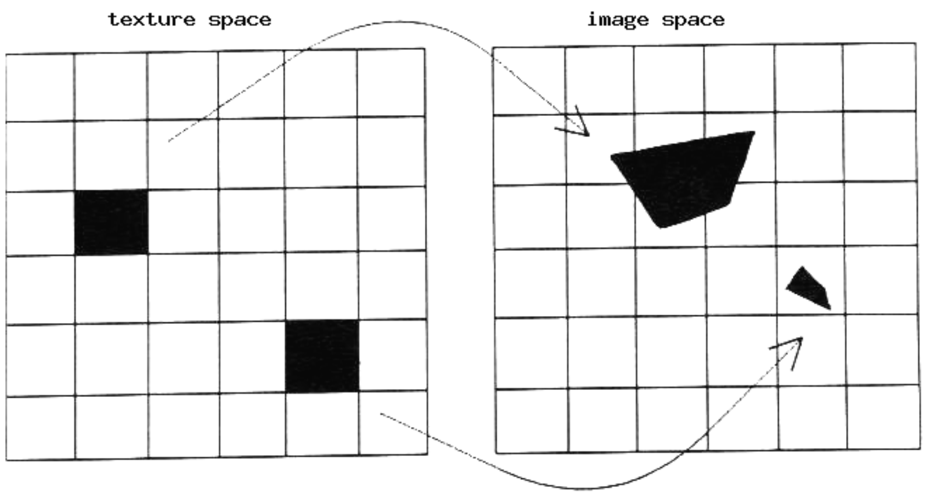
- **Forward mapping**
  - Object surface parameterization
  - Projective transformation
- **Inverse mapping**
  - Find corresponding pre-image/footprint of each pixel in texture
  - Integrate over pre-image

Forward Mapping

- **Maps each texel to its position in the image**
- **Uniform sampling of texture space does not guarantee uniform sampling in screen space**
- **Possibly used if**
  - The texture-to-screen mapping is difficult to invert
  - The texture image does not fit into memory

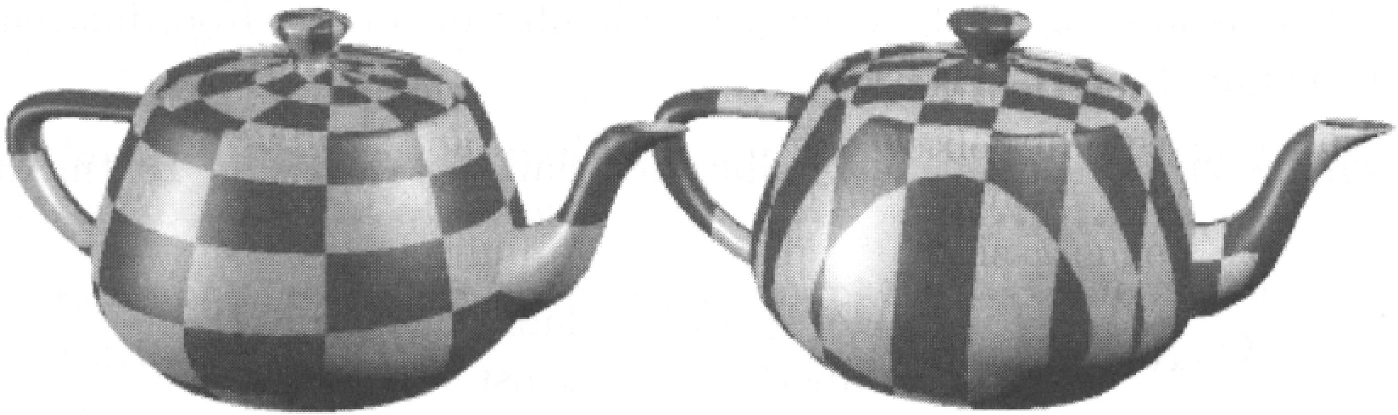
Texture scanning:

```
for v
  for u
    compute x(u,v) and y(u,v)
    copy TEX[u,v] to SCR[x,y]
(or in general
     rasterize image of TEX[u,v])
```



Surface Parameterization

- **To apply textures we need 2D coordinates on surfaces**
  → Parameterization
- **Some objects have a natural parameterization**
  – Sphere: spherical coordinates $(\varphi, \theta) = (2\pi\ u, \pi\ v)$
  – Cylinder: cylindrical coordinates $(\varphi, z) = (2\ \pi\ u, H\ v)$
  – Parametric surfaces (such as B-spline or Bezier surfaces → later)
- **Parameterization is less obvious for**
  – Polygons, implicit surfaces, …



Triangle Parameterization

- **Triangle is a planar object**
  - Has implicit parameterization (e.g. barycentric coordinates)
  - But we need more control: Placement of triangle in texture space
- **Assign texture coordinates** (u,v) **to each vertex** (x_o,y_o,z_o)
- **Apply viewing projection** $(x_o,y_o,z_o) \rightarrow (x,y)$
- **Yields full texture transformation (warping)** $(u,v) \rightarrow (x,y)$

$$x = \frac{au+bv+c}{gu+hv+i} \qquad y = \frac{du+ev+f}{gu+hv+i}$$

  - In homogeneous coordinates (by embedding (u,v) as (u',v',1))

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} u' \\ v' \\ q \end{bmatrix} \qquad (x,y) = (x'/w, y'/w) \qquad (u,v) = (v'/w, v'/w)$$

  - Transformation coefficients determined by 3 pairs $(u,v) \rightarrow (x,y)$
    - Three linear equations
    - Invertible iff neither set of points is collinear

Triangle Parameterization II

- **Given**

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} u' \\ v' \\ q \end{bmatrix}$$

- **the inverse transform** ***(x,y)→(u,v)*** **is defined as**

$$\begin{bmatrix} u' \\ v' \\ q \end{bmatrix} = \begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} \begin{bmatrix} x' \\ y' \\ w \end{bmatrix} \qquad \begin{bmatrix} u' \\ v' \\ q \end{bmatrix} = \begin{bmatrix} ei-fh & ch-bi & bf-ce \\ fg-di & ai-cg & cd-af \\ dh-eg & bg-ah & ae-bd \end{bmatrix} \begin{bmatrix} x' \\ y' \\ w \end{bmatrix}$$

- **Coefficients must be calculated for each triangle**
  - Rasterization
    - Incremental bilinear update of (u',v',q) in screen space
    - Using the partial derivatives of the linear function (i.e. constants)
  - Ray tracing
    - Evaluated at every intersection

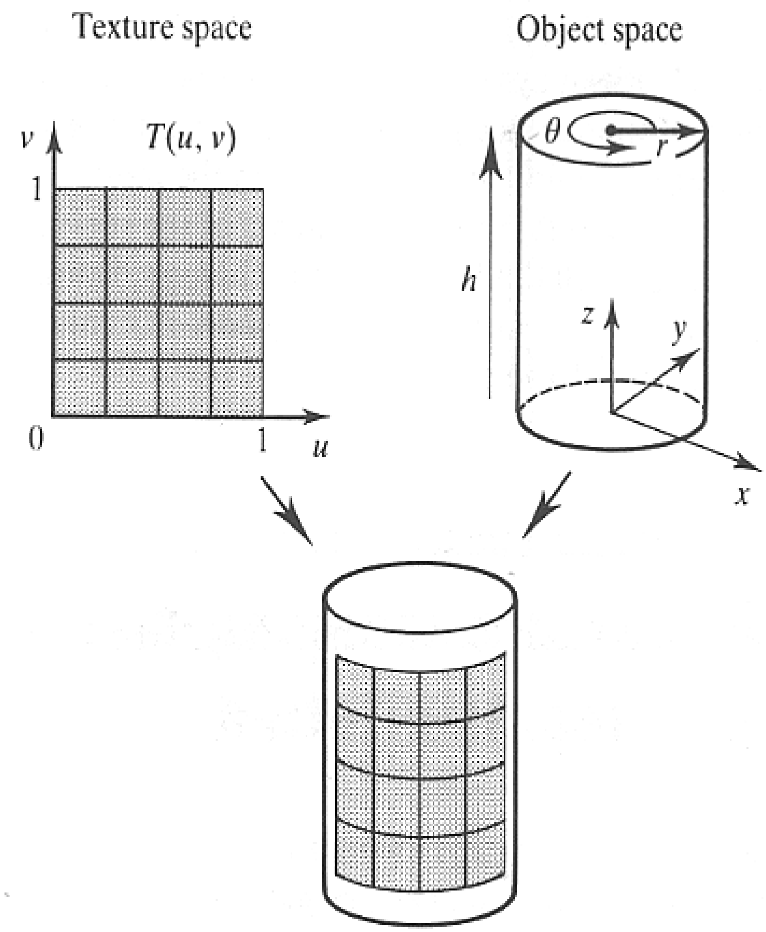
Cylinder Parameterization

- **Transformation from texture space to the cylinder parametric representation can be written as:**

$$(\theta, h) = (2\pi u, vH)$$

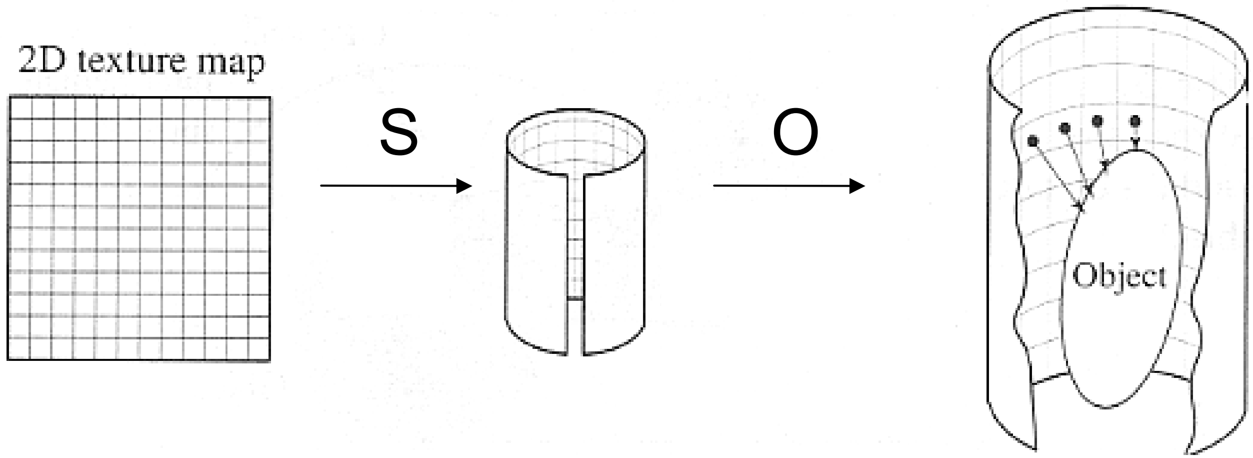
- **where H is the height of the cylinder.**
- **The surface coordinates in the Cartesian reference frame can be expressed as:**

