# Computer Graphics
## - Rasterization & Clipping -

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# Overview

- **Last lecture:**
  - Camera Transformations
  - Projection

- **Today:**
  - Rasterization of Lines and Triangles
  - Clipping

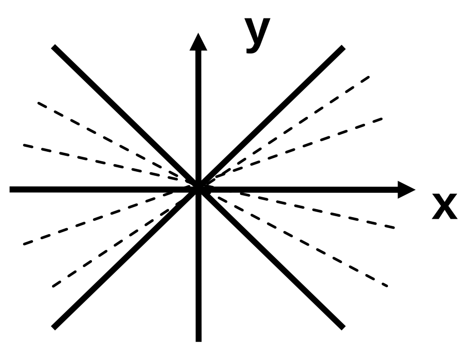
- **Next lecture:**
  - OpenGL

# Rasterization

- **Definition**
  - Given a primitive (usually 2D lines, circles, polygons), specify which pixels on a raster display are covered by this primitive
  - Extension: specify what part of a pixel is covered
    → filtering & anti-aliasing
- **OpenGL lecture**
  - From an application programmer‘s point of view
- **This lecture**
  - From a graphics package implementer‘s point of view
- **Usages of rasterization in practice**
  - 2D-raster graphics
    - e.g. Postscript
  - 3D-raster graphics
  - 3D volume modeling and rendering
  - Volume operations (CSG operations, collision detection)
  - Space subdivision
    - Construction and traversing

# Rasterization

- **Assumption**
  - Pixels are sample ***points*** on a 2D-integer-grid
    - OpenGL: integer-coordinate bottom left; X11, Foley: in the center
  - Simple raster operations
    - Just setting pixel values
      - Antialiasing later
  - Endpoints at pixel coordinates
    - simple generalization with fixed point
  - Limiting to lines with gradient $|m| \leq 1$
    - Separate handling of horizontal and vertical lines
    - Otherwise exchange of x & y: $|1/m| \leq 1$
  - Line size is one pixel
    - $|m| \leq 1$: 1 pixel per column (X-driving axis)
    - $|m| > 1$: 1 pixel per row (Y-driving axis)



# Lines: As Functions

- **Specification**
  - Initial and end points: $(x_0, y_0)$, $(x_e, y_e)$
  - Functional form: **y = mx + B with m = dy/dx**
- **Goal**
  - Find pixels whose distance to the line is smallest
- **Brute-Force-Algorithm**
  - It is assumed that +X is the driving axis

```
for xi = x0 to xe
  yi = m * xi + B
  setpixel(xi, Round(yi))    // Round(yi)=Floor(yi+0.5)
```

- **Comments**
  - Variables m and $y_i$ must be calculated in floating-point
  - Expensive operations per pixel (e.g. in HW)

# Lines: DDA

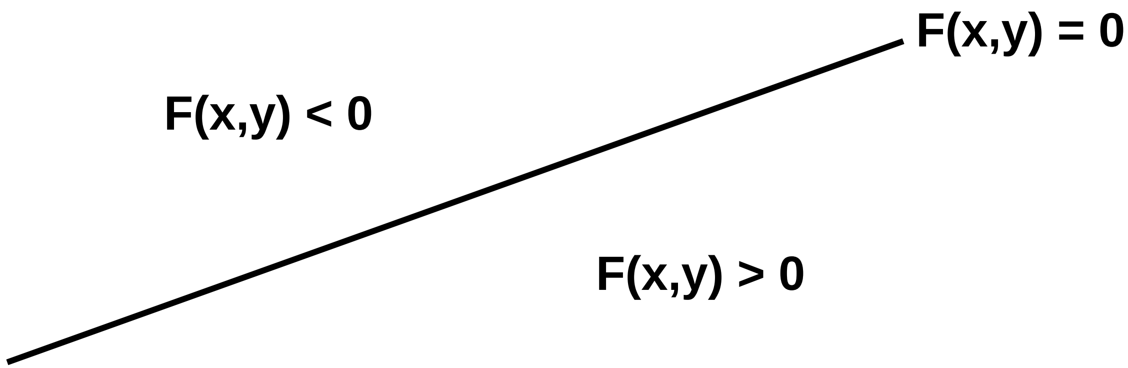
- **DDA: Digital Differential Analyzer**
  - Origin of solvers for simple incremental differential equations (the Euler method)
    - Per step in time: `x´ = x + dx/dt, y´ = y + dy/dt`
- **Incremental algorithm**
  - Per pixel
    - $x_{i+1} = x_i + 1$
    - $y_{i+1} = m\ (x_i + 1) + B = y_i + m$
    - $\mathtt{setpixel}(x_{i+1},\ \mathtt{Round}(y_{i+1}))$
- **Remark**
  - Utilization of line coherence trough incremental calculation
    - Avoid the costly multiplication
  - Accumulates error over the length of the line
  - Floating point calculations may be moved to fixed point
    - Must control accuracy of fixed point representation

# Lines: Bresenham (´63)

- **DDA analysis**
  - Critical point: decision by rounding up or down
  - Integer-based decision through implicit functions
- **Implicit version**

$$F(x,y) = dy\,x - dx\,y + dx\,B = 0$$

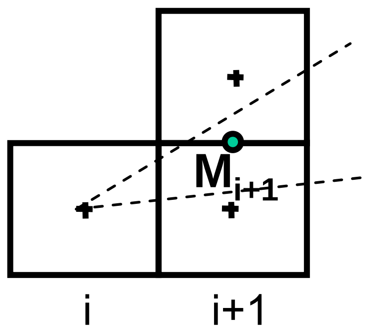
$$F(x,y) = ax + by + c = 0 \quad \text{where } a = dy, b = -dx, c = Bdx$$



# Lines: Bresenham

- **Decision variable (the midpoint formulation)**
  - Measures the vertical distance of midpoint from line:

    $d_{i+1} = F(M_{i+1}) = F(x_i+1,\ y_i+1/2) = a(x_i+1) + b(y_i+1/2) + c$



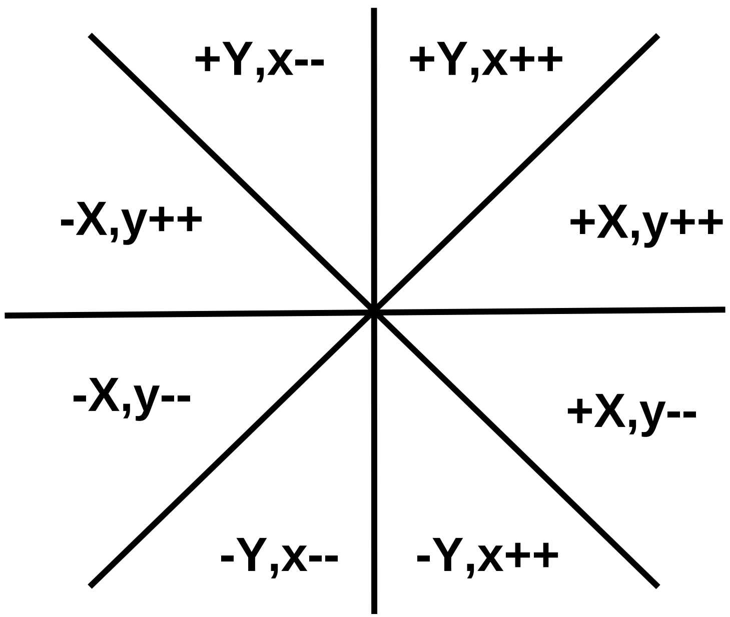

- **Preparations for the next pixel**
  - if ($d_i \leq 0$)
    - $d_{i+1} = d_i + a = d_i + dy$ // incremental calculation
  - else
    - $d_{i+1} = d_i + a + b = d_i + dy – dx$
    - y= y + 1
  - x = x +1

