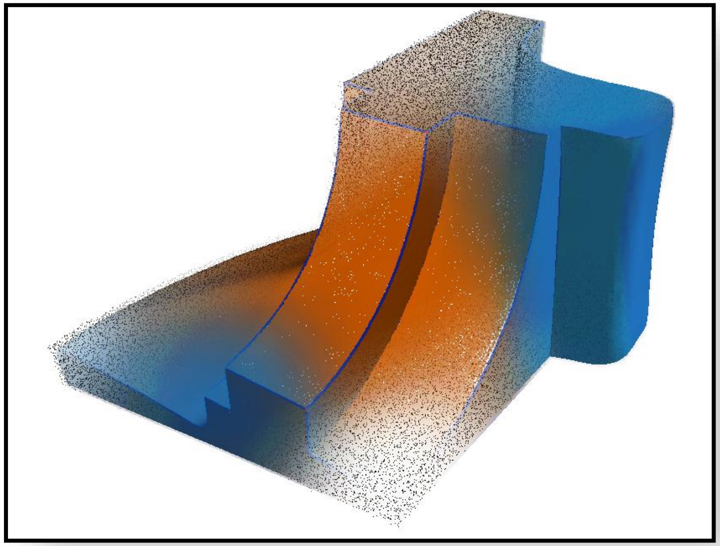
Statistical Geometry Processing

Winter Semester 2011/2012



Shape Spaces and Surface Reconstruction

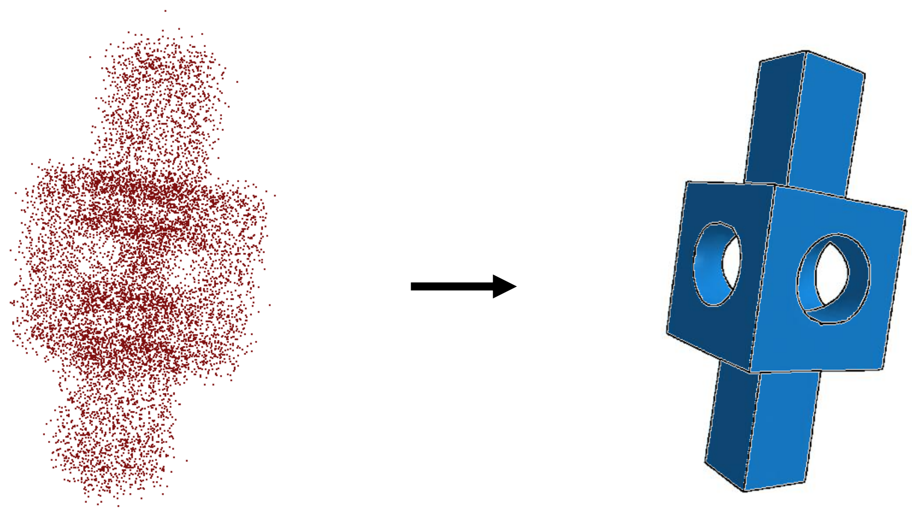
UNIVERSITÄT DES SAARLANDES

CLUSTER OF EXCELLENCE

max planck institut informatik

Part I: Mesh Denoising

Surface Reconstruction



Goal: Surface reconstruction from noisy point clouds

- **Input:** Noisy raw scanner data
- **Output:** “Nice” surface

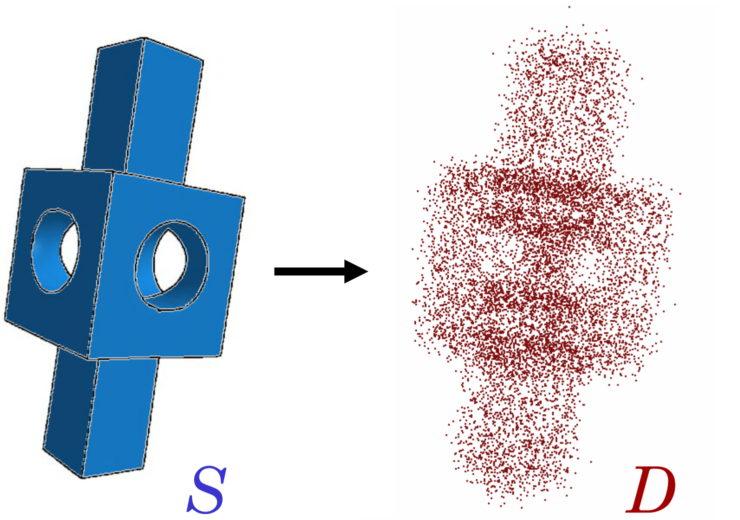
Statistical Model

Bayesian reconstruction

- Probability space
  $\Omega = \Omega_S \times \Omega_D$
- S – original model
  D – measurement data
- Bayes' rule:

$$P(S \mid D) = \frac{P(D \mid S)\, P(S)}{P(D)}$$

- Find most likely S



Bayesian Approach

measurement model
("likelihood")

prior assumptions

$$P(S|D) = \frac{P(D|S)\,P(S)}{\cancel{P(D)}}$$

optimize (best S)

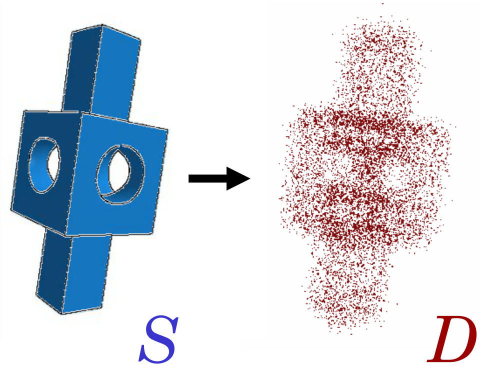
Candidate reconstruction *S* – 

Measured data *D* – 

Computational Framework

Negative log-posterior

Compute maximum a posteriori (MAP) solution



$$E(S \mid D) \sim E(D \mid S) + E(S)$$

measurement potential

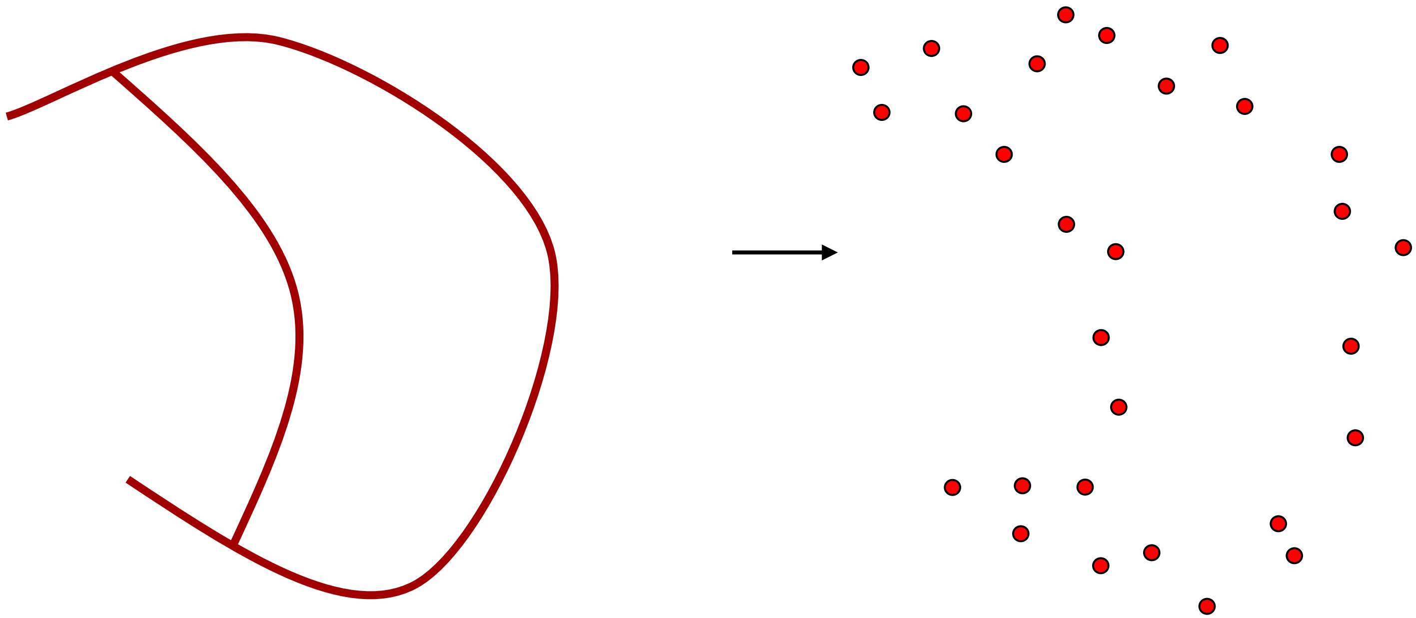
data fitting

prior potential

reasonable reconstruction?