$$x_o = r\cos\theta$$
$$y_o = r\sin\theta$$
$$z_o = h$$



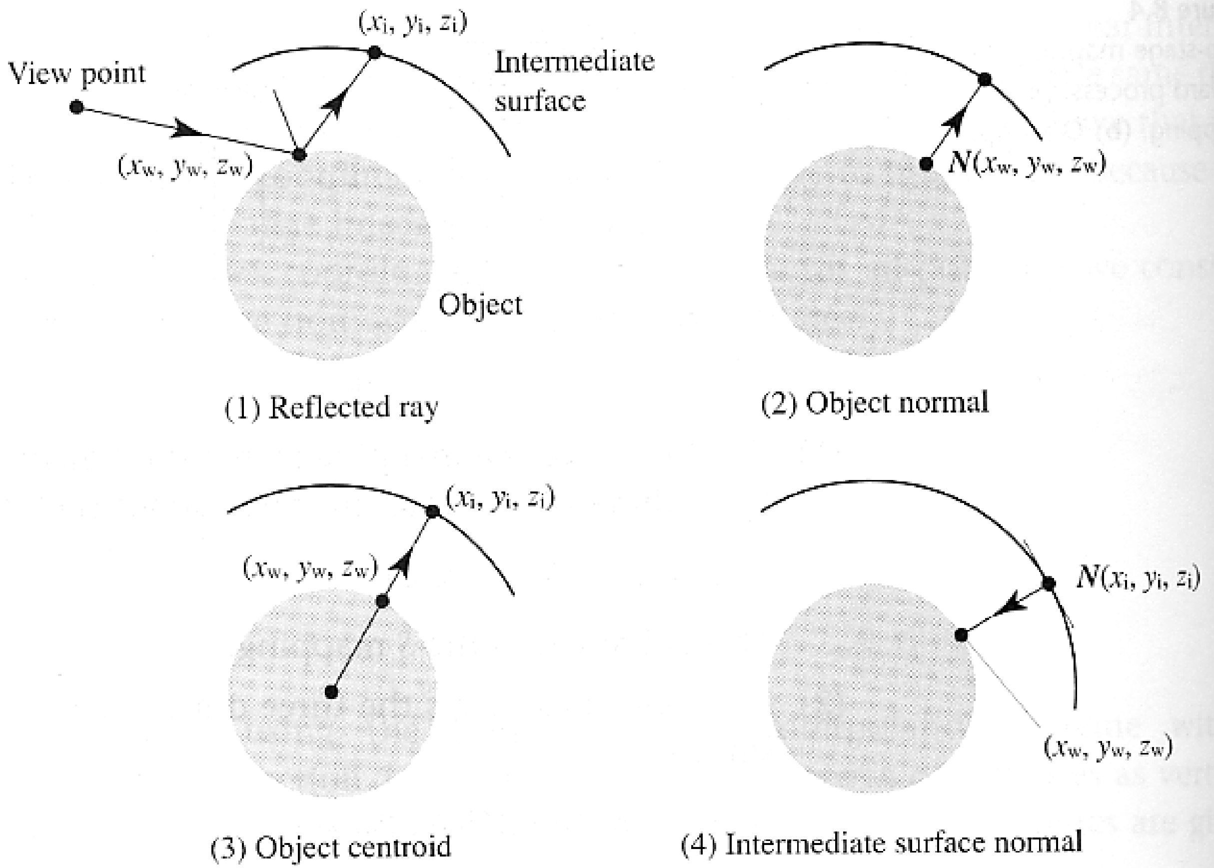
Two-Stage Mapping

- **Inverse Mapping for arbitrary 3D surfaces too complex**
- **Approximation technique is used:**
  - Mapping from 2D texture space to a simple 3D intermediate surface (S mapping)
    - Should be a reasonable approximation of the destination surface
    - E.g.: plane, cylinder, sphere, cube, ...
  - Mapping from the intermediate surface to the destination object surface (O mapping)



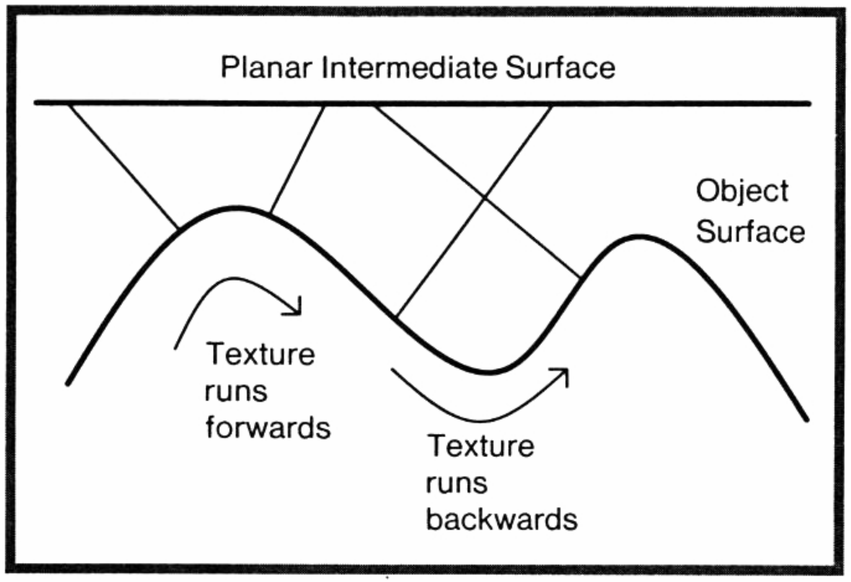
O-Mapping

- **Determine point on intermediate surface through**
  – Reflected view ray
    - Reflection or environment mapping
  – Normal mapping
  – Line through object centroid
  – Shrinkwrapping
    - Forward mapping
    - Normal mapping from intermediate surface



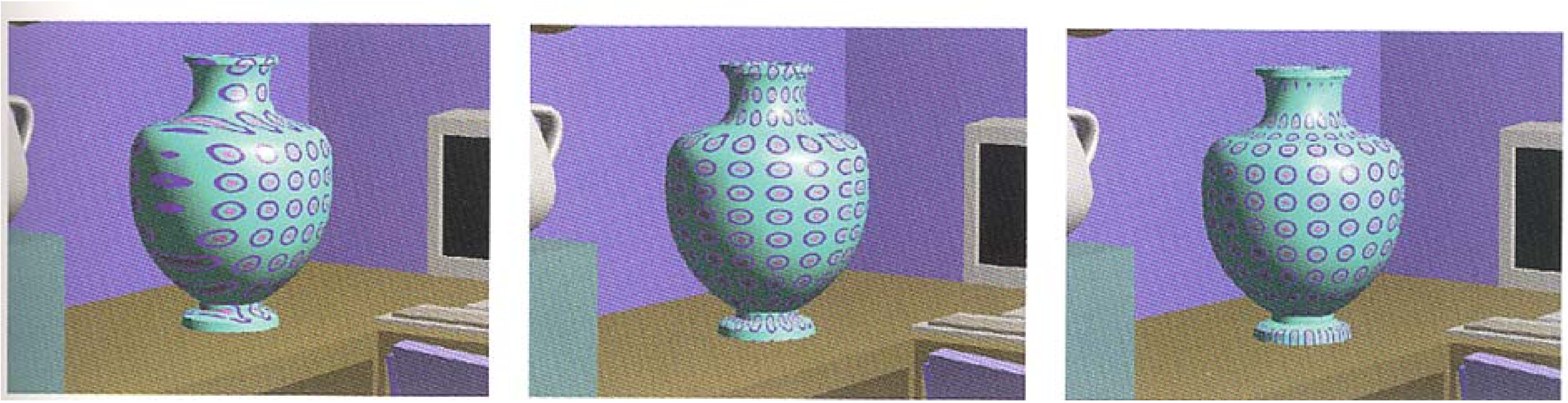
Two-Stage Mapping: Problems

- **Problems**
  - May introduce undesired texture distortions if the intermediate surface differs too much from the destination surface
  - Still often used in practice because of its simplicity



Surface concavities can cause the texture pattern to reverse if the object normal mapping is used.

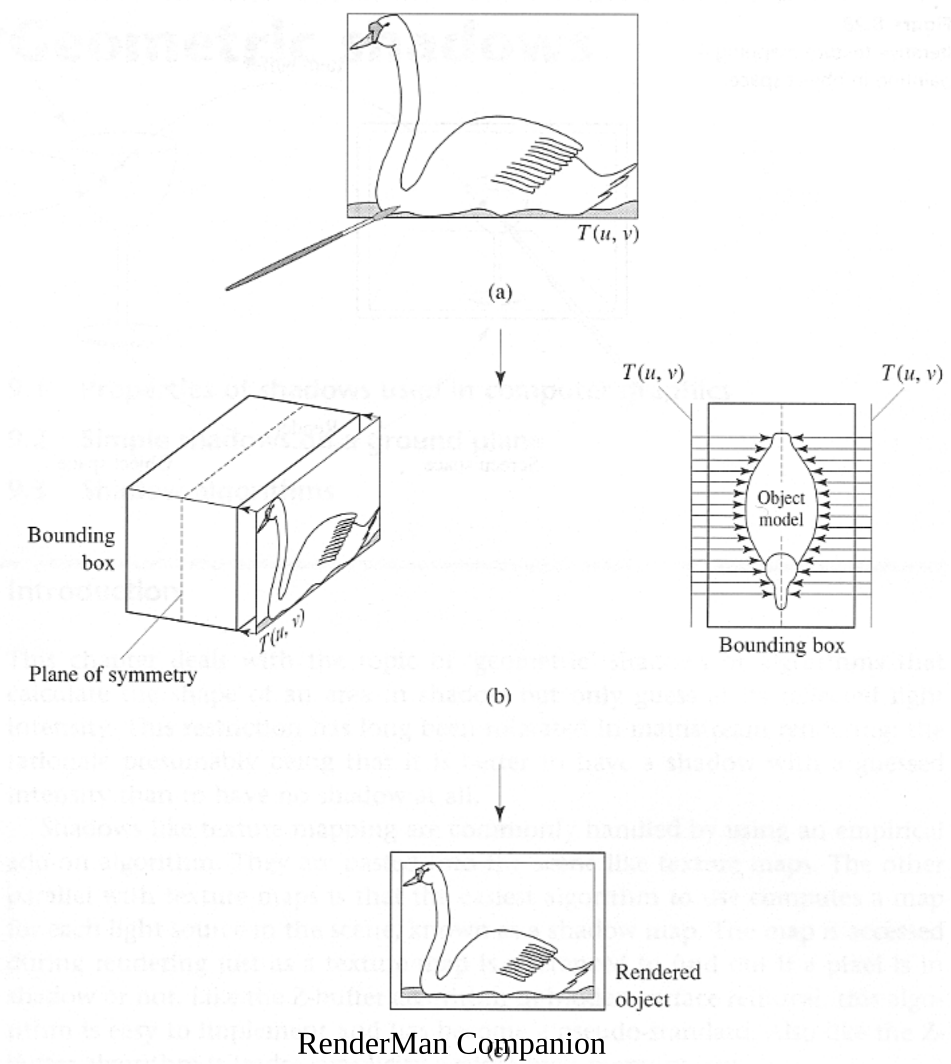
Two-Stage Mapping: Example



- **Different intermediate surfaces**
- **Plane**
  – Strong distortion where object surface normal ⊥ plane normal
- **Cylinder**
  – Reasonably uniform mapping (symmetry !)
- **Sphere**
  – Problems with concave regions

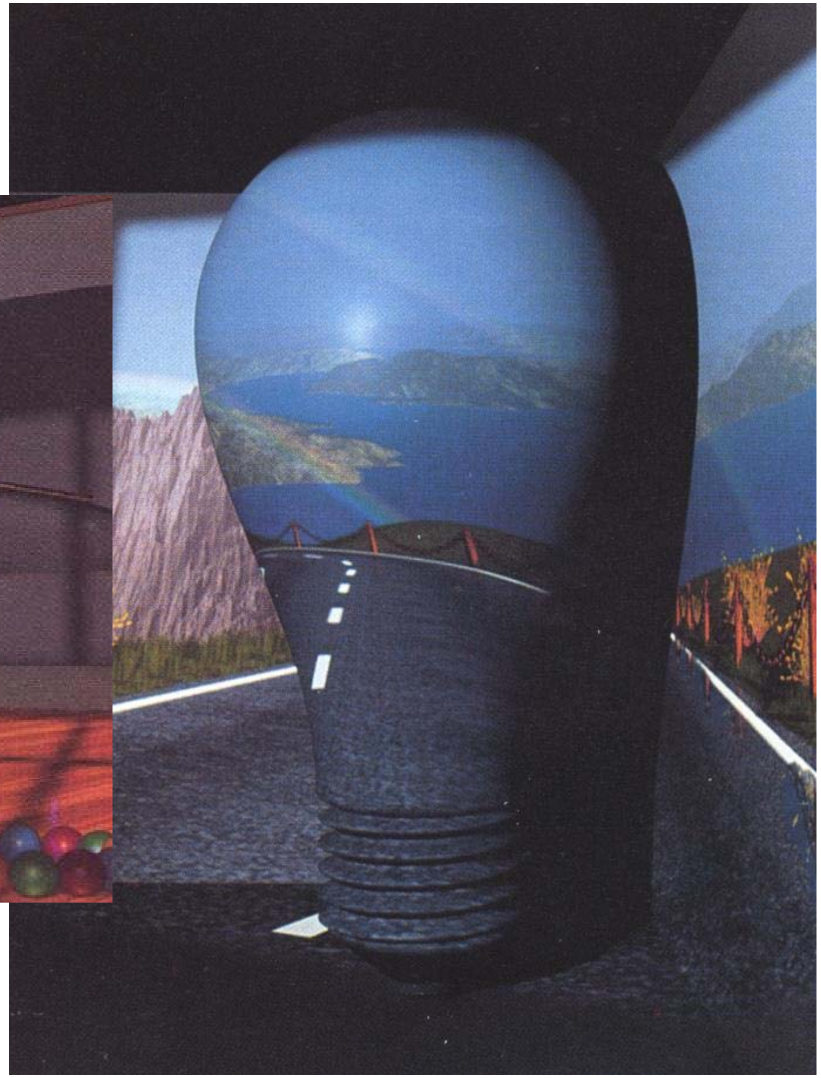
Projective Textures

- **Project texture onto object surfaces**
  - Slide projector
- **Parallel or perspective projection**
- **Use photographs as textures**
- **Multiple images**
  - View-dependent texturing
- **Perspective Mapping**