# Lines: Integer Bresenham

- **Initialization**
  - $d_{start} = F(x_0+1, y_0+1/2) = a(x_0+1) + b(y_0+1/2) + c$
    $= ax_0 + by_0 + c + a + b/2 = F(x_0, y_0) + a + b/2$
    $= a + b/2$
  - Because $F(x_0, y_0)$ is zero by definition (line goes through end point)
    - Pixel is always set
- **Elimination of fractions**
  - Any positive scale factor maintains the sign of F(x,y)
  - $F(x_0, y_0) = 2(ax_0 + by_0 + c) \rightarrow d_{start} = 2a + b$
- **Observation:**
  - When the start and end points have integer coordinates then **b= dx** and **a= -dy** have also integer values
  - Floating point computation can be eliminated
  - No accumulated error

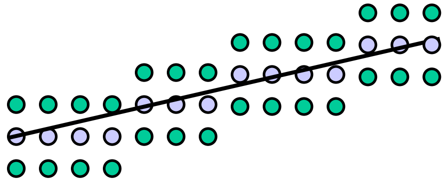
# Lines: Arbitrary Directions

- **8 different cases**
  - Driving (active) axis: ±X or ±Y
  - Increment/decrement of y or x, respectively



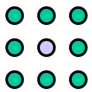
# Thick Lines

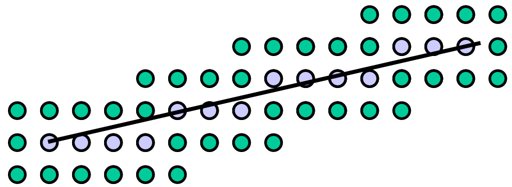
- **Pixel replication**



  - Problems with even-numbered widths,
  - Varying intensity of a line as a function of slope

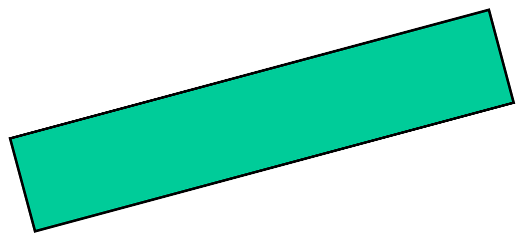
- **The moving pen**





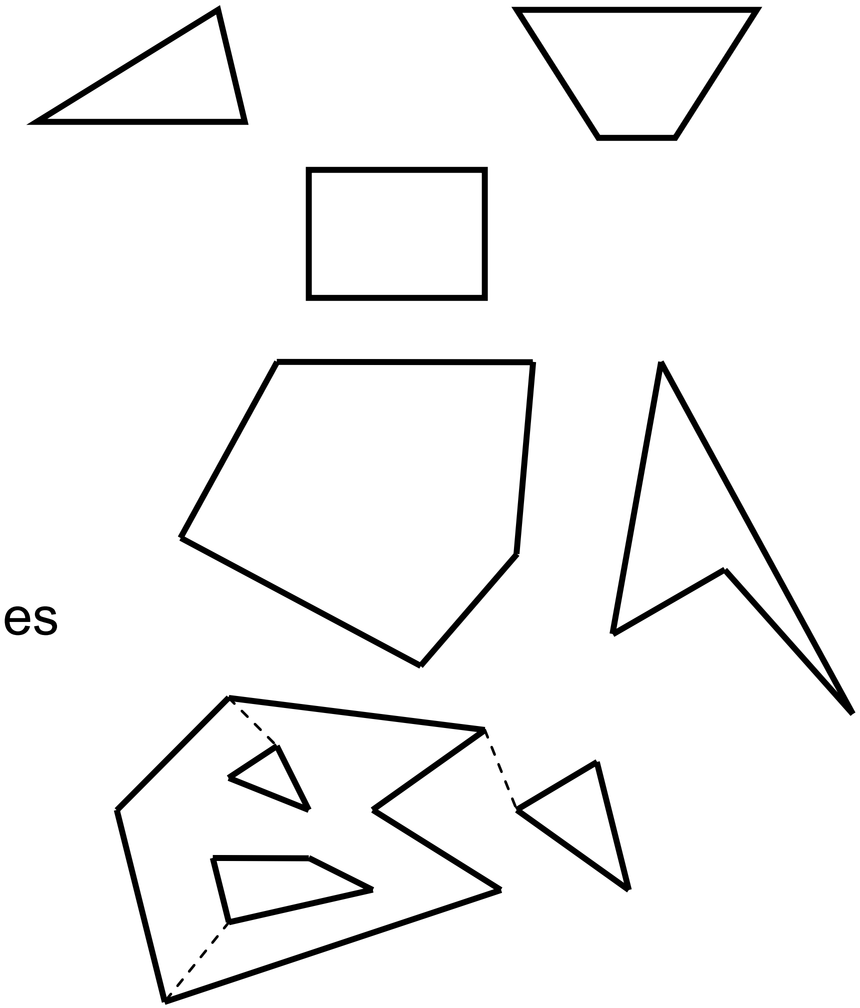
  - For some pen footprints the thickness of a line might change as a function of its slope
  - Should be as „round“ as possible

- **Filling areas between boundaries**



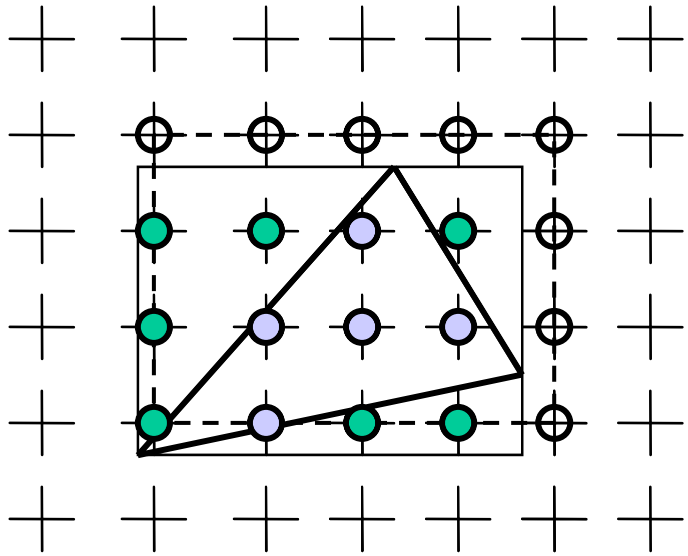
# Reminder: Polygons

- **Types**
  - Triangles
  - Trapezoids
  - Rectangles
  - Convex polygons
  - Concave polygons
  - Arbitrary polygons
    - Holes
    - Non-coherent
- **Two approaches**
  - Polygon tessellation into triangles
    - edge-flags for internal edges
  - Direct scan-conversion



