# Statistical Geometry Processing
## Winter Term 2011/2012

### Assignment Sheet #4: Bayesian Mesh Denoising – Bunny Restoration

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**Remark:** Again this is a mixed theory / practice assignment. Please prepare one write-up *per person* for the parts marked as "theory" and hand them in at the beginning of the tutorial course (this is mandatory).

### Assignment 4.1: Bunny Distortion Field (practice) (score: 10%)

Reuse the GeoX experiment from Assignment 3.1 that can load meshes from file. Again, import the "Stanford Bunny" model (you might also try the cow or other graphics-affine animals) and display it as a triangle mesh. In addition to the bunny mesh, prepare also functionality to create a cube that is tessellated regularly with triangles (the user should be able to set the tessellation density).

We now want to simulate data acquisition noise: Implement the following tools

- Adding Gaussian to the mesh vertices; the user should be able to set the noise parameters.
- Optional (see Assignment 4.4): Marking a local region of vertices as "invalid" or "missing". For visualization, they could be set to some constant value (for example, [0,0,0]), or marked in a special color.

We now want to reconstruct the data using a simple probabilistic model.

### Assignment 4.2: Denoising Model (theory) (score: 40%)

We want to reconstruct the input mesh from the distorted data. We know that the data we have has been distorted by independent, random, Gaussian noise. Furthermore, we need prior assumptions on how the original shape should have looked like. We can combine this in a Bayesian estimation approach:

$$\underbrace{P(Model \mid Data)}_{\text{want to optimize the model}} \sim \underbrace{P(Data \mid Model)}_{\substack{\text{likelihood/data term} \\ \text{Gaussian error model}}} \underbrace{P(Model)}_{\substack{\text{shape prior} \\ \text{assumptions}}} \qquad (1)$$

Our task is to optimize the posterior probability $P(Model|Data)$ for an unknown model (i.e., unknown vertex positions). This quantity is proportional to the product of the likelihood of the data $P(Data|Model)$ and the prior distribution on the space of all shapes.

**(a)** We now need to formulate these terms.

- Formulate the data term using the knowledge of how the data was corrupted (ignoring missing "invalid" data for now). **Instructions:** Remember that you just added Gaussian noise to the original model vertices.
- For the prior, we need assumptions on how original models look without being too specific. A common approach is to assume smoothness. We implement this here using a "Laplacian" prior: We assume that for every vertex, it is most likely that it is located at the mean of its direct neighbors (the mean of all points connected through an edge of the triangulation, excluding the point considered itself). Formulate an error model that assumes a radially symmetric Gaussian distribution around this attracting point.

We now want to make this model handy for optimization. We therefore take the negative of the logarithm of equation (1) and obtain a big quadratic objective function. Please note that we can throw away all constants and factors that do not depend on the solution (the vertex position of the reconstructed model). Prepare a write-up for this objective function and its derivation.

**(b)** Study the resulting quadric. Answer the following questions (no formal proof necessary, intuitive arguments are sufficient):

- Looking at the data term: What is the solution (maximum of Eq. 1) without a regularizer?
- Looking at the Laplacian regularizer: Which surfaces have zero energy?
- What is the space of minimal energy for the Laplacian regularizer? How many degrees of freedom are undetermined? So how much information is necessary in addition to make the quadratic optimization problem fully determined?
- Is the quadric SPD? Why?

Prepare a short write-up for all of these questions.

**(c)** Finally, we want to solve for the reconstructed bunny: For this, we need to compute the gradient of the log-space objective function and set it to zero. Derive the gradient analytically, in other words, compute (formally, on paper) the linear system that needs to be solved to reconstruct the bunny vertex positions.

## **Assignment 4.3:** Implementation (practice) (score: 40%)

Use the theoretical model from Assignment 4.2 to implement a mesh-denoising algorithm that removes noise from the vertices 3D meshes (the mesh structure itself is assumed to be known). In other words, implement the technique developed theoretically in Assignment 4.2.

**Hints:**

- The simplest solution is iteratively computing the gradient and performing gradient descent. However this is slow and not really elegant for quadratic objective functions.
- A better solution is to express the linear system using a sparse matrix for the quadric and a dense right hand side (GeoX: DynamicVector / SparseMatrix). You can then use an iterative solver (conjuage gradients for sparse matrices is provided by GeoX, which is reasonably efficient here).

## **Assignment 4.4:** Extensions (theory & practice) (score: 10%)

Implement an extension of the technique. You can choose from the following suggestions (or come up with your own; in that case, talk to us about this before doing it):

- Augment the model such that it can handle missing data (pretend that the vertex positions marked in Assignment 1 are completely unknown; this is the simplest extension).
- Robust regularizer: $L_1$ norm instead of $L_2$ norm in regularizer (reweighted least-squares).
- Robust data term: $L_1$ norm for data term (reweighted least-squares)
- Mesh resampling: Can you modify the scheme such that the model does not do denoising, but only distributes points evenly across the surface? Hint: Run iteratively, project points back on the mesh after each iteration.

You have solved the assignment if you implement one out of these. Nevertheless, additional experiments help understanding this type of approaches better. You should add some brief explanations to your write-up according to the extension you pursued.

**Remark:** This is a rather complex assignment. Do not hesitate to drop by to discuss personally how to approach the problem in detail. Due to the holiday season, remember to start early enough. We would expect to have multiple meetings before the assignment is finalized.