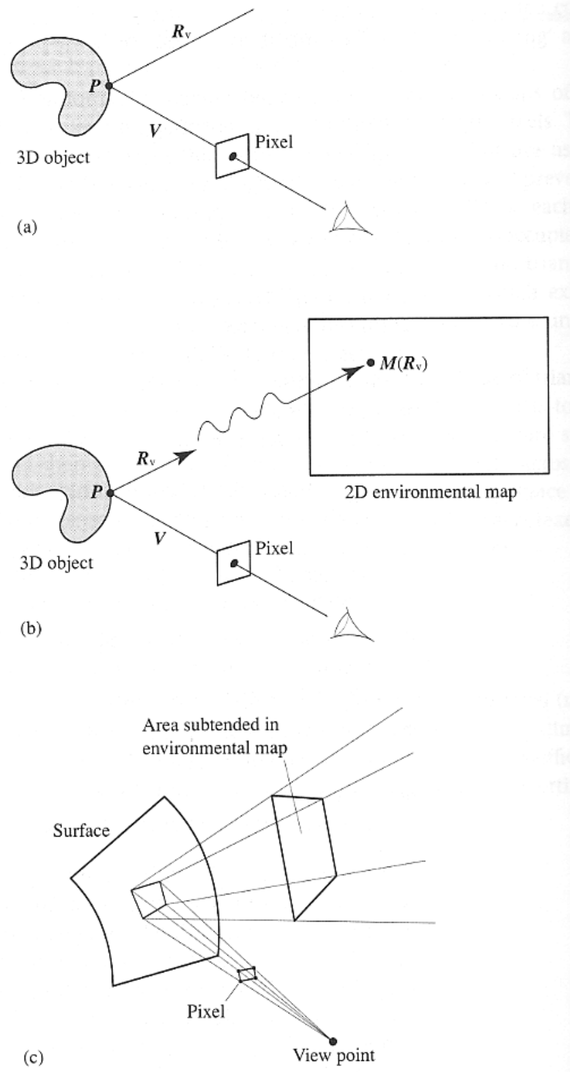
RenderMan Companion

Projective Texturing: Examples



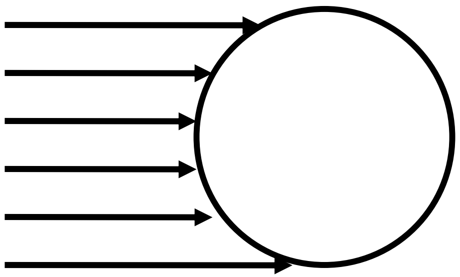
Reflection Mapping

- **Also called Environment Mapping**
- **Mirror reflections**
  - Surface curvature: beam tracing
  - Map filtering
- **Reflection map parameterization**
  - Intermediate surface in 2-stage mapping
  - Often cube, sphere, or double paraboloid
- **Assumption: Distant illumination**
  - Parallax-free illumination
  - No self-reflections, distortion of near objects
- **Option: Separate map per object**
  - Often necessary to be reasonable accurate
  - Reflections of other objects
  - Maps must be recomputed after changes



Reflection Map Acquisition

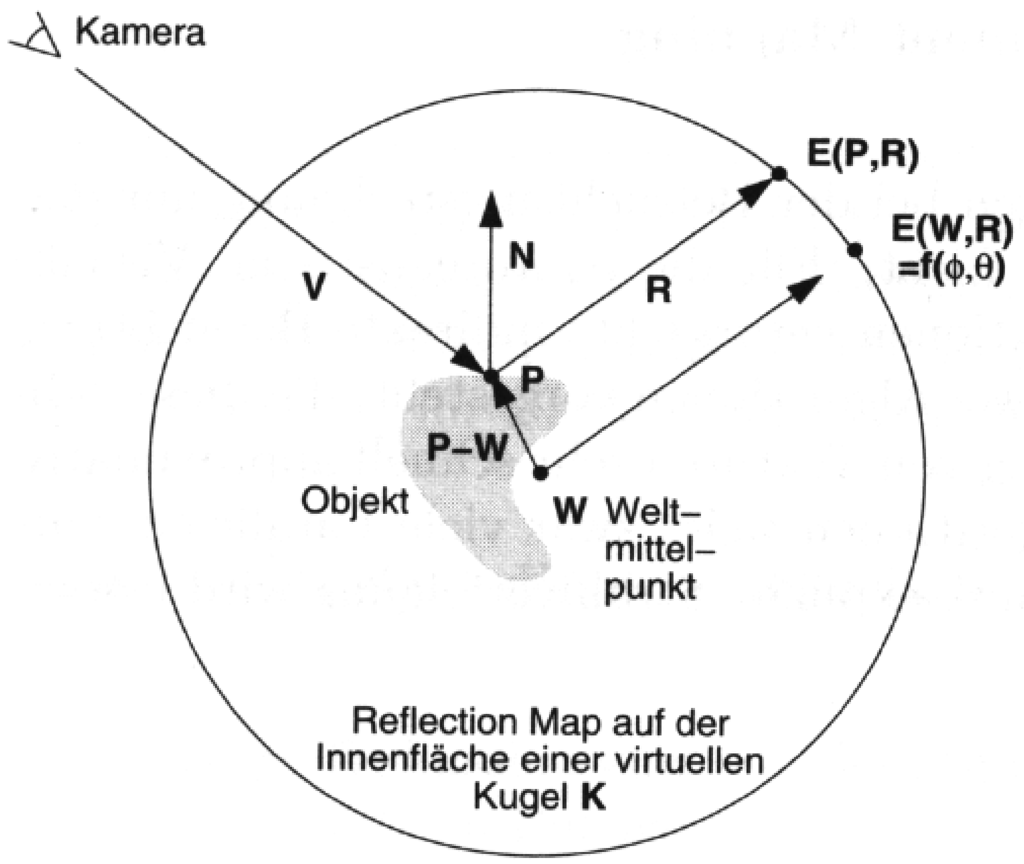
- **Generating spherical maps (original 1982/83)**
  - i.e. photo of a reflecting sphere (gazing ball)



Peter Chou

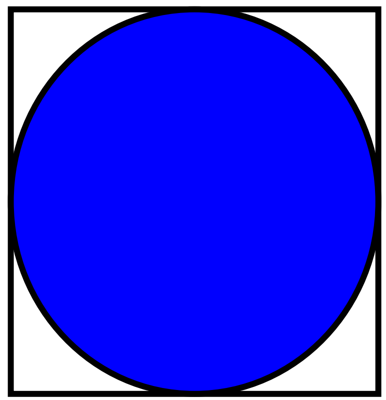
Reflection Map Rendering

- **Spherical parameterization**
- **O-mapping using reflected view ray intersection**

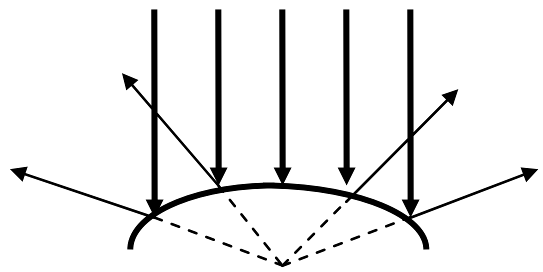


Reflection Map Parameterization

- **Spherical mapping**
  - Single image
  - Bad utilization of the image area
  - Bad scanning on the edge
  - Artifacts, if map and image do not have the same direction

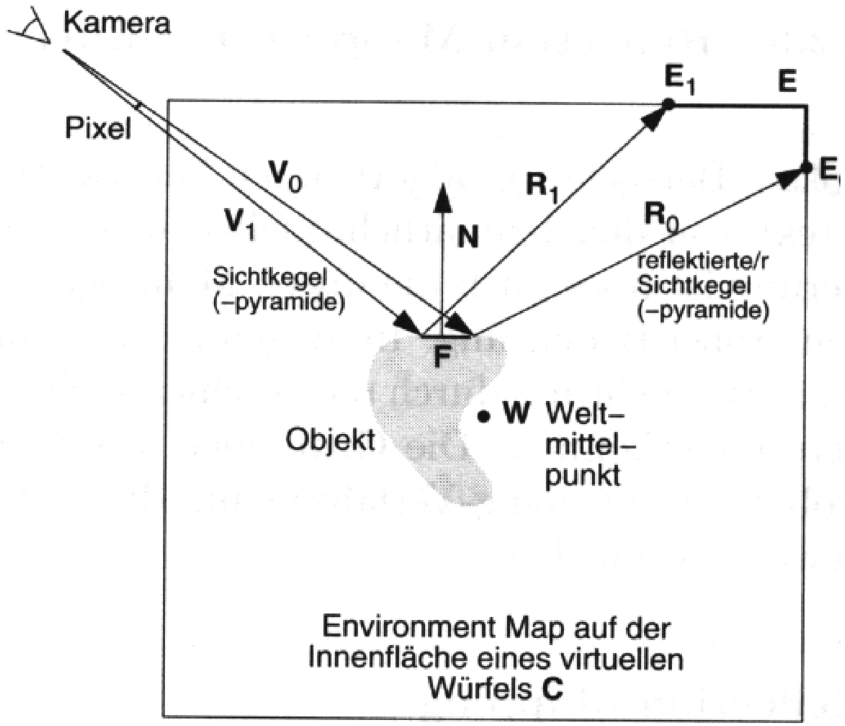


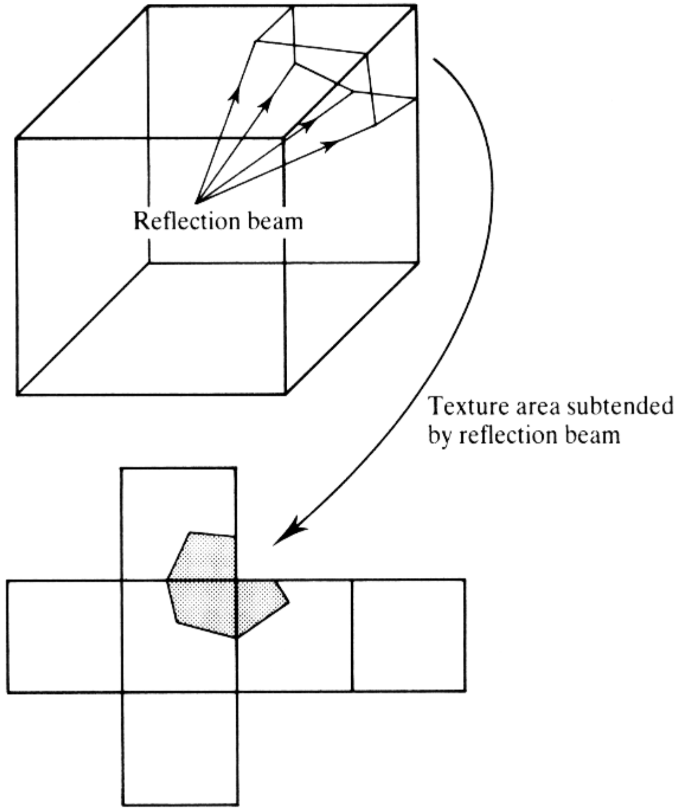
- **Double parabolic mapping**
  - Subdivide in 2 images (facing and back facing side)
  - Less bias on the edge
  - Arbitrarily reusable
  - Supported by OpenGL extensions