# Triangle Rasterization

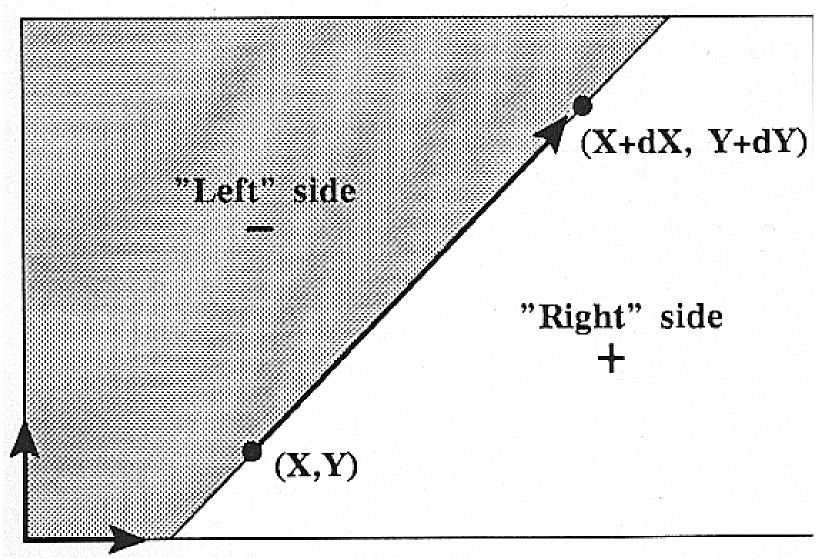
```
Raster3_box(vertex v[3])
{
  int x, y;
  bbox b;
  bound3(v, &b);
  for (y= b.ymin; y < b.ymax; y++)
    for (x= b.xmin; x < b.xmax; x++)
      if (inside(v, x, y))
        fragment(x,y);
}
```



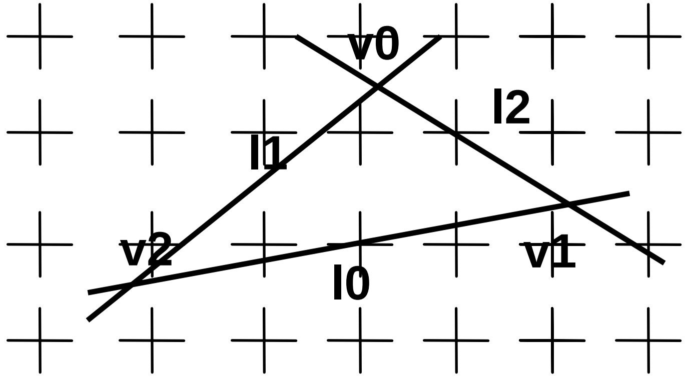
- **Brute-Force algorithm**
- **Possible approaches for dealing with scissoring**
  - Iterate over intersection of scissor box and bounding box, then test against triangle (as above)
  - Iterate over triangle, then test against scissor box

# Incremental Rasterization

- **Approach**
  - Implicit edge functions to describe the triangle $F_i(x,y) = ax+by+c$
  - Point inside triangle, if every $F_i(x,y) \le 0$
  - Incremental evaluation of the linear function F by adding a or b



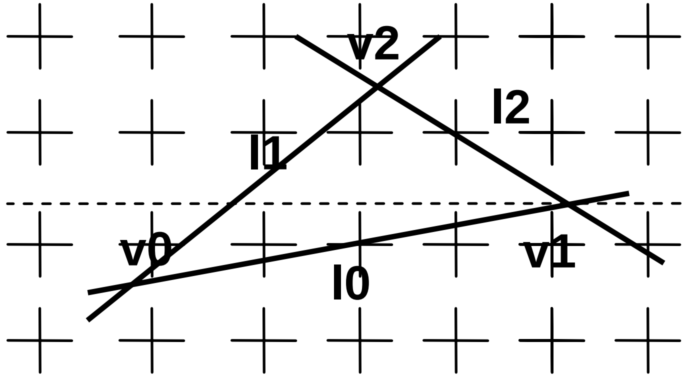
# Incremental Rasterization




```
Raster3_incr(vertex v[3])
{
  edge l0, l1, l2;
  value d0, d1, d2;
  bbox b;
  bound3(v, &b);
  mkedge(v[0],v[1],&l2);
  mkedge(v[1],v[2],&l0);
  mkedge(v[2],v[0],&l1);

  d0 = l0.a * b.xmin + l0.b * b.ymin + l0.c;
  d1 = l1.a * b.xmin + l1.b * b.ymin + l1.c;
  d2 = l2.a * b.xmin + l2.b * b.ymin + l2.c;

  for( y=b.ymin; y<b.ymax, y++ ) {
    for( x=b.xmin; x<b.xmax, x++ ) {
      if( d0<=0 && d1<=0 && d2<=0 ) fragment(x,y);
      d0 += l0.a; d1 += l1.a; d2 += l2.a;
    }
    d0 += l0.a * (b.xmin - b.xmax) + l0.b; . . . }
}
```

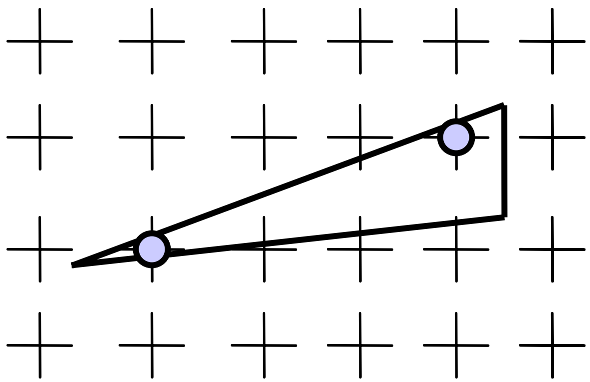
# Triangle Scan Conversion

```
Raster3_scan(vert v[3])
{
  int y;
  edge l, r;
  value ybot, ymid, ytop;

  ybot = ceil(v[0].y);
  ymid = ceil(v[1].y);
  ytop = ceil(v[2].y);

  differencey(v[0],v[2],&l,ybot);
  differencey(v[0],v[1],&r,ybot);

  for( y=ybot; y<ymid; y++ ) {
    scanx(l,r,y);
    l.x += l.dxdy; r.x += r.dxdy;
  }
  differencey(v[1],v[2],&r,ymid);
  for( y=ymid; y<ytop; y++ ) {
    scanx(l,r,y);
    l.x += l.dxdy; r.x += r.dxdy;
  }
}
```

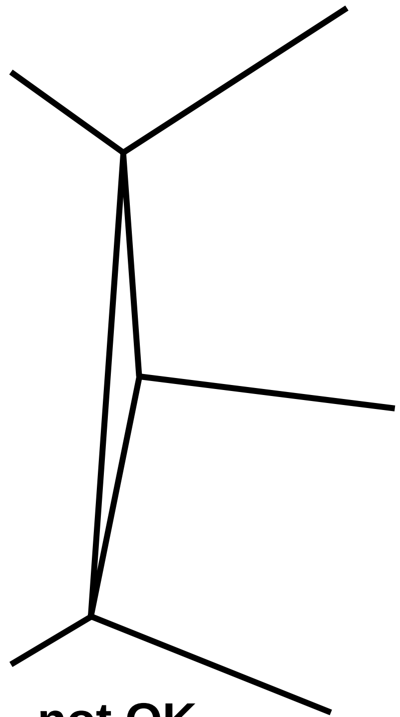



```
differencey(vert a, vert b,
            edge* e, int y) {
   e->dxdy=(b.x-a.x)/(b.y-a.y);
   e->x=a.x+(y-a.y)*e->dxdy;
}

scanx(edge l, edge r, int y){
  lx= ceil(l.x);
  rx= ceil(r.x);
  for (x=lx; x < rx; x++)
    // ggf. Scissor-Test
    fragment(x,y);
}
```

