# Culprits and Islands









4 July 2014 (TADA)

### Service Announcement #1

#### Introduction

Is DM science?DM in action

#### Tensors

Introduction to tensors
Tensors in DM
Special topics in tensors