Reflection Map Parameterization

- **Cubical environment map, cube map, box map**
  - Enclose object in cube
  - Images on faces are easy to compute
  - Poorer filtering at edges
  - Support in OpenGL





Reflection Mapping

Terminator II motion picture

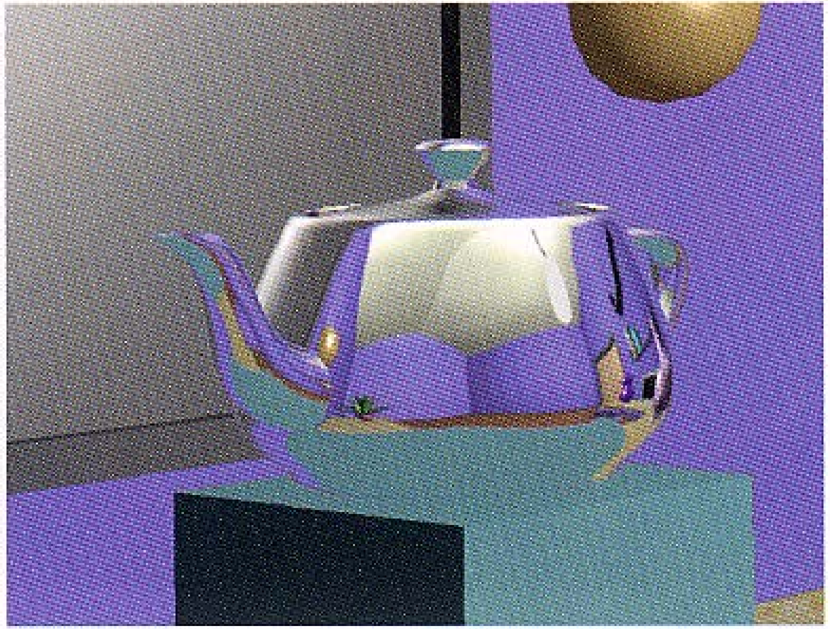
Reflection Mapping Example II

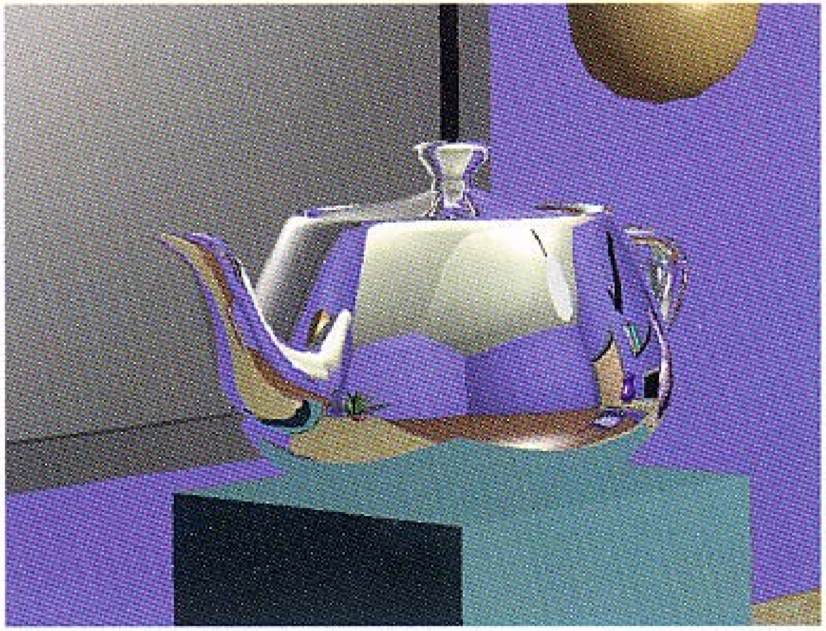
- **Reflection mapping with Phong reflection**
  - Two maps: diffuse & specular
  - Diffuse: index by surface normal
  - Specular: indexed by reflected view vector

RenderMan Companion

Ray Tracing vs. Reflection Mapping

- **Differences ?**





Recursive Ray Tracing

- **How to fake it with reflection mapping?**

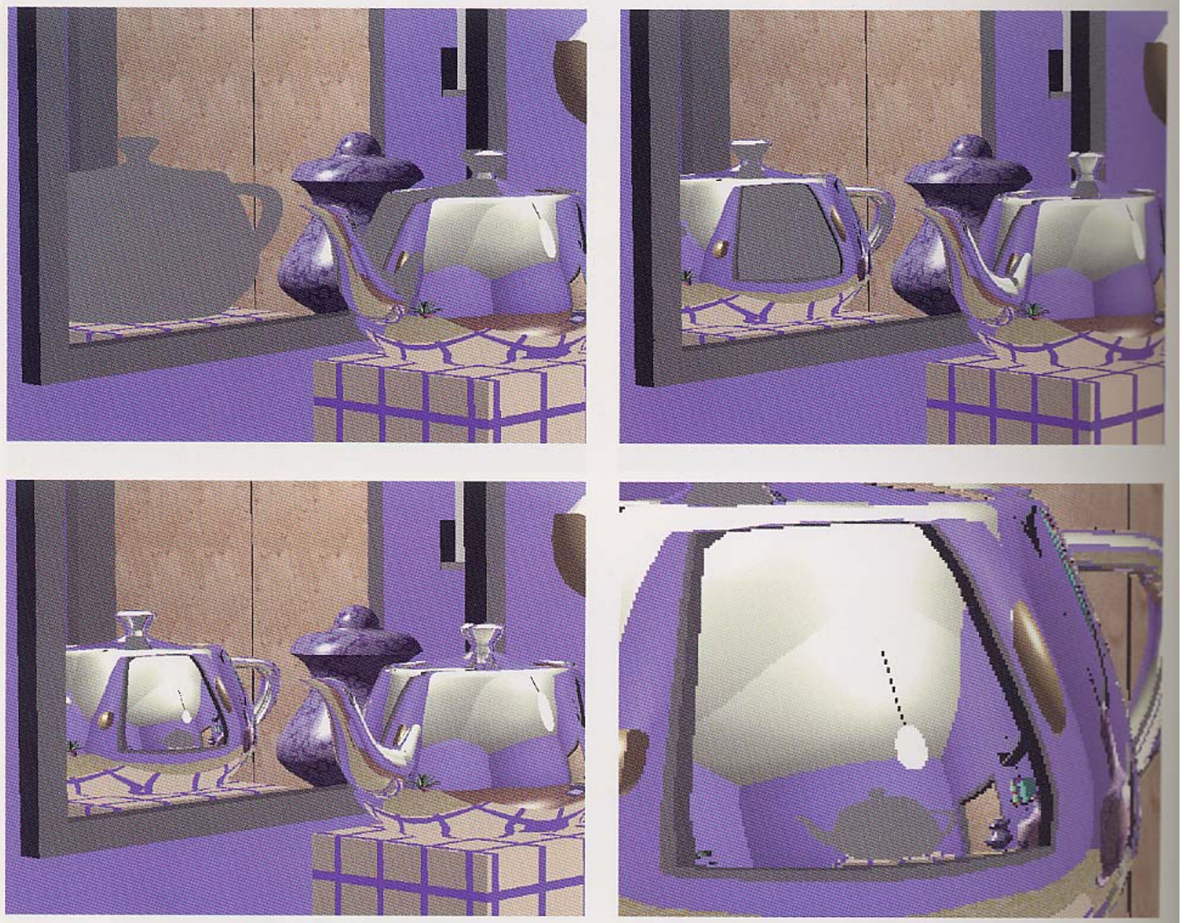
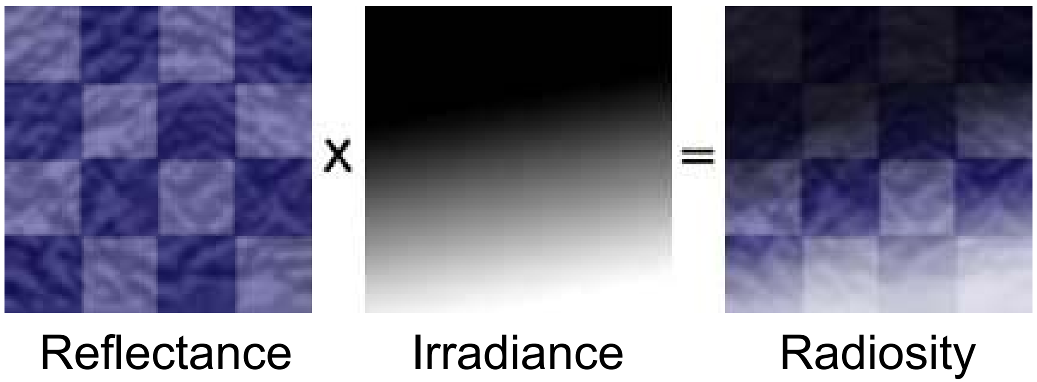
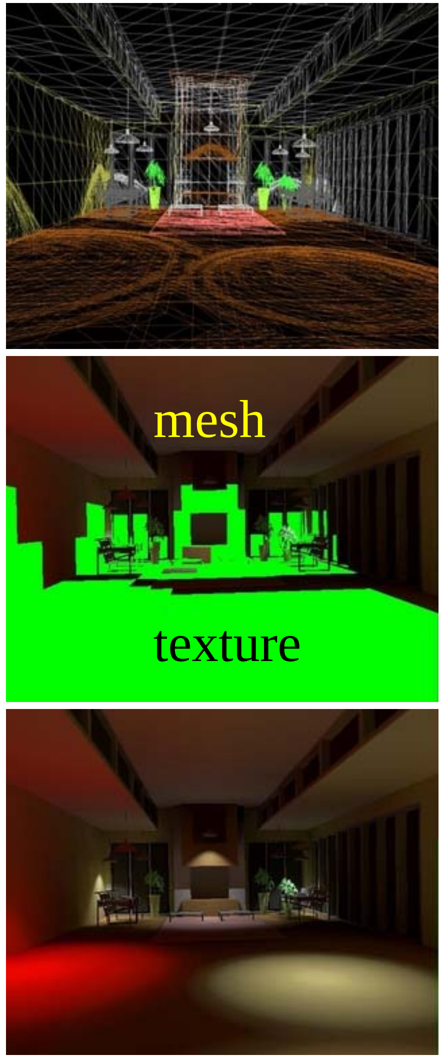


Figure 18.11
A recursive depth demonstration. The trace terminates at depth 2, 3, 4 and 5 (zoom image) respectively. 'Unassigned' pixels are coloured grey. Bad aliasing as a function of recursive depth (the light cable) is apparent.

Light Maps

- **Light maps (i.e. in Quake)**
  – Pre-calculated illumination (local irradiance)
    - Often very low resolution
  – Multiplication of irradiance with base texture
    - Diffuse reflectance only
  – Provides surface radiosity
    - View-independent
  – Animated light maps
    - Animated shadows, moving light spots etc.