# Gap and T-Vertices



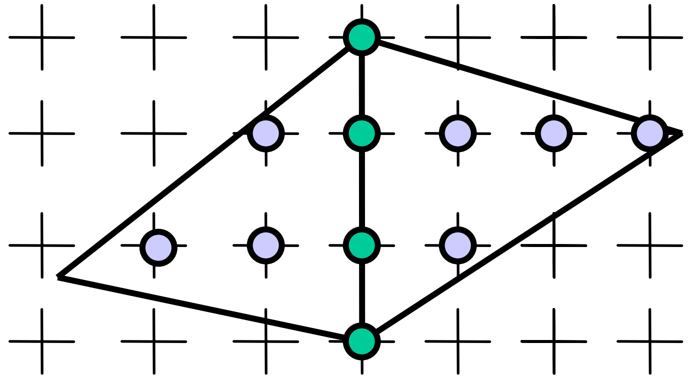
**OK**



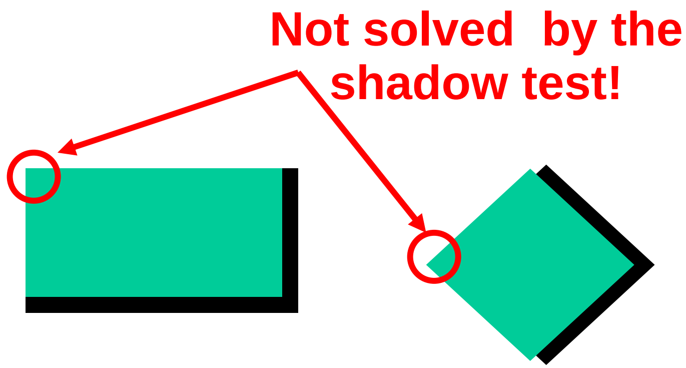
**not OK**
**Modeling problem**

# Problem on Edges

- **Singularity**
  - If term d= ax+by+c = 0
  - Multiple pixels for d <= 0:
    - Problem with some algorithms
      - transparency, XOR, CSG, ...
  - Missing pixels for d < 0:



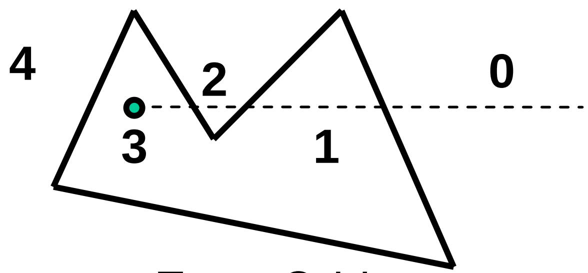
- **Partial solution: shadow test**
  - Pixels are not drawn on the right and bottom edges
  - Pixels are drawn on the left and upper edges



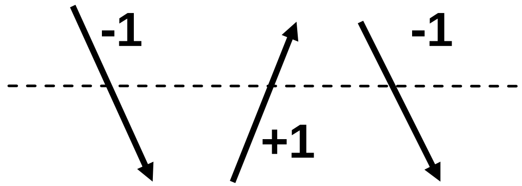

```
inside(value d, value a, value b) { // ax + by + c = 0
  return (d < 0) || (d == 0 && !shadow(a,b));
shadow(value a, value b) {
  return (a > 0) || (a == 0 && b > 0) }
```

# Inside-Outside Tests

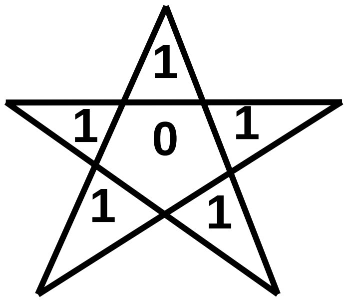
- **What is the interior of a polygon?**
  - Jordan Curve Theorem
    - Any continuous *simple* closed curve in the plane, separates the plane into two disjoint regions, the inside and the outside, one of which is bounded.
  - Even-odd rule (odd parity rule)
    - Counting the number of edge crossings with a ray starting at the queried point **P**
    - Inside, if the number of crossings is odd
  - Nonzero winding number rule
    - Signed intersections with a ray
    - Inside, if the number is not equal to zero
  - Differences only in the case of non-simple curves (self-intersection)



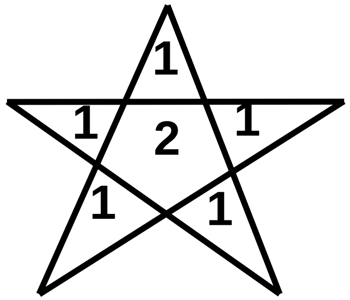

Even-Odd




Winding



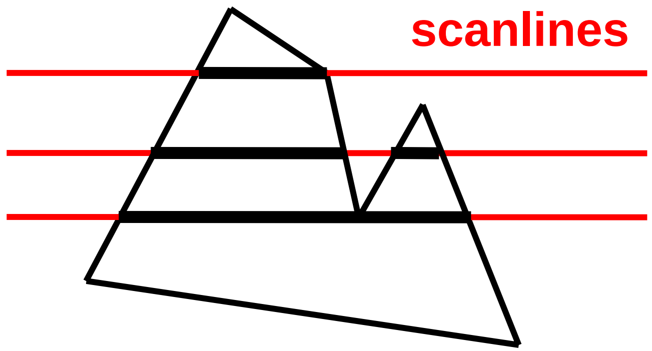

Even-Odd

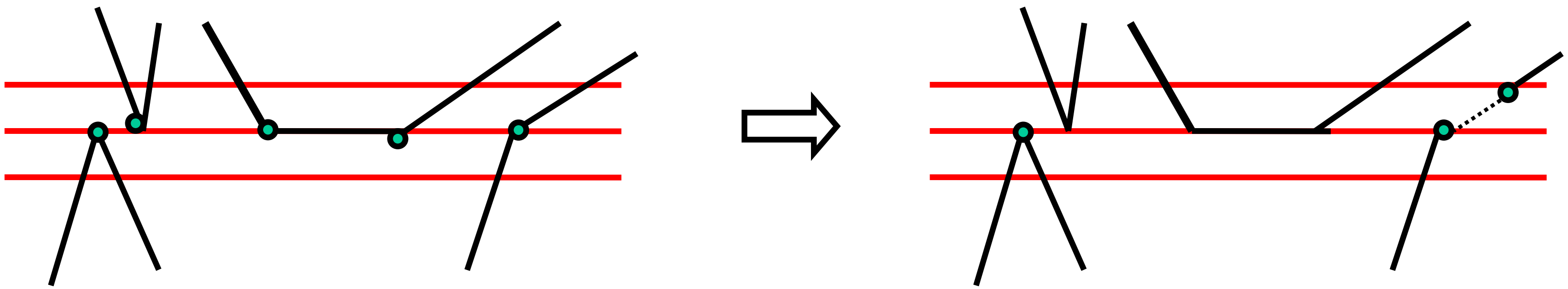



Winding

# Polygon Scan-Conversion

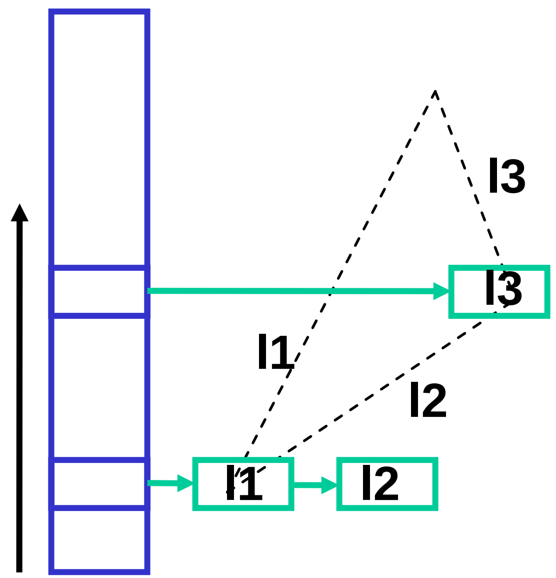
- **Special cases**
  - Edge along a scanline
    - shadow test:
      - draw the upper edge
      - skip the bottom edge
  - Vertex at a scanline
    - If edges sharing the vertex are located on the **same side** of the scanline – properly handled
    - If edges sharing the vertex are located on the **opposite sides** of the scanline – one edge (bottom) is shortened: the $y_{min}/y_{max}$ rule
    - Complex situations
      - In general use randomization: Offset point by ε



