Statistical Geometry Processing
Winter Term 2011/2012

Assignment Sheet #1: Inverse Problems

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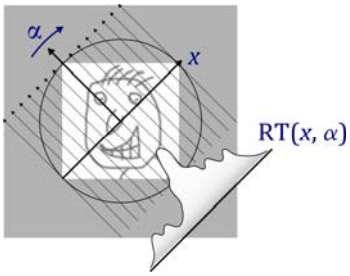

Figure 1: Randon transform – for each angle α, a 1D projection is obtained by integrating over the 2D image along parallel lines. The Radon transformation consists of all such 1D projections, varying $\alpha \in [0...180°]$.

Assignment 1.1: The Radon Transform (score: 50%)

(a) Implement a 2D Radon transform: Prepare a GeoX experiment that allows you to load an image and compute its Radon transform. The Radon transform is obtained forming lines that cross the image and store their respective line integrals, as shown in Figure 1.

(b) Compute the matrix that maps the image pixels to the transformation.

Assignment 1.2: The Inverse Radon Transform (score: 50%)

(a) Analyze the properties of this transformation by considering the SVD of the transformation matrix. In particular, examine the singular values and the corresponding singular vectors.

(b) Visualize the kernel of the transform: provide example images that are (almost) mapped to zero by the Radon transform. On the contrary, give also examples of images that are reproduced with little attenuation. *Hint:* Look at the (right type of) singular vectors.

(c) Compute an inverse Radon Transform using the pseudo inverse (SVD-based matrix inverse with singular values suitably clamped / excluded). Apply the inversion under different levels of (artificially added) noise.

Remarks

GeoX: An updated version of GeoX is provided online that integrates the OpenCV implementation for a dense SVD. Because of the dense matrix computations, only transformations up to about 64 x 64 pixels can be handled in reasonable computation time. The new version also provides an implementation of the image viewer discussed in Assignment #0.

Support: In case you run into difficulties solving this assignment (conceptually or implementation-wise), do not hesitate to contact us. The easiest way to get feedback is to contact us via email to make an appointment for a personal meeting.