Statistical Model

Generative Model:



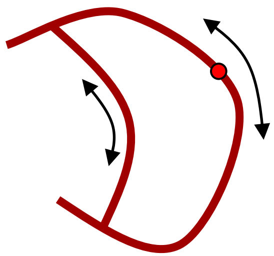
original curve / surface

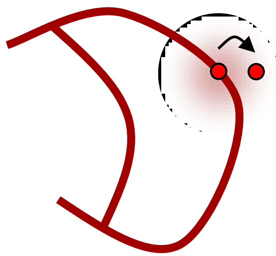
noisy sample points

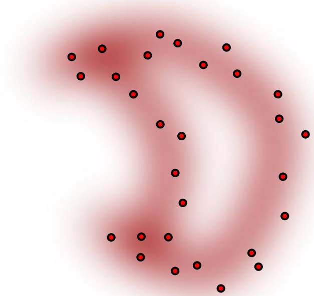
Statistical Model

Generative Model:

1. Determine sample point (uniform)
2. Add noise (Gaussian)

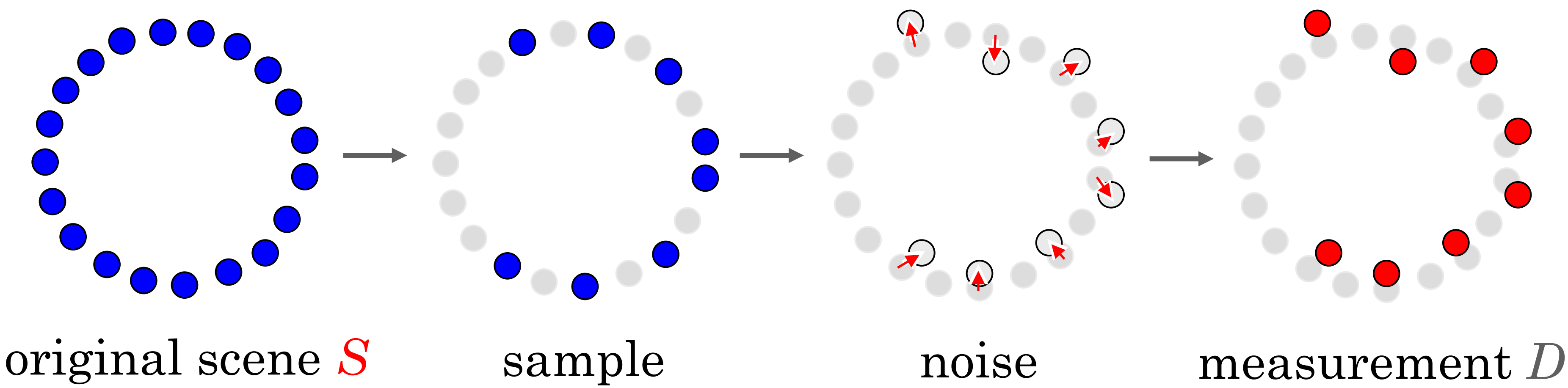

sampling


Gaussian noise


many samples


distribution
(in space)

Denoising: Vertex Displacement



Measurement Model (Assignment #4):

1. **Sampling:** choose subset of measured points (known)
2. **Noise:** shift measured points randomly according to (known) $p_{noise}(x_1,...,x_m)$

Measurement Model

Noise Model

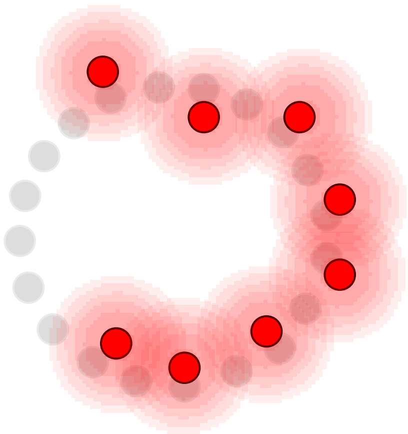
- Most simple: Independent, Gaussian noise
- Negative log-likelihood:

$$-\log p(D \mid S) = \frac{1}{2}\sum_{i=1}^{m}(s_i - d_i)^{\mathrm{T}}\Sigma_i^{-1}(s_i - d_i) + c$$

Why do We Need Priors?