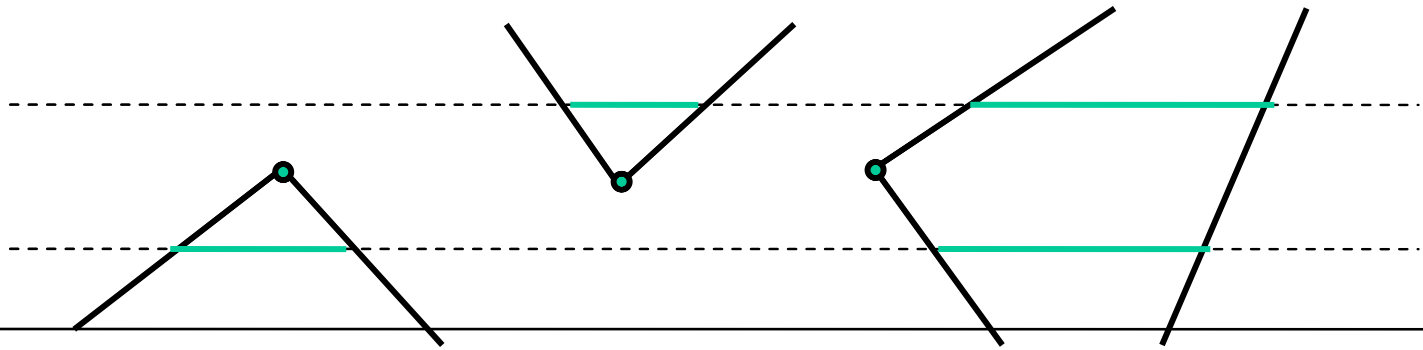
# Scanline Algorithm

- **Incremental algorithm**
  - Use the odd-even parity rule to detemine that a point is inside a polygon
  - Utilization of coherence
    - along the edges
    - on scanlines
    - „sweepline-algorithm“
  - Edge-Table initialization :
    - Bucket sort (one bucket for each scanline)
    - Edges ordered by xmin
    - Linked list of edge-entries
      - ymax
      - xmin
      - dx/dy
      - link to triangle data



# Scanline Algorithm

- **For each scan line**
  - Update the Active-Edge-Table
    - Linked-list of entries
      - Link to edge-entries,
      - x, horizontal increment of depth, color, etc
    - Remove edges if theirs ymax is reached
    - Insert new edges (from Edge-Table)
  - Sorting
    - Incremental update of x
    - Sorting by X-coordinate of the intersection point with scanline
  - Filling the gap between pairs of entries

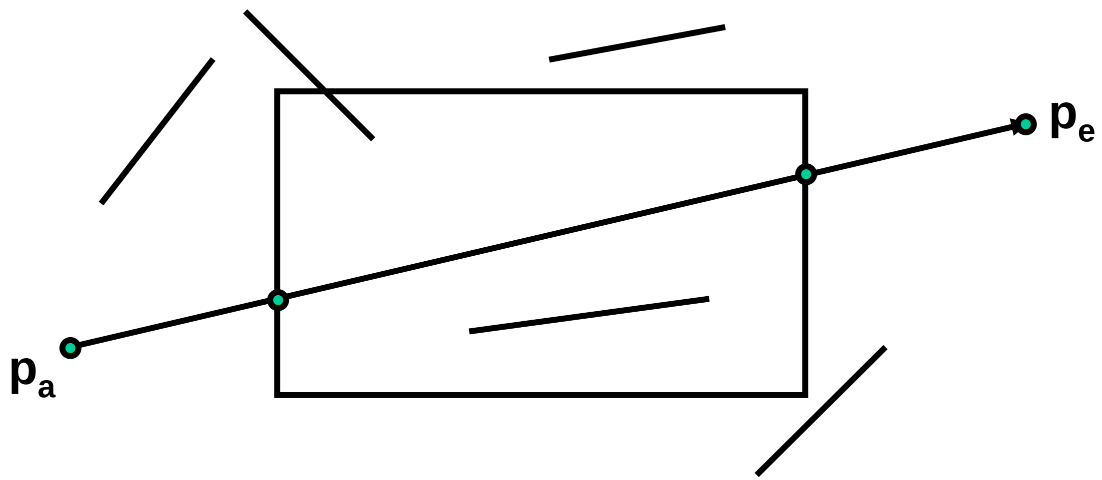


# Clipping

- **Motivation**
  - Happens after transformation from 3D to 2D
  - Many primitives will fall (partially) outside of display window
    - E.g. if standing inside a building
  - Eliminates non-visible geometry early in the pipeline
  - Must cut off parts outside the window
    - Cannot draw outside of window (e.g. plotter)
    - Outside geometry might not be representable (e.g. in fixed point)
  - Must maintain information properly
    - Drawing the clipped geometry should give the correct results
    - Type of geometry might change
      - Cutting off a vertex of a triangle produces a quadrilateral
      - Might need to be split into triangle again
    - Polygons must remain closed after clipping

# Line clipping

- **Definition Clipping:**
  - Cut off parts of objects, which lie outside/inside of a defined region.
  - Often: Clipping against a viewport (2D) or a canonical view-volume (3D)

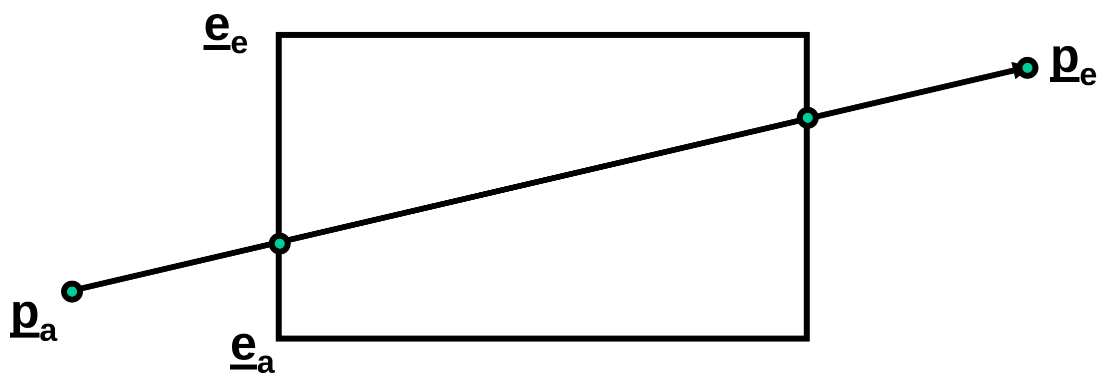


# Brute-force method

- **Brute-Force line clipping at the viewport**
  - If both points $\underline{p}_a$ and $\underline{p}_e$ are inside,
    - Accept the whole line
  - Otherwise, clip the line at each edge

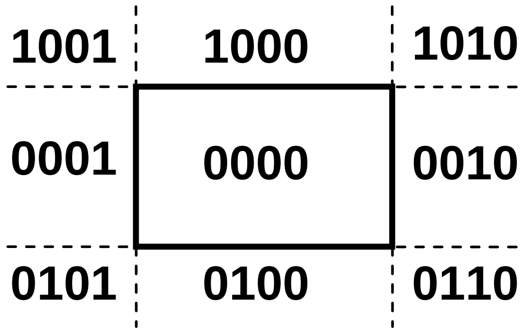
$$\underline{p} = \underline{p}_a + t_{line}(\underline{p}_e - \underline{p}_a) = \underline{e}_a + t_{edge}(\underline{e}_e - \underline{e}_a)$$

