Statistical Geometry Processing
Winter Semester 2011/2012

Shape Spaces and Surface Reconstruction
Part I: Mesh Denoising
Surface Reconstruction

Goal: Surface reconstruction from noisy point clouds

- **Input:** Noisy raw scanner data
- **Output:** “Nice” surface
Bayesian reconstruction

- Probability space
  \[ \Omega = \Omega_S \times \Omega_D \]
- \( S \) – original model
- \( D \) – measurement data
- Bayes’ rule:
  \[
  P(S \mid D) = \frac{P(D \mid S) P(S)}{P(D)}
  \]
- Find most likely \( S \)
Bayesian Approach

\[ P(S|D) = \frac{P(D|S) \cdot P(S)}{P(D)} \]

measurement model ("likelihood")

prior assumptions

optimize (best S)

Candidate reconstruction \( S \) –

Measured data \( D \) –
Computational Framework

Negative log-posterior

Compute maximum a posteriori (MAP) solution

\[
E(S \mid D) \sim E(D \mid S) + E(S)
\]

measurement potential

prior potential

reasonable reconstruction?

data fitting
Statistical Model

Generative Model:

original curve / surface

noisy sample points
Statistical Model

Generative Model:

1. Determine sample point (uniform)
2. Add noise (Gaussian)

- sampling
- Gaussian noise
- many samples
- distribution (in space)
Denoising: Vertex Displacement

Measurement Model (Assignment #4):

1. **Sampling**: choose subset of measured points (known)
2. **Noise**: shift measured points randomly according to (known) $p_{\text{noise}}(x_1, \ldots, x_m)$
Measurement Model

Noise Model

- Most simple: Independent, Gaussian noise
- Negative log-likelihood:

\[- \log p(D | S) = \frac{1}{2} \sum_{i=1}^{m} (s_i - d_i)^T \Sigma_i^{-1} (s_i - d_i) + c\]
Why do We Need Priors?

No Reconstruction without Priors

- Measurement itself has highest probability

measurement $D$
Priors

Shape Prior

• Generic Prior
  - Smooth surfaces

• Example (assignment sheet):
  - Points are expected to lie at the mean of their neighbors
  - “Laplacian” prior:

\[
E(S) = E(x_1, \ldots, x_n) \sim \sum_{i=1}^{n} \left( x_i - \frac{1}{|N(i)|} \sum_{j \in N(i)} x_j \right)^2
\]

• Formal integrability of \( P(S) \)
  - Limit to bounding box, large Gaussian window
  - Omit in practice
Denoising Model

Data fitting

\[ E(D \mid S) \sim \sum_i \text{dist}(S, d_i)^2 \]

Prior: Smoothness

\[ E_s(S) \sim \int_S \text{curv}(S)^2 \]
Parametrization

- Need to know neighborhood
- Here, we assume this is known (denoising vs. full reconstruction)

Optimization

- Minimize $E(S|D)$
- Here: Solve linear system
Example

data  →  optimized  →  mesh
Extensions

Piecewise smooth objects

- Additional (heuristic) segmentation step
- Modify priors at edges
- Man-made objects
MRF Structure

Markov Random Field (MRF)

data $D$

reconstruction $S$

data fitting (per node)

smoothness
(local neighborhoods)
Shape Spaces
Shape Spaces

Mesh Denoising

- Fixed topology (fixed mesh)
- $n$ vertices can move around
- Space: $\mathbb{R}^{3n}$
- On this space:
  - Probability density
    \[ p(x), p : \mathbb{R}^{3n} \rightarrow \mathbb{R}^+ \]
  - Alternatively: energy
    \[ E(x) = -\log p(x), \quad E(x) : \mathbb{R}^{3n} \rightarrow \mathbb{R}^+ \]
  - Minimize $E$, maximize $p$
  - $E$ does not need to integrate to one (more general)
General Concept

General shape spaces:

- Mapping from sphere to $\mathbb{R}^3$ (fixed topology)
- Implicit functions in $\mathbb{R}^3$
  - General topology
  - But redundancy for off-surface points
- Point-based models
  - Topology implicit
  - Hard to capture
- How to describe more specific priors?
  - Our model is a stationary MRF (typical choice)
  - “Space of all people”, “Space of all houses”?