No Reconstruction without Priors

- Measurement itself has highest probability

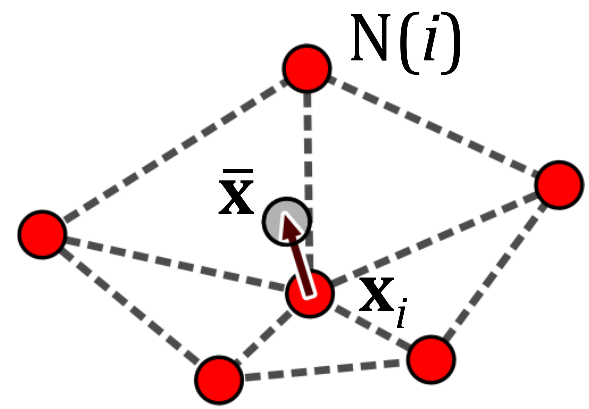


measurement D

Priors

Shape Prior

- Generic Prior
  - Smooth surfaces
- Example (assignment sheet):
  - Points are expected to lie at the mean of their neighbors
  - "Laplacian" prior:



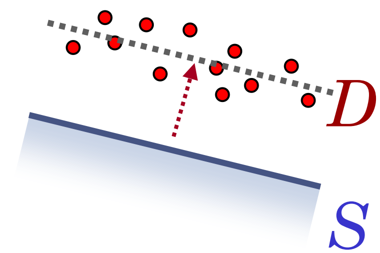
$$E(S) = E(\mathbf{x}_1, \dots, \mathbf{x}_n) \sim \sum_{i=1}^{n} \Big(\mathbf{x}_i - \underbrace{\frac{1}{|N(i)|} \sum_{j \in N(i)} \mathbf{x}_j}_{\bar{\mathbf{x}}}\Big)^2$$

- Formal integrability of P(S)
  - Limit to bounding box, large Gaussian window
  - Omit in practice

Denoising Model

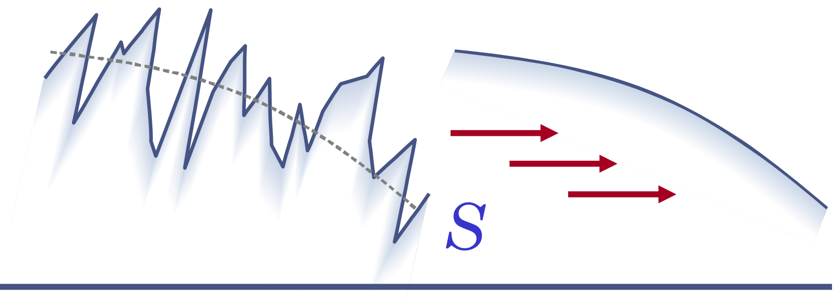
Data fitting

$E(D \mid S) \sim \Sigma_i \, \mathrm{dist}(S, d_i)^2$



Prior: Smoothness

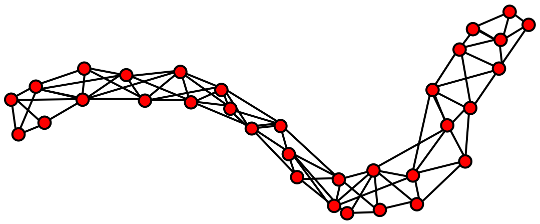
$E_s(S) \sim \int_S \mathrm{curv}(S)^2$



Parametrization

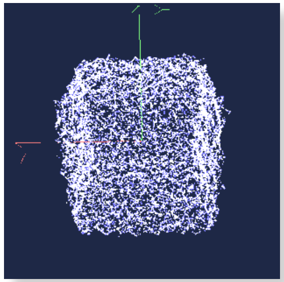
Parametrization

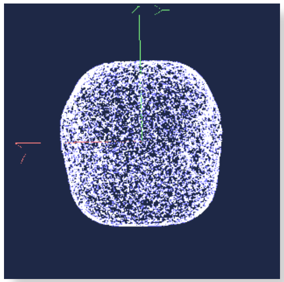
- Need to know neighborhood
- Here, we assume this is known (denoising vs. full reconstruction