- Intersection point, if $0 \leq t_{line}, t_{edge} \leq 1$
- Pick up suitable end points from the intersection points for the line



# Cohen-Sutherland (´74)

- **Advantage: divide and conquer**
  - Efficient trivial accept and trivial reject
  - Non-trivial case: divide and test
- **Outcodes of points:**
  - Bit encoding (**o**ut**c**ode, OC)
    - Each edge defines a half space
    - Set bit, if point is outside
- **Trivial cases**
  - Trivial accept:
    - $(OC(p_a)\ \mathbf{OR}\ OC(p_e)) = 0$
  - Trivial reject:
    - $(OC(p_a)\ \mathbf{AND}\ OC(p_e)) \neq 0$
  - Edges has to be clipped to all edges where bits are set:
    - $OC(p_a)\ \mathbf{XOR}\ OC(p_e)$



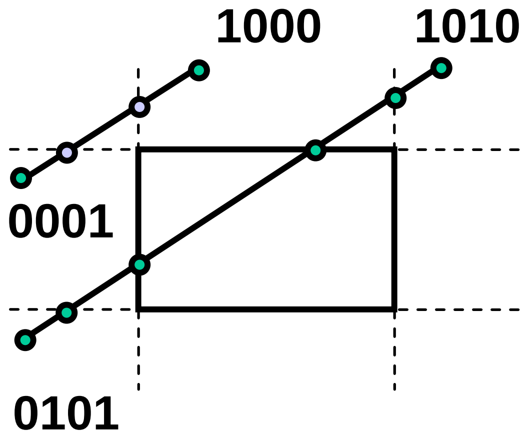

Bit order: **Top, Bottom, Right, Left**

**Viewport ($x_{min}$, $y_{min}$, $x_{max}$, $y_{max}$)**

# Cohen-Sutherland

- **Clipping**

```
... // trivial cases
for each vertex p
  oc= OC(p)
  for each edge e
    if (oc[e]) {
      p= cut(p,e);
      oc= OC(p);
    }
  Reject, if point outside
```



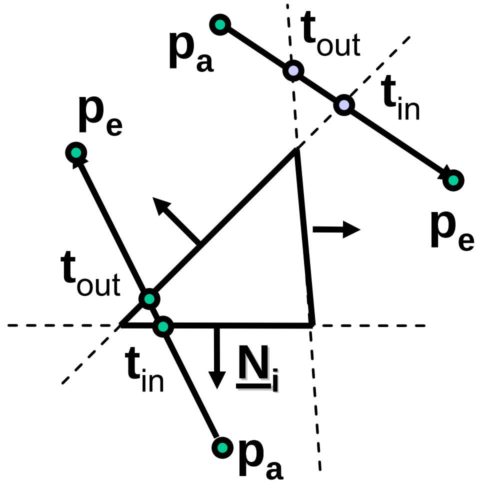

- **Intersection calculation for $x=x_{min}$**

$$\frac{y-y_a}{y_e-y_a}=\frac{x-x_a}{x_e-x_a}$$

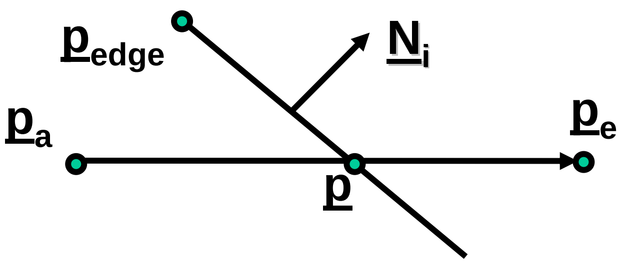
$$y=y_a+(x-x_a)\frac{y_e-y_a}{x_e-x_a}$$

# Cyrus-Beck (´78) Clipping against Polygons

- **Parametric line-clipping algorithm**
  - Only convex polygons: max. 2 intersection points
  - Use edge orientation
- **Idea:**
  - Clipping line $\underline{p}_a + t_i(\underline{p}_e - \underline{p}_a)$ with each edge
  - Intersection points sorted by parameter $t_i$
  - Select
    - $t_{in}$: entry point $((\underline{p}_e - \underline{p}_a)\cdot\underline{N}_i < 0)$ with largest $t_i$ and
    - $t_{out}$: exit point $((\underline{p}_e - \underline{p}_a)\cdot\underline{N}_i > 0)$ with smallest $t_i$
  - If $t_{out} < t_{in}$, line lies completely outside




- **Intersection calculation:**




$$\left(p - p_{edge}\right)\cdot N_i = 0$$

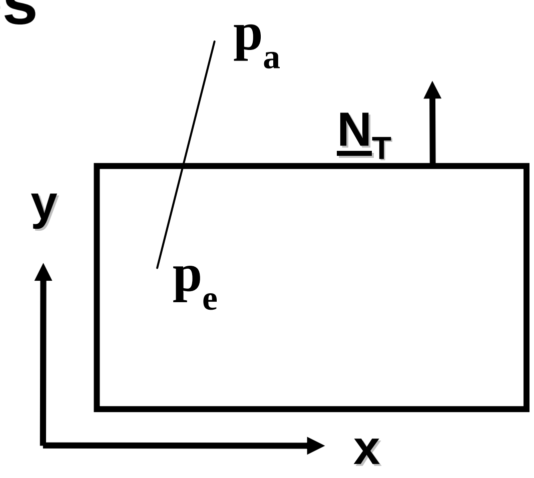
$$t_i\left(p_e - p_a\right)\cdot N_i + \left(p_a - p_{edge}\right)\cdot N_i = 0$$

$$t_i = \frac{\left(p_{edge} - p_a\right)\cdot N_i}{\left(p_e - p_a\right)\cdot N_i}$$

# Liang-Barsky (´84)

- **Cyrus-Beck for axis-parallel rectangles**
  - Using Window-Edge-Coordinates (with respect to an edge T)

$$WEC_T(p) = (p - p_T) \cdot N_T$$




- **Example: top (y= $y_{max}$)**

$$N_T = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, p_a - p_T = \begin{pmatrix} x_a - x_{max} \\ y_a - y_{max} \end{pmatrix}$$

$$t_T = \frac{(p_a - p_T) \cdot N_T}{(p_a - p_e) \cdot N_T} = \frac{WEC_T(p_a)}{WEC_T(p_a) - WEC_T(p_e)} = \frac{y_a - y_{max}}{y_a - y_e}$$