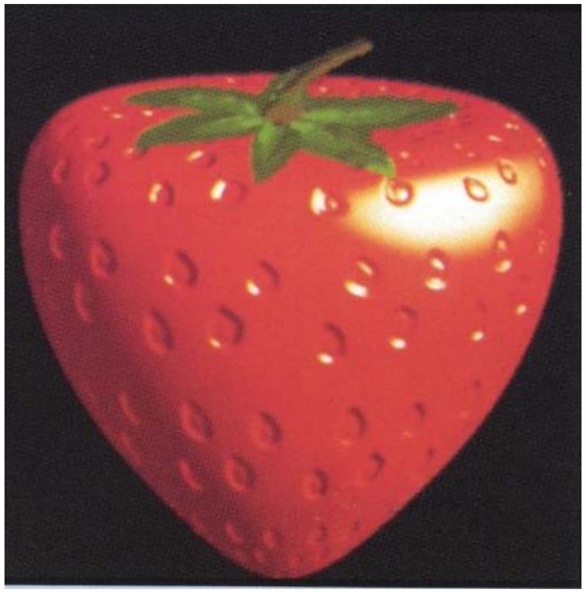


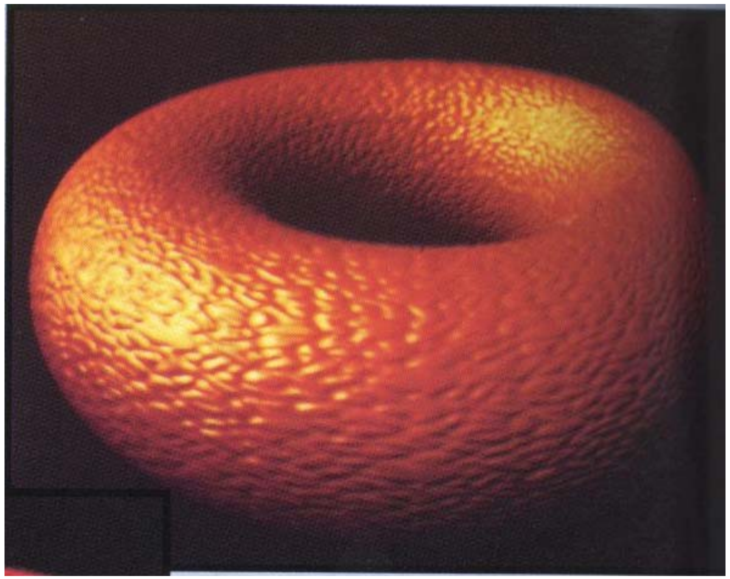


Representing radiosity in a mesh or texture

Bump Mapping

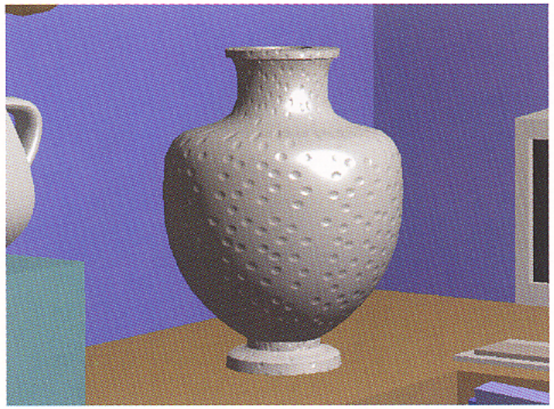
- **Modulation of the normal vector**
  - Surface normals changed only
    - Influences shading only
    - No self-shadowing, contour is ***not*** altered

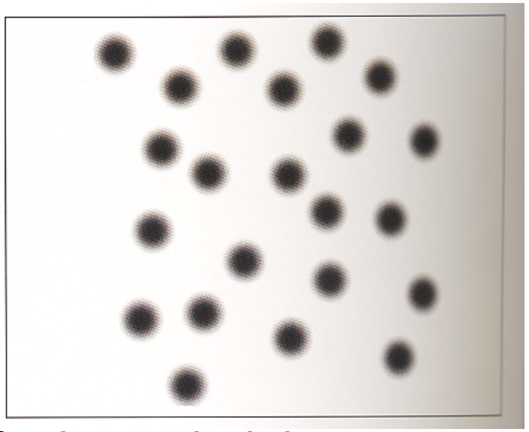




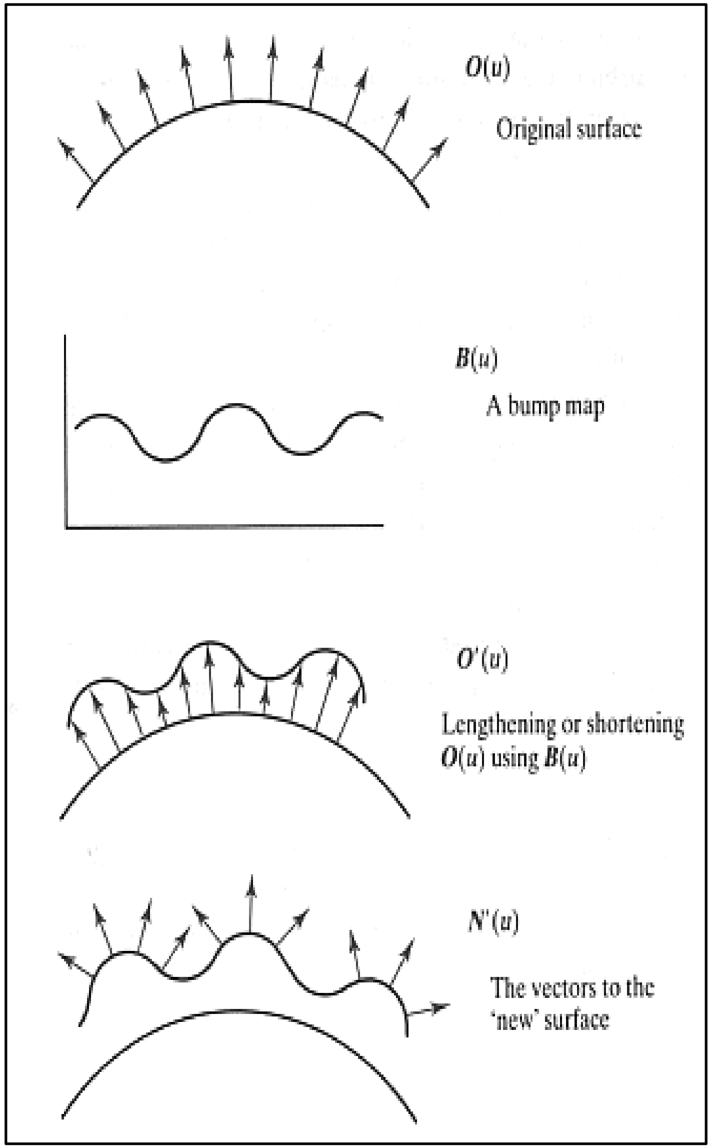
Bump Mapping

- **Original surface *O(u,v)***
  - Surface normals are known
- **Bump map *B(u,v)* $\in$ *R***
  - Surface is offset in normal direction according to bump map intensity
  - New normal directions *N'(u,v)* are calculated based on *virtually* displaced surface *O'(u,v)*
  - Originals surface is rendered with new normals *N'(u,v)*





Grey-valued texture used for bump height



Bump Mapping

$$O'(u, v) = O(u, v) + B(u, v)\frac{N}{|N|}$$

Now differentiating this equation gives:

$$O'_u = O_u + B_u\frac{N}{|N|} + B\left(\frac{N}{|N|}\right)_u$$

$$O'_v = O_v + B_v\frac{N}{|N|} + B\left(\frac{N}{|N|}\right)_v$$

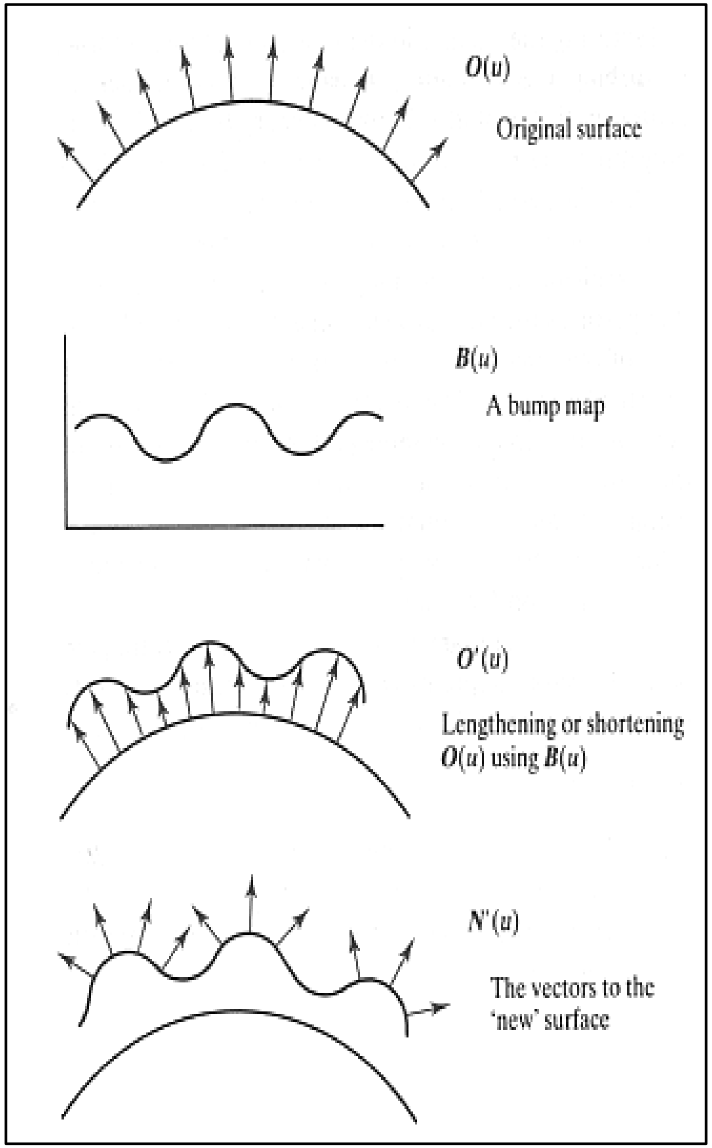
If B is small (that is, the bump map displacement function is small compared with its spatial extent) the last term in each equation can be ignored and

$$N'(u, v) = O_u \times O_v + B_u\left(\frac{N}{|N|} \times O_v\right) + B_v\left(O_u \times \frac{N}{|N|}\right) + B_u B_v\left(\frac{N \times N}{|N|^2}\right)$$

The first term is the normal to the surface and the last term is zero, giving:

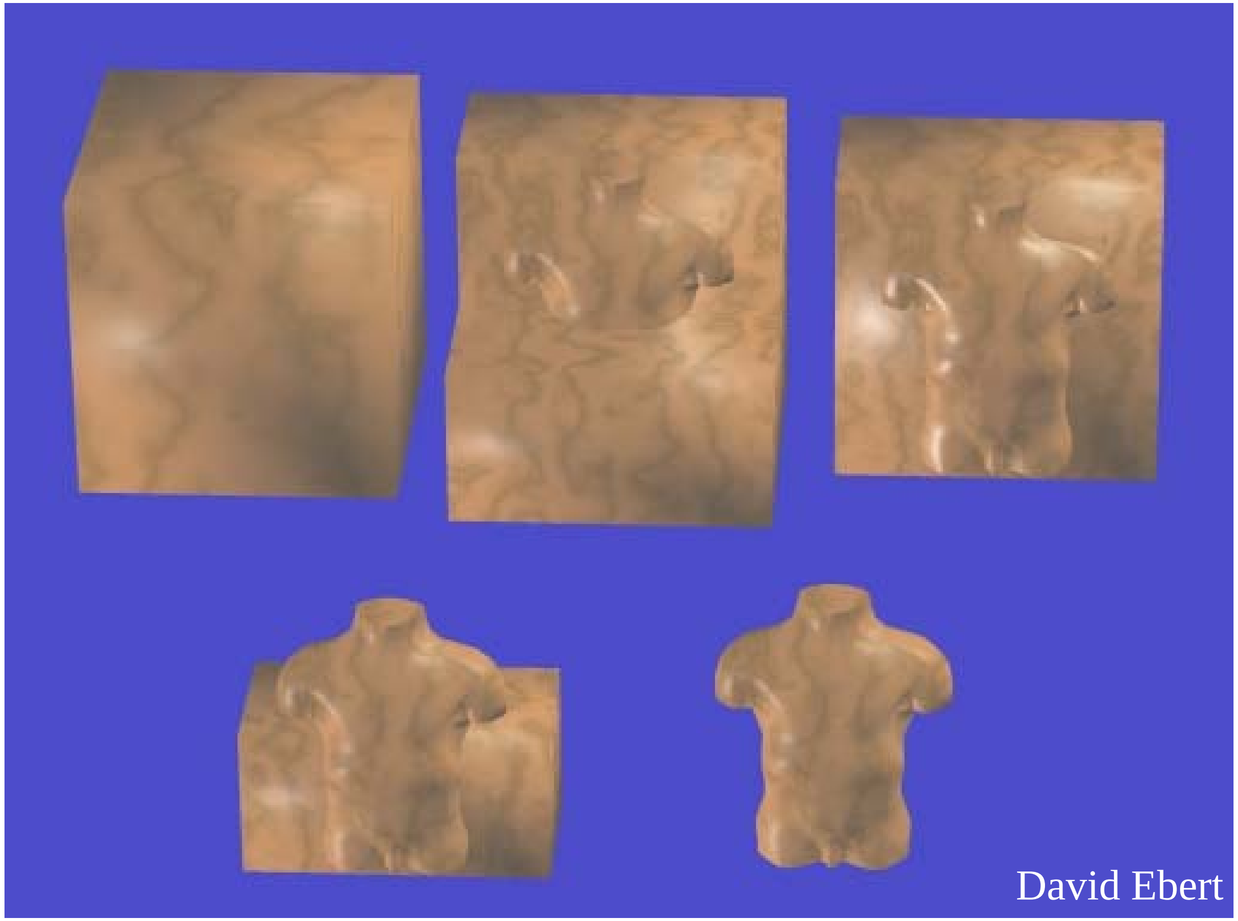
$$D = B_u(N \times O_v) - B_v(N \times O_u)$$

$N' = N + D$



3-D Textures

- **“Carving object shape out of material block”**



Texture Examples

- **Solid 3D textures (wood, marble)**
- **Bump map (middle)**

RenderMan Companion

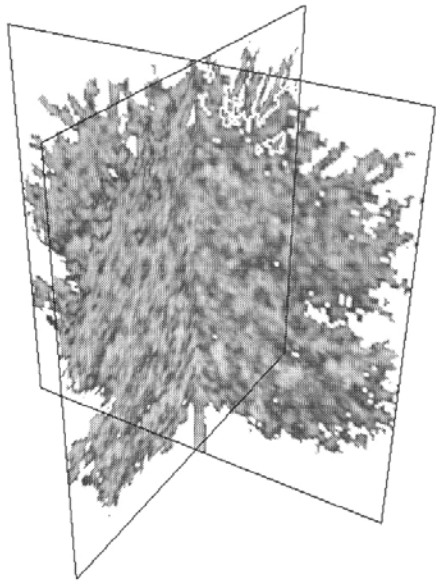
Texture Examples

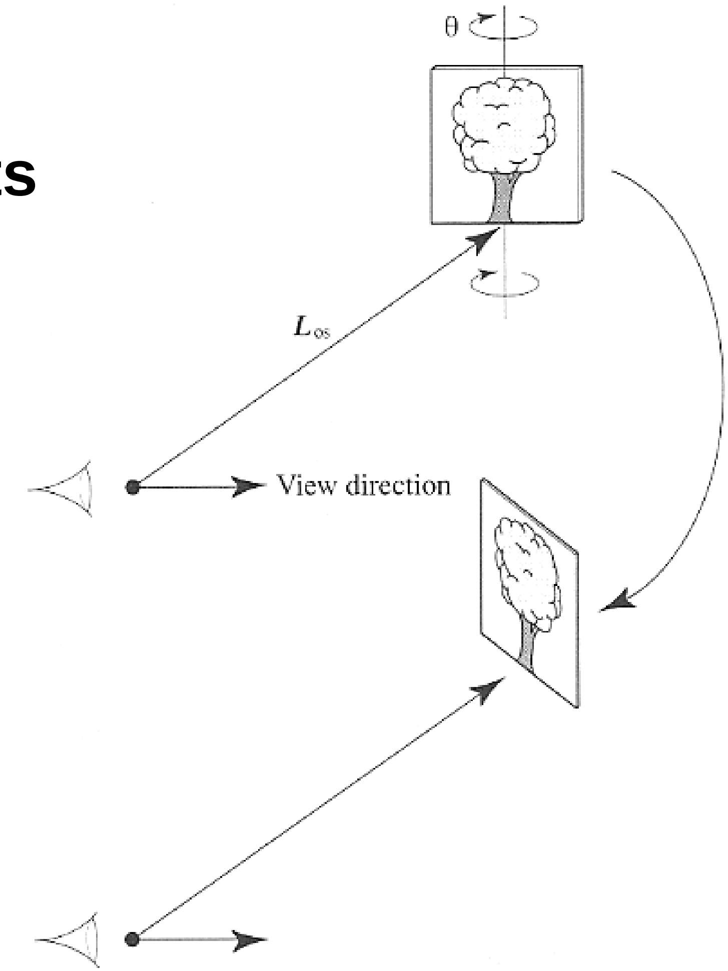
- **Complex optical effects**
  - Combination of multiple textures

RenderMan Companion

Billboards

- **Single textured polygons**
  - Often with transparency texture
- **Rotates, always facing viewer**
- **Used for rendering distant objects**
- **Best results if approximately radially or spherically symmetric**





Procedural Methods

Texture Maps vs. Procedural Textures

- **Texture maps (photos, simulations, videos, ...)**
  - Simple acquisition
  - Illumination „frozen“ during acquisition
  - Limited resolution, aliasing
  - High memory requirements
  - Mapping issues
- **Procedural textures**
  - Non-trivial programming
  - Flexibility & parametric control
  - Unlimited resolution
  - Anti-aliasing possible
  - Low memory requirements
  - Low-cost visual complexity
  - Can adapt to arbitrary geometry

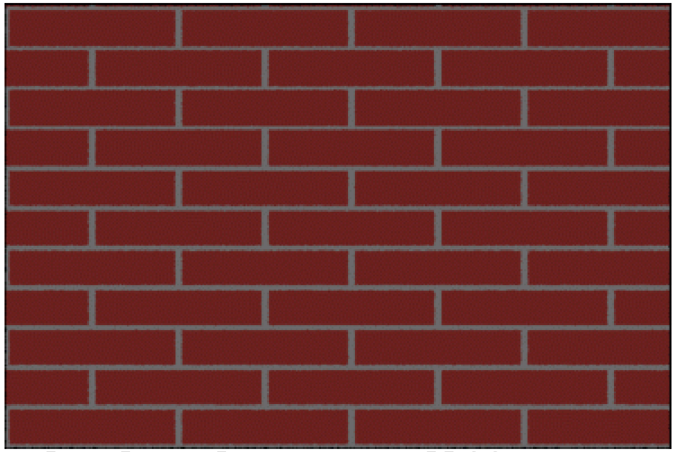
Ken Perlin

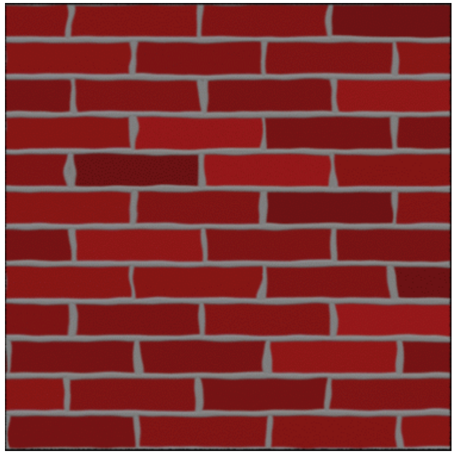
Procedural Textures

- **Function of some shading parameter, e.g.**
  - world space, texture coordinates, ...
- **Texturing: evaluation of function on object surface**
  - Ray tracing: At intersection point with surface
- **Observation: Textures of natural objects**
  - Similarity between patches at different locations
    - Repetitiveness, coherence (e.g. skin of a tiger)
  - Similarity on different resolution scales
    - Self-similarity
  - But never completely identical
    - Additional disturbances, turbulence, noise
- **Goal: Generic procedural texture function**
  - Mimics statistical properties of natural textures
  - Purely empirical approach
    - Looks convincing, but has nothing to do with material's physics

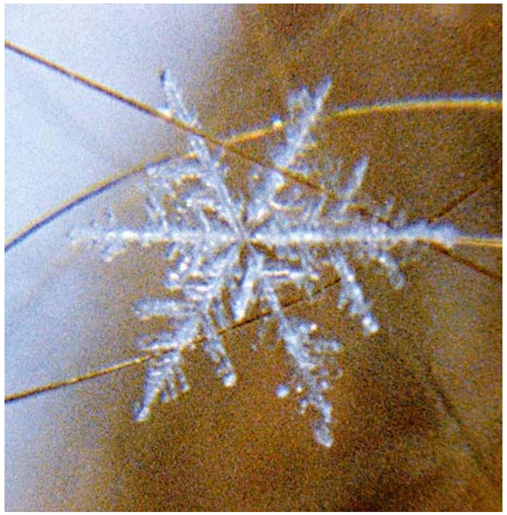
Texture Examples

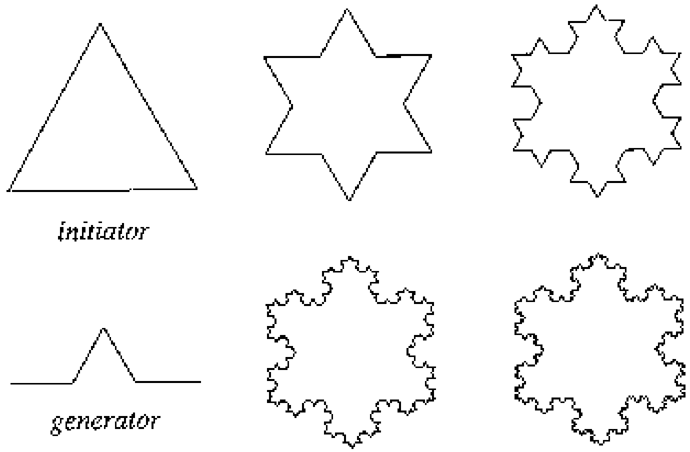
- **Translational similarity**





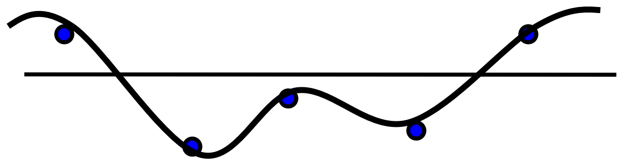
- **Similarity on different scales**



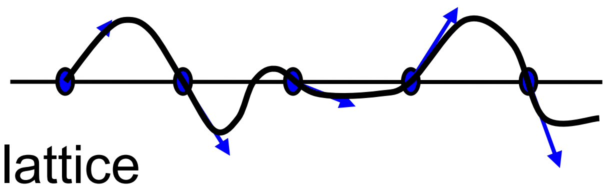


3D / Solid Noise: Perlin Noise

- **Noise(x,y,z)**
  - Statistical invariance under rotation
  - Statistical invariance under translation
  - Roughly one specific frequency



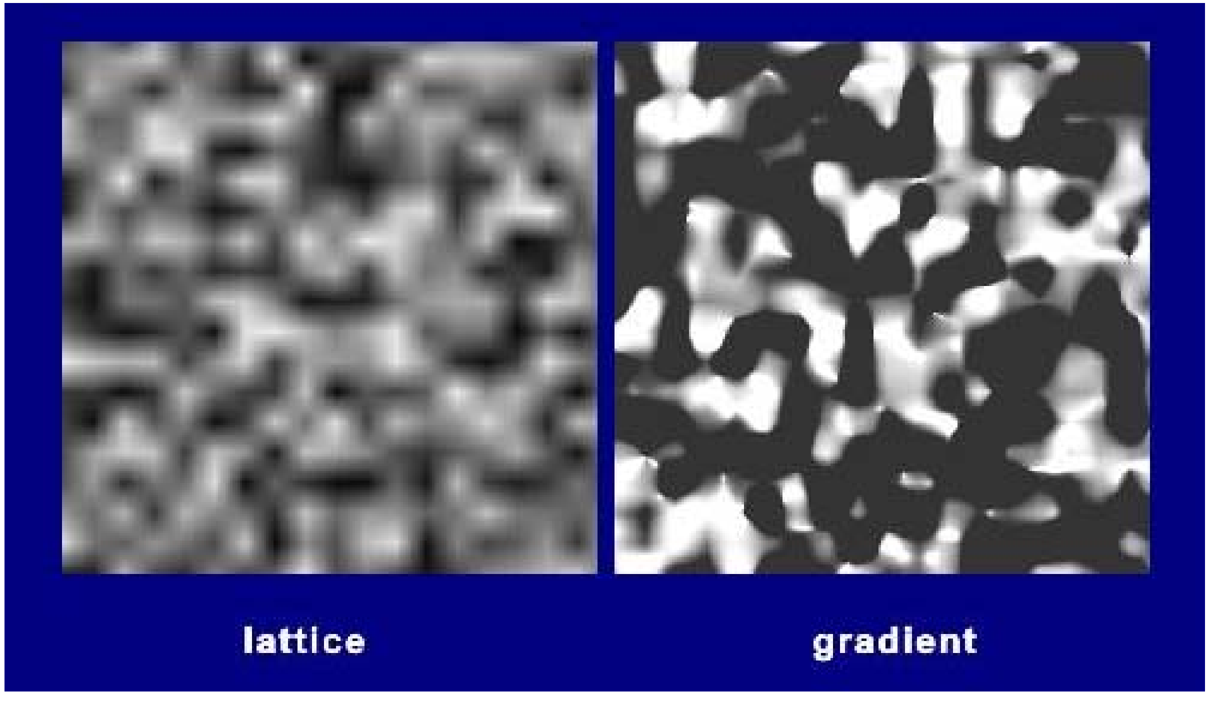
- **Integer lattice (i,j,k)**
  - Value noise: Random number at lattice
    - Look-up table or hashing function into hash map
  - Gradient lattice noise
    - Random (hashed) gradient vectors
  - Fixed fundamental frequency of ~1 Hz over lattice



- **Evaluation at (x,y,z)**
  - Tri-linear interpolation
  - Cubic interpolation (Hermite spline → later)
- **Unlimited domain due to lattice and hashing**
- **Also see**
  - **http://www.cs.cmu.edu/~mzucker/code/perlin-noise-math-faq.html**

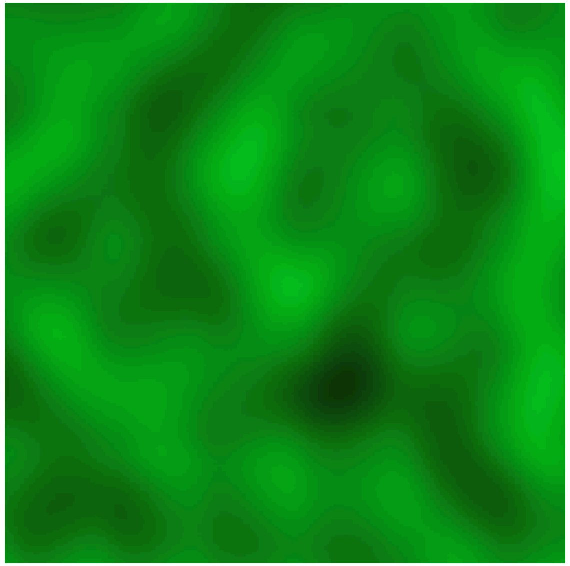
Gradient vs. Value Noise

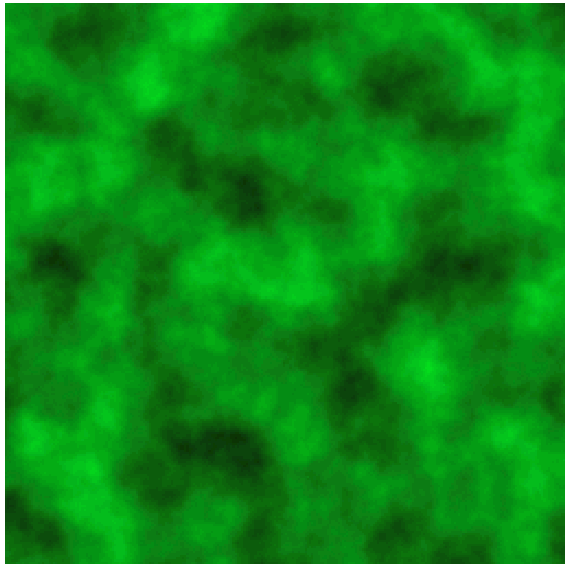
- **Gradient noise better than value noise**
  - Less regularity artifacts
  - More high frequencies in noise spectrum
  - Even tri-linear interpolation produces good results



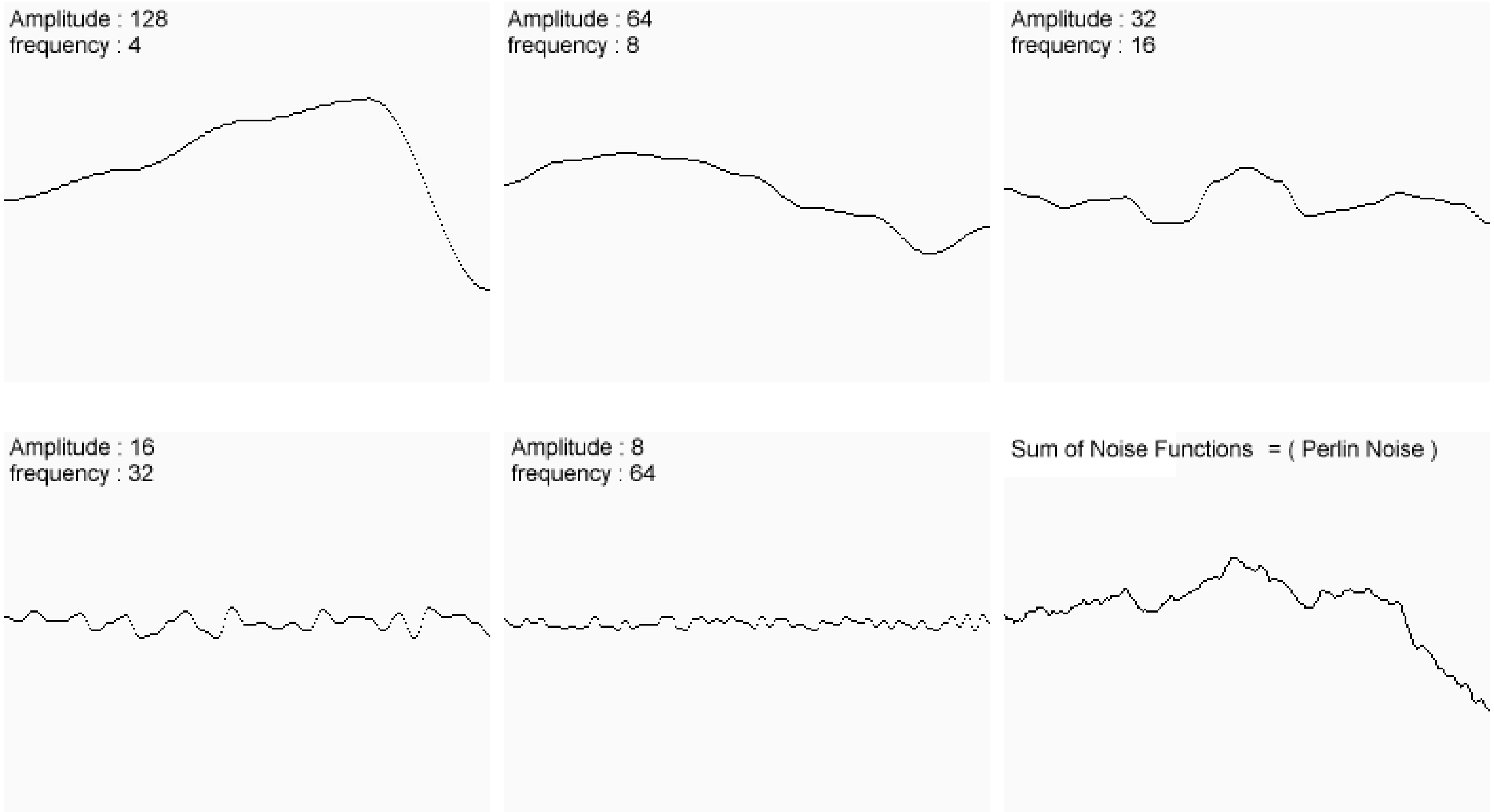
Turbulence Function

- **Noise function**
  - "White" frequency spectrum
- **Natural textures**
  - Decreasing power spectrum towards high frequencies
- **Turbulence from noise**
  - $\text{Turbulence}(x) = \sum_{i=0}^{k} \text{abs}(\text{noise}(2^i x) / p^i)$
  - persistence p typically p=2
  - Summation truncation
    - $1/2^{k+1}$ < size of one pixel (band limit)
  - 1. Term: noise(x)
  - 2. Term: noise(2x)/2
  - …
  - Power spectrum: 1/f
  - (Brownian motion has $1/f^2$)