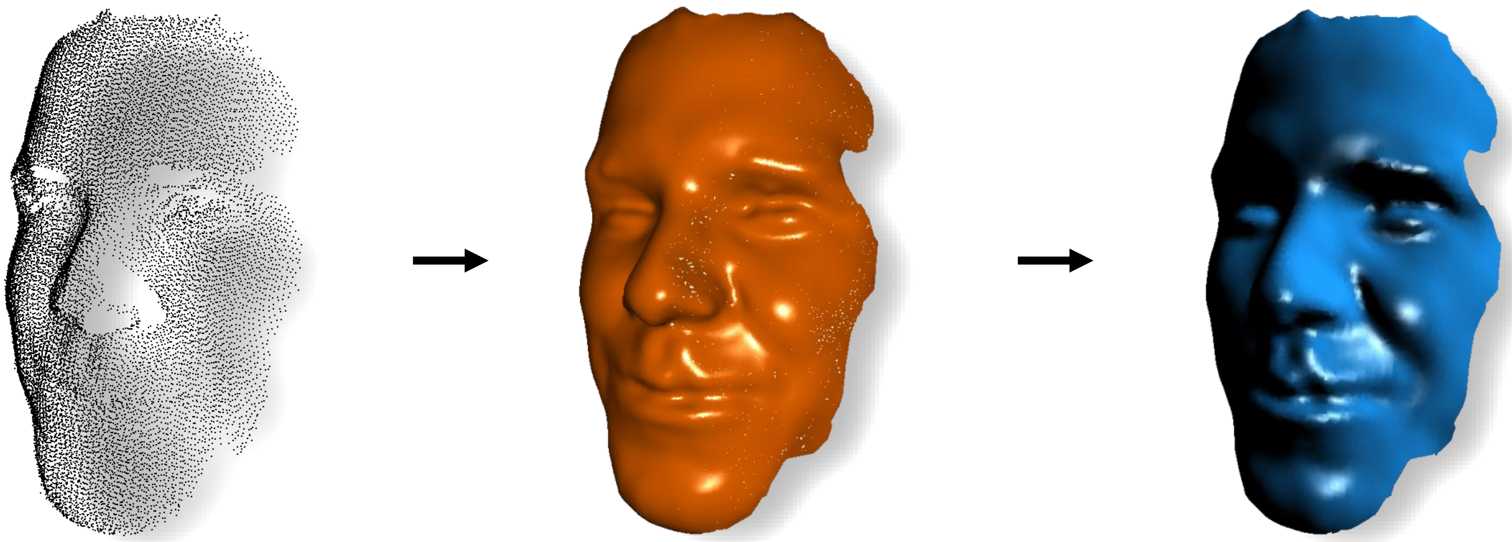
Optimization

- Minimize $E(S|D)$
- Here: Solve linear system





Example

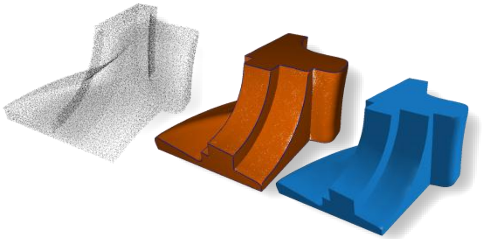


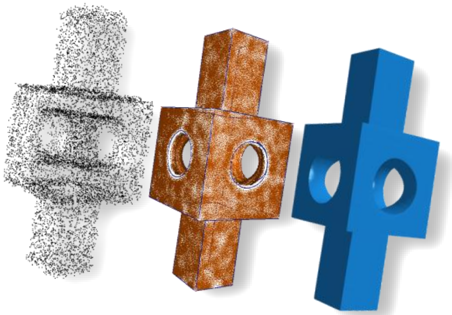
data optimized mesh

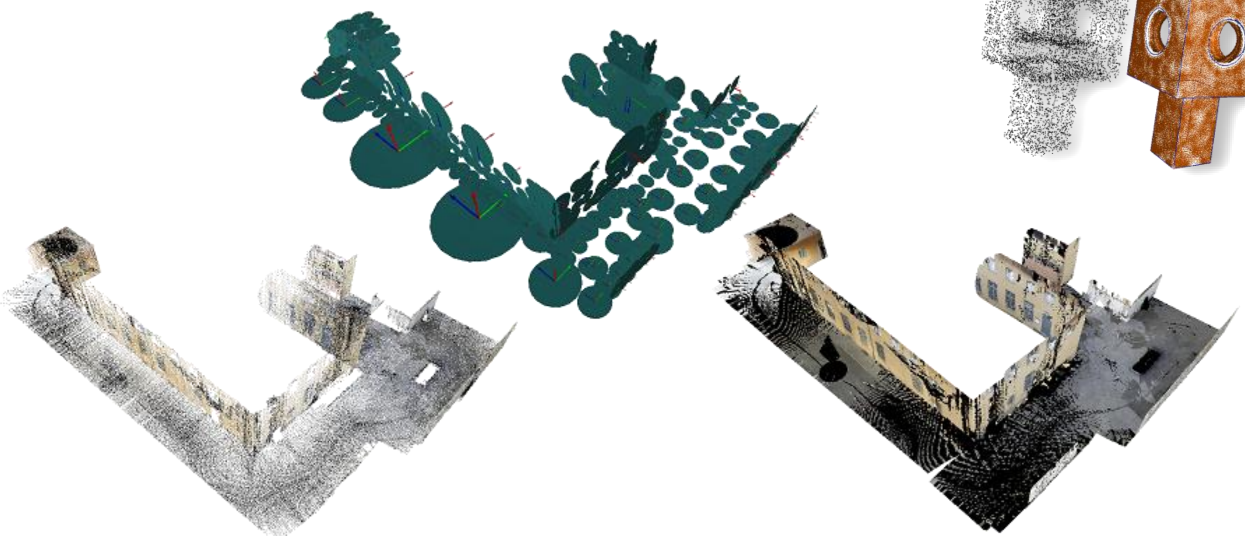
Extensions

Piecewise smooth objects

- Additional (heuristic) segmentation step
- Modify priors at edges
- Man-made objects