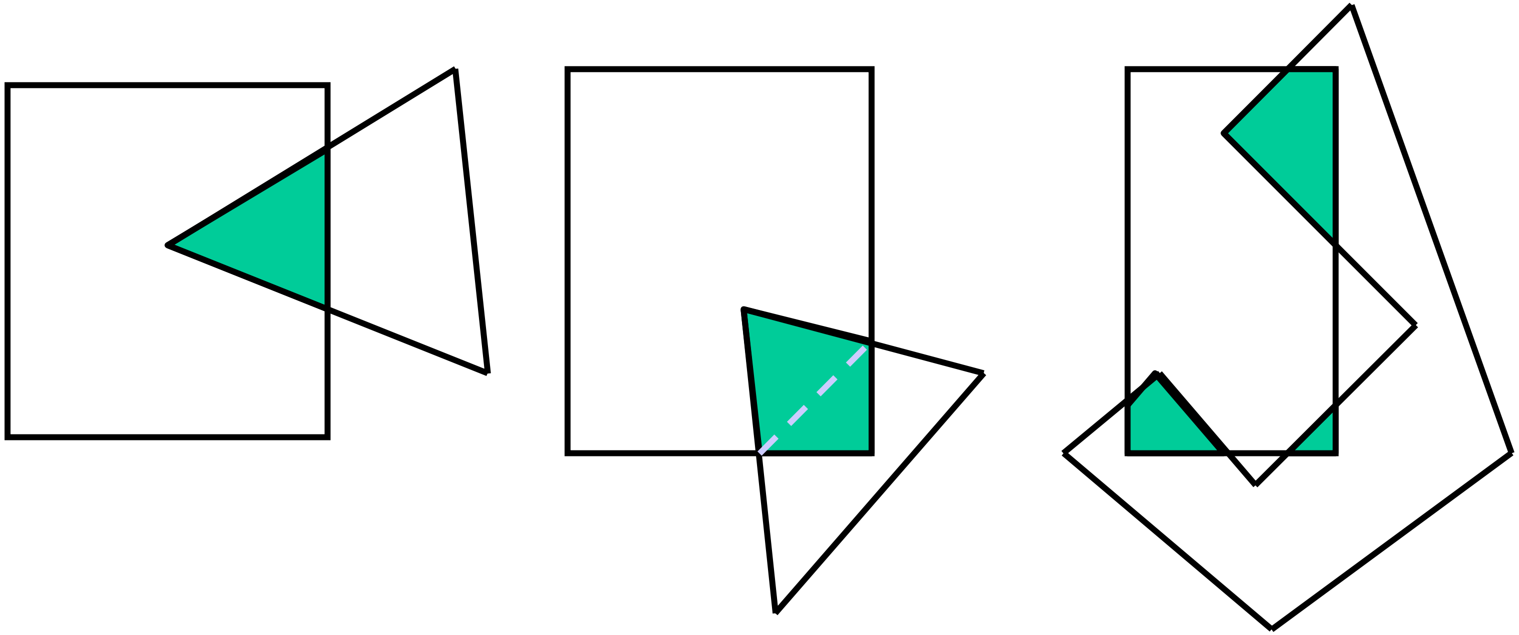
  - Window-Edge-Coordinate (WEC): Decision function for an edge
    - Directed distance to edge
      - Only sign matters, similar to Cohen-Sutherland opcode
    - Sign of the dot product determines whether the point is in or out
    - Normalization unimportant

# Line clipping - Summary

- **Cohen-Sutherland, Cyrus-Beck, and Liang-Barsky algorithms readily extend to 3D**
- **Cohen-Sutherland algorithm**
  - \+ Efficient when a majority of lines can trivially  accepted or rejected
    - Very large clip rectangles: almost all lines inside
    - Very small clip rectangles: almost all lines outside
  - – Repeated clipping for remaining lines
  - – Testing for 2D/3D point coordinates
- **Cyrus-Beck (Liang-Barsky) algorithms**
  - \+ Efficient when many lines must be clipped
  - \+ Testing for 1D parameter values
  - – Testing intersections always for all clipping edges (in the Liang-Barsky trivial rejection testing possible)

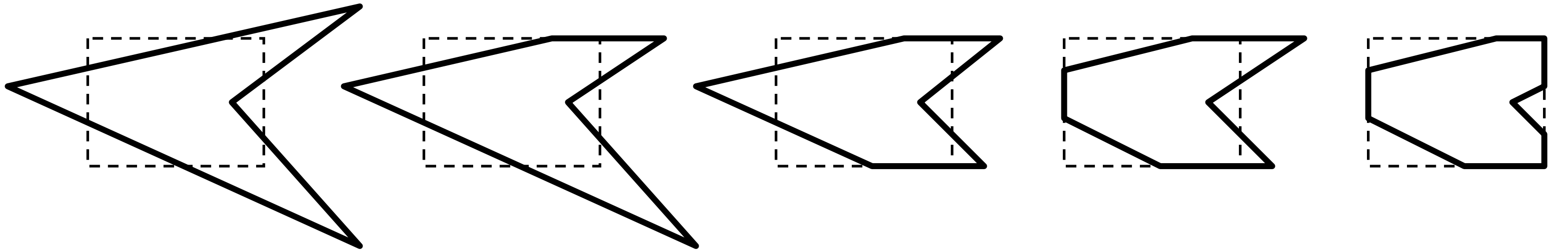
# Polygon Clipping

- **Extending line clipping**
  - Polygons have to remain closed
    - Filling, hatching, shading, ...