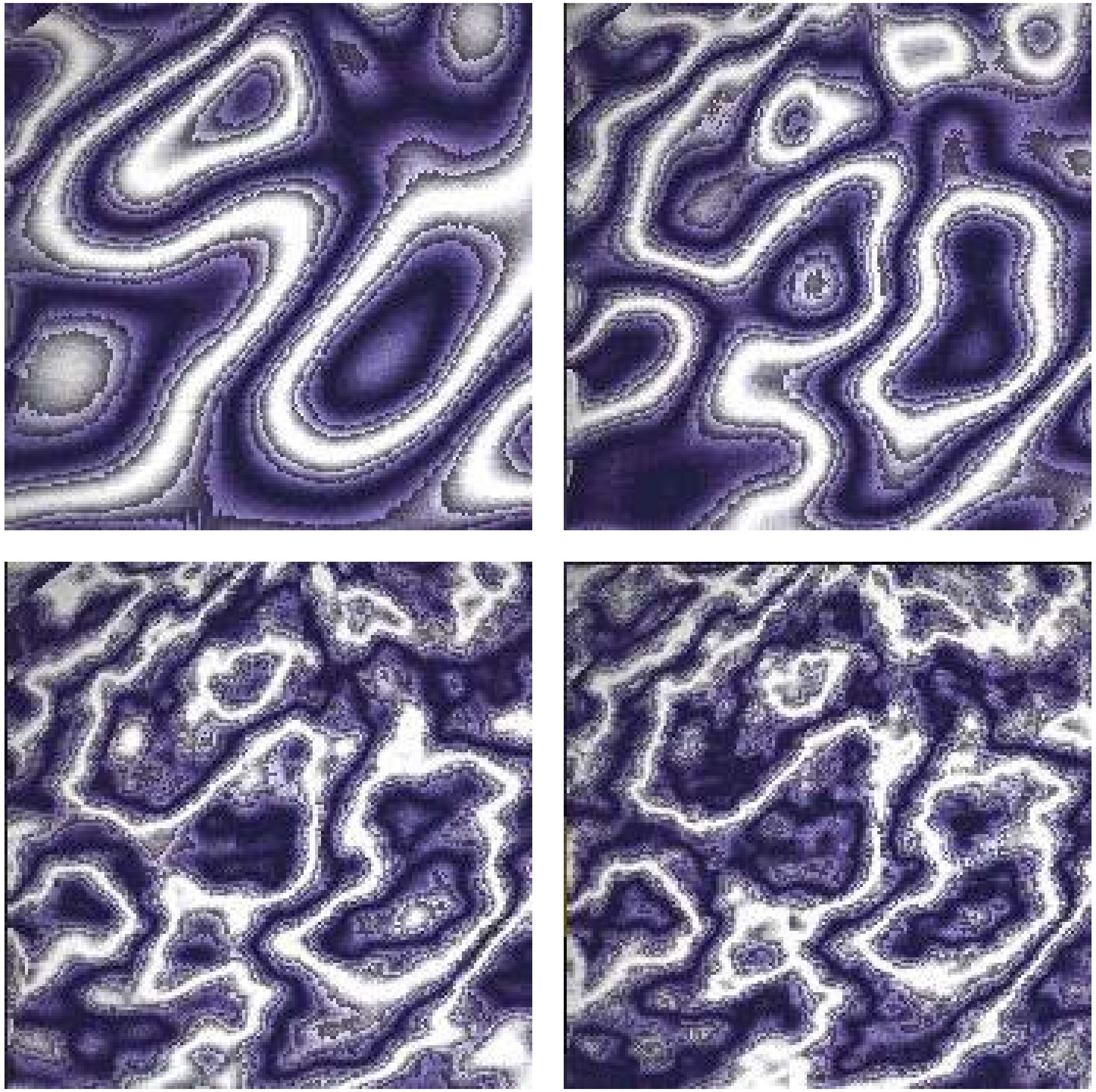




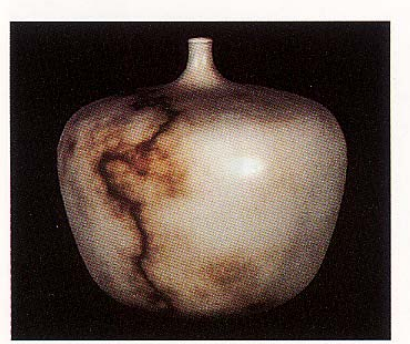
Synthesis of Turbulence (1D)



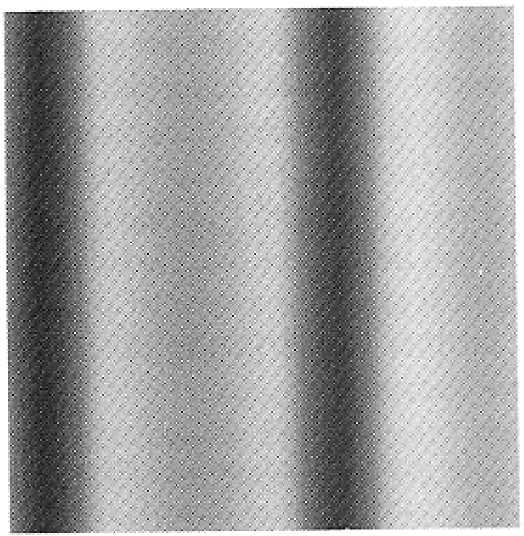
Synthesis of Turbulence (2D)

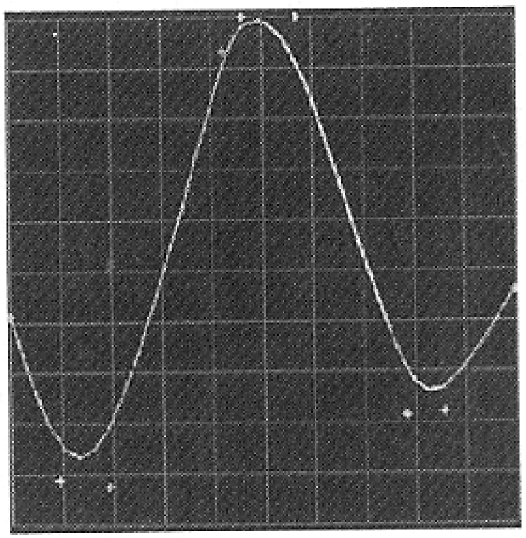


Example: Marble Texture Function



- **Overall structure: alternating layers of white and colored marble**
  - $f_{marble}(x,y,z)$:=marble_color(sin(x))
  - marble_color : transfer function (see lower left)
- **Realistic appearance: simulated turbulence**
  - $f_{marble}(x,y,z)$:=marble_color(sin(x+turbulence(x,y,z)))
- **Moving object: turbulence function also transformed**





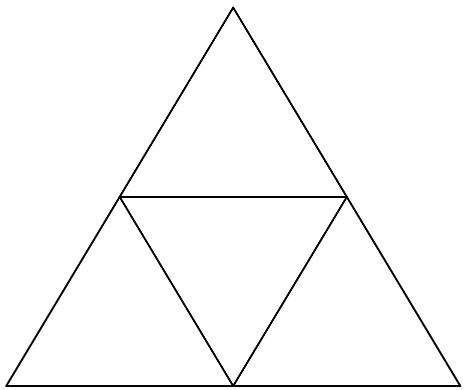
Further Procedural Texturing Applications

- **Bark**
  - Turbulated sawtooth function
  - Bump mapping
- **Clouds**
  - White blobs
  - Turbulated transparency along edge
  - Transparency mapping
- **Animation**
  - Vary procedural texture function's parameters over time

Fractal Landscapes

- **Procedural generation of geometry**
- **Complex geometry at virtually no memory cost**
  - Can be difficult to ray trace !!

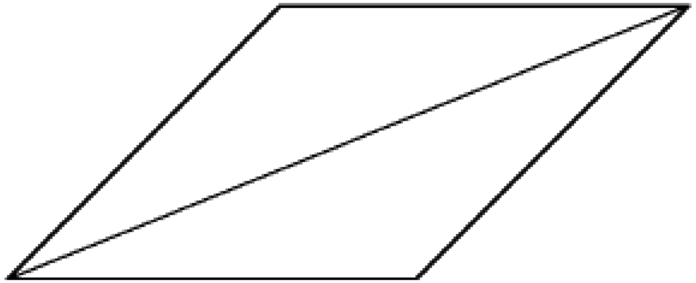
Fractal Landscapes

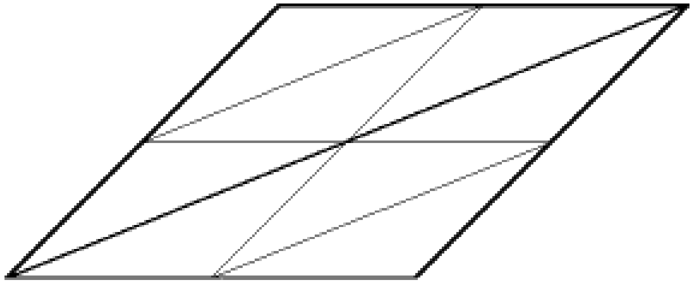


- **Coarse triangle mesh approximation**
- **1:4 triangle subdivision**
  - Vertex insertion at edge-midpoints
- **New vertex perturbation**
  - Random displacement along normal
  - Scale of perturbation depends on subdivision level
    - Decreasing power spectrum
    - Parameter models surface roughness
- **Recursive subdivision**
  - Level of detail (LOD) determined by # subdivisions
- **All done inside renderer !**
  - LOD generated locally when/where needed (bounding box test)
  - Minimal I/O cost (coarse mesh only)

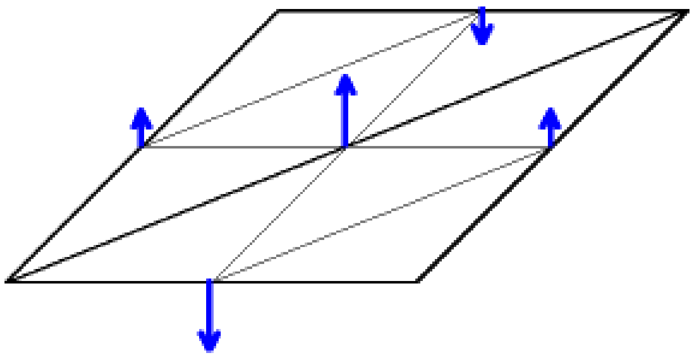
Fractal Landscapes

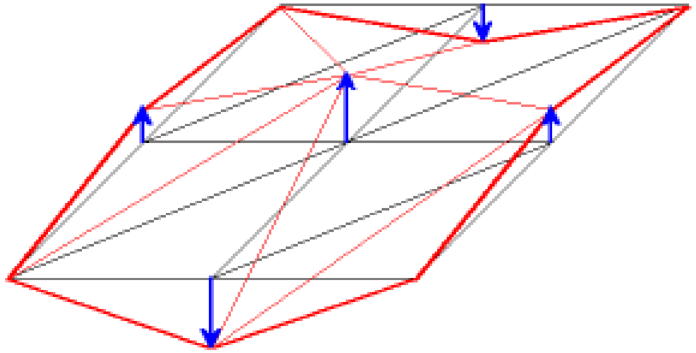
- **Triangle subdivision**
  - Insert new vertices at edge midpoints
  - 1:4 triangle subdivision





- **Vertex displacement**
  - Along original triangle normal

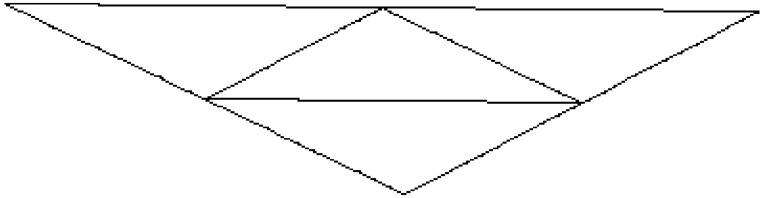


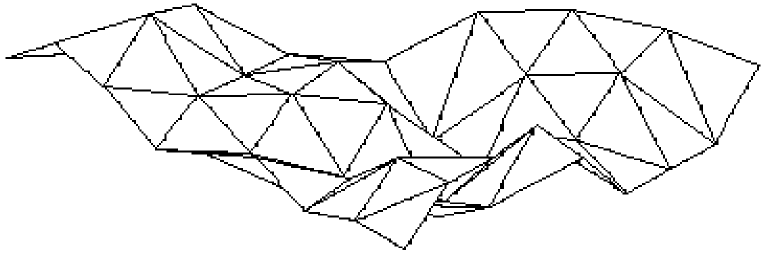


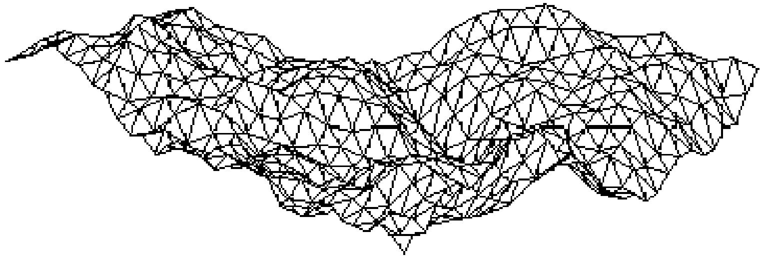
Courtesy http://www.uni-paderborn.de/SFB376/projects/a2/zBufferMerging/

Fractal Landscape Generation

- **Base mesh**
- **Repeated subdivision & vertex displacement**
- **Shading**
- **+ Water surface**
- **+ Fog**
- **+ …**







Courtesy http://www.uwp.edu/academic/computer.science/morris.csci/CS.320/Week.11/Ch11b.www/Ch11b.html

Fractal Landscape Ray Tracing

- **Fractal terrain generated on-the-fly**
- **Problem: where is the ray-surface interaction ?**
  – Triangle mesh not a-priori known
- **Solution: bounding boxes**
  – Maximum possible bounding box around each triangle
  – Decreasing displacement amplitude: finite bounding box
- **Algorithm**
  – Intersect ray with bounding box
  – Subdivide corresponding triangle
  – Compute bounding boxes of 4 new triangles
  – Test against 4 new bounding boxes
  – Iterate until termination criterion fulfilled (LOD / pixel size)