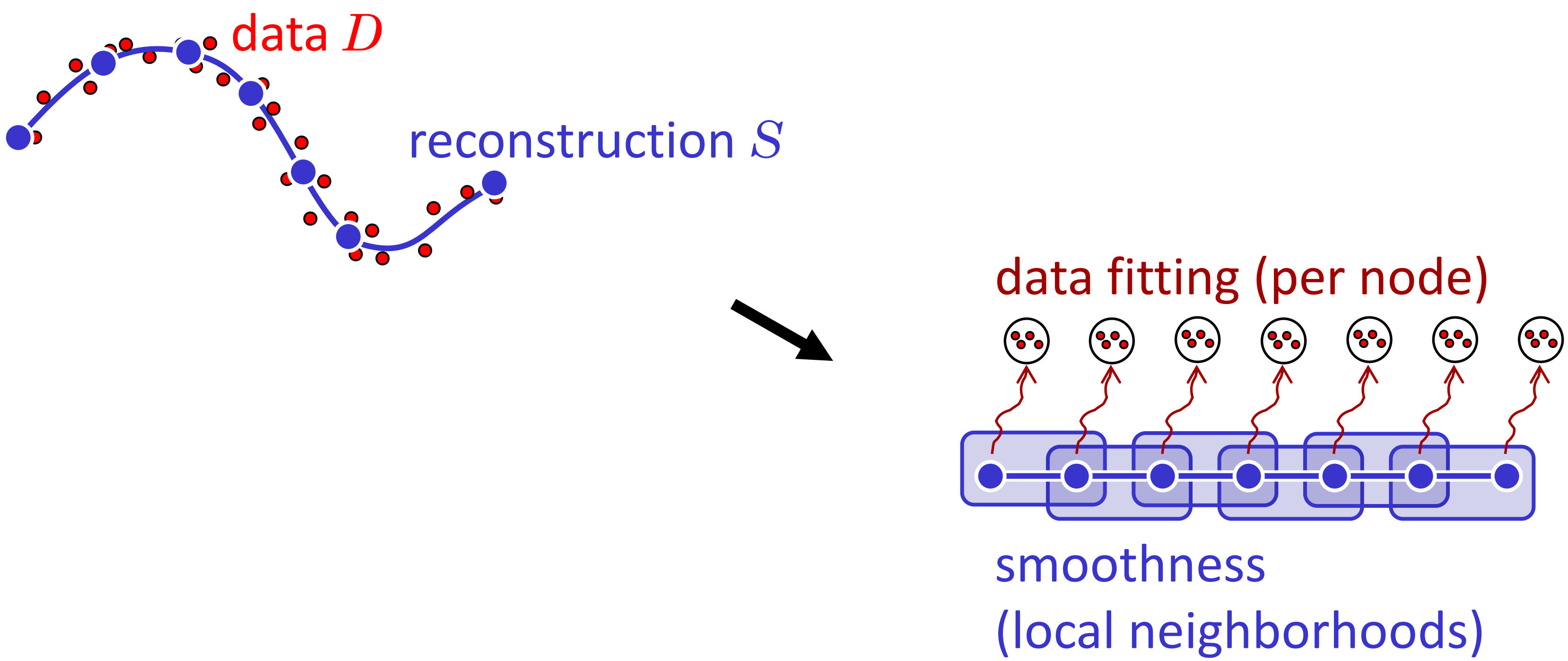






MRF Structure

Markov Random Field (MRF)

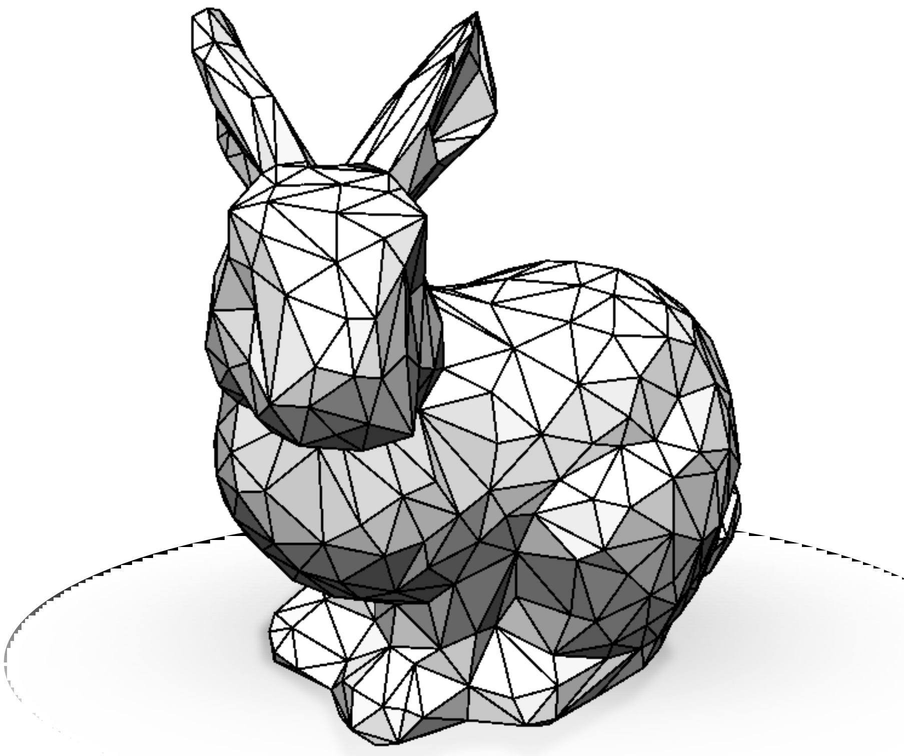


Shape Spaces

Shape Spaces

Mesh Denoising

- Fixed topology (fixed mesh)
- *n* vertices can move around
- Space: $\mathbb{R}^{3n}$
- On this space:
  - Probability density $p(\mathbf{x}), p: \mathbb{R}^{3n} \rightarrow \mathbb{R}^{+}$
  - Alternatively: energy $E(\mathbf{x}) = -\log p(\mathbf{x})$, $E(\mathbf{x}): \mathbb{R}^{3n} \rightarrow \mathbb{R}^{+}$
  - Minimize E, maximize p
  - E does not need to integrate to one (more general)



General Concept

General shape spaces:

- Mapping from sphere to $\mathbb{R}^3$ (fixed topology)
- Implicit functions in $\mathbb{R}^3$
  - General topology
  - But redundancy for off-surface points
- Point-based models
  - Topology implicit
  - Hard to capture
- How to describe more specific priors?
  - Our model is a stationary MRF (typical choice)
  - “Space of all people”, “Space of all houses”?