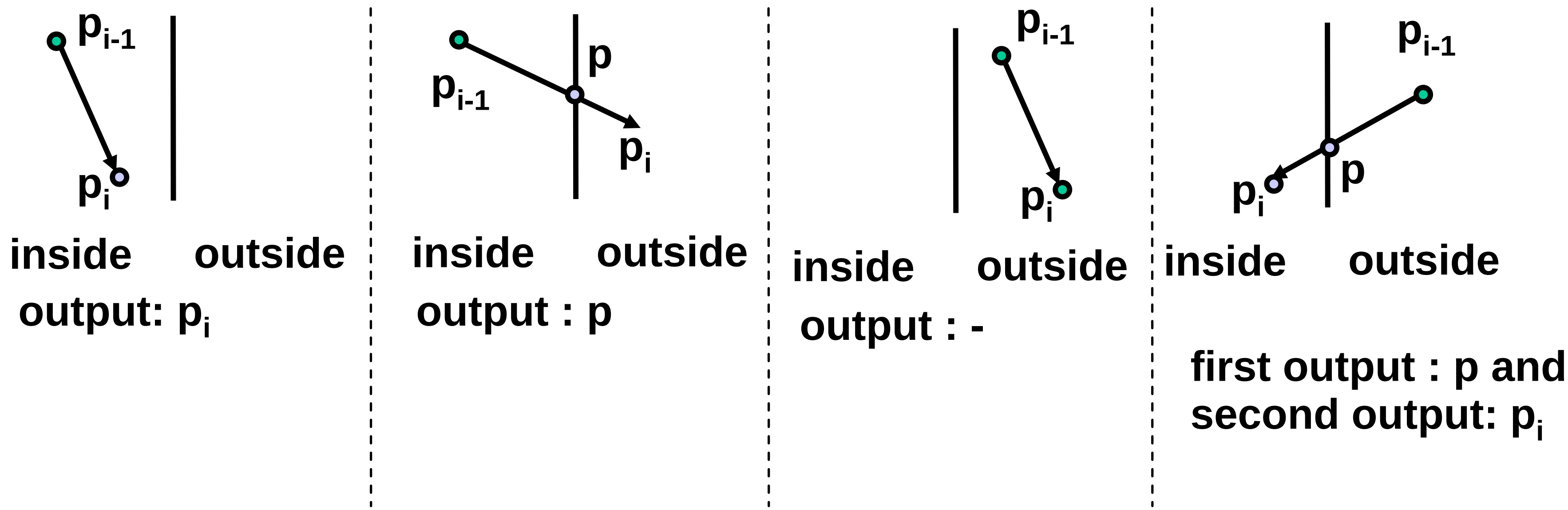


# Sutherland-Hodgeman (´74)

- **Idea:**
  - Iterative clipping against each clipping line

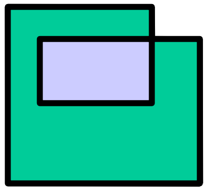


  - Local operations on $p_{i-1}$ and $p_i$



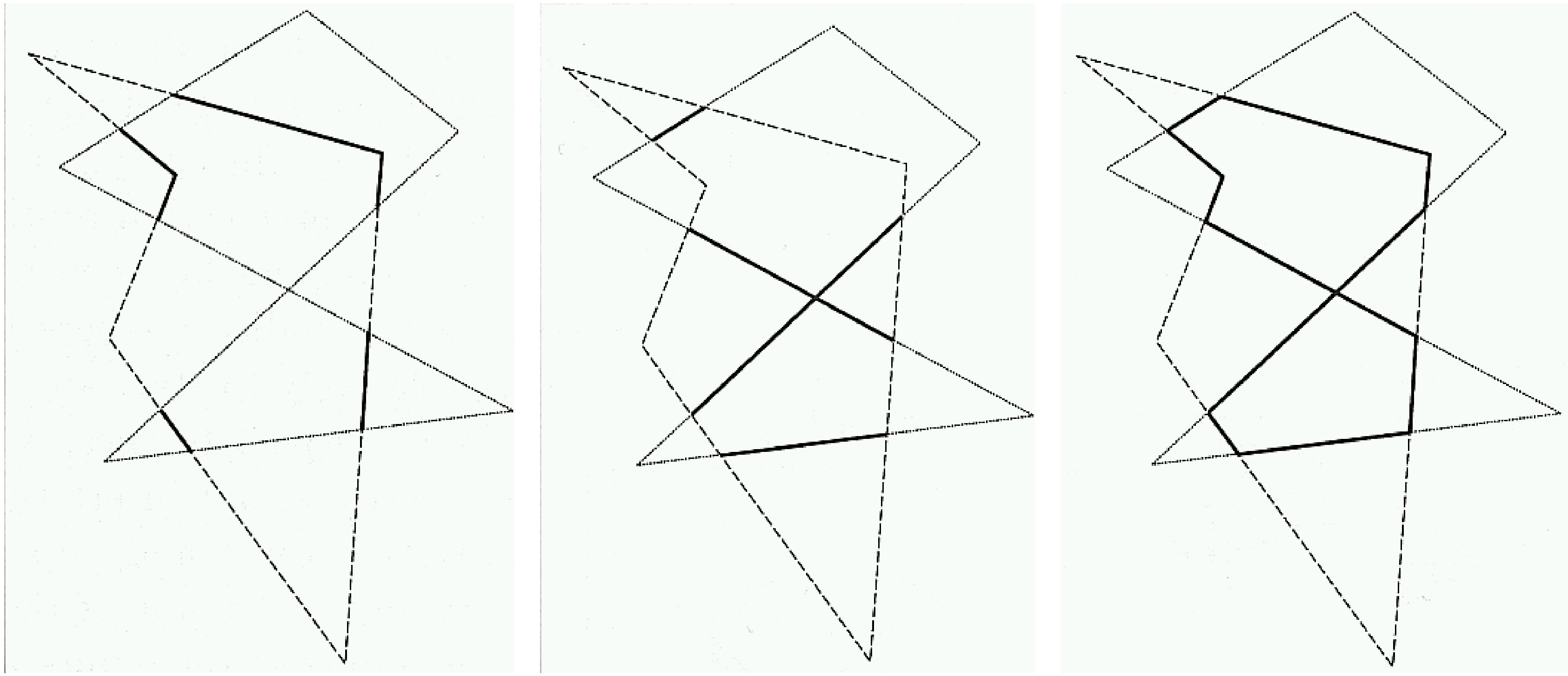
# Other clipping algorithms

- **Weiler & Atherton (´77)**
  - Arbitrary concave polygons with holes against each other
- **Vatti (´92)**
  - Also with self-overlap
- **Greiner & Hormann (TOG ´98)**
  - Simpler and faster as Vatti
  - Also supports boolean operations
  - Idea:
    - Odd winding number rule
      - Intersection with the polygon leads to a winding number $\pm 1$
    - Walk along both polygons
    - Alternate winding number
    - Mark point of entry and point of exit
    - Combine results



**Non-zero WN: In**
**Even WN: Out**

# Greiner & Hormann



A in B B in A (A in B) ∪ (B in A)

# 3D Clipping against View Volume

- **Requirements**
  - Avoid unnecessary rasterization
  - Avoid overflow on transformation at fixed point !
- **Clipping against viewing frustum**
  - Enhanced Cohen-Sutherland with 6-bit outcode
  - After perspective division
    - $-1 < y < 1$
    - $-1 < x < 1$
    - $-1 < z < 0$
  - Clip against side planes of the viewing frustum
  - Works analogous with Liang-Barsky or Sutherland-Hodgeman

# 3D Clipping against View Volume

- **Clipping in homogeneous coordinates**
  - Avoid division by w
  - Inside test with a linear distance function (WEC)
    - Left: $X/W > -1$ ➔ $W+X= WEC_L(\underline{p}) > 0$
    - Top: $Y/W < 1$ ➔ $W-Y= WEC_T(\underline{p}) > 0$
    - Back: $Z/W > -1$ ➔ $W+Z= WEC_B(\underline{p}) > 0$
    - ...
  - Intersection point calculation (before homogenizing)
    - Test: $WEC_L(\underline{p}_a) > 0$ and $WEC_L(\underline{p}_e) < 0$
    - Calculation:

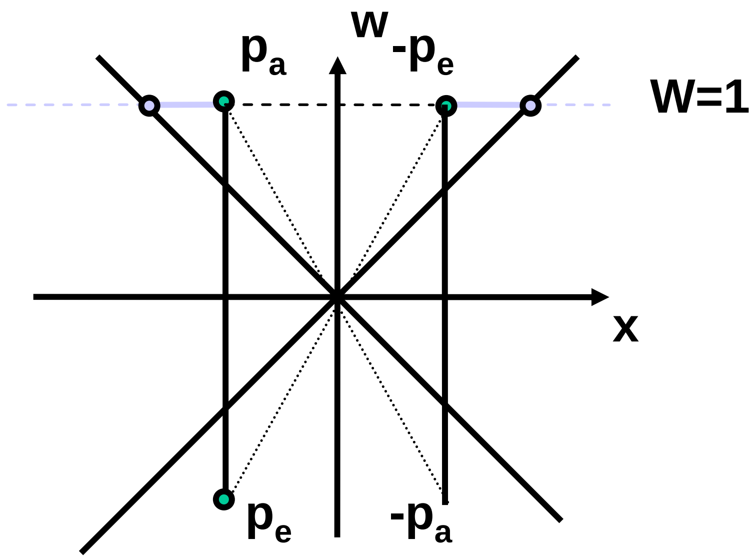
$$WEC(\underline{p}_a + t(\underline{p}_e - \underline{p}_a)) = 0$$

$$W_a + t(W_e - W_a) + X_a + t(X_e - X_a) = 0$$

$$t = \frac{W_a + X_a}{(W_a + X_a) - (W_e + X_e)} = \frac{WEC_L(\underline{p}_a)}{WEC_L(\underline{p}_a) - WEC_L(\underline{p}_e)}$$

# Problems with Homog. Coord.

- **Negative w**
  - Points with $w < 0$ or lines with $w_a < 0$ and $w_e < 0$
    - Negate and continue
  - Lines with $w_a \cdot w_e < 0$ (NURBS)
    - Line moves through infinity
      - External line
    - Clipping two times
      - Original Line
      - Negated line
    - Generates up to two segments



# Practical Implementations

- **Combining clipping and scissoring**
  - Clipping is expensive and should be avoided
    - Intersection calculation
    - Variable number of new points
  - Enlargement of clipping region
    - Larger than viewport, but
    - Still avoiding overflow due to fixed-point representation
  - Result
    - Less clipping
    - Applications should avoid drawing objects which are lying outside of the viewing frustum
    - Objects which are lying partially outside will be clipped implicitly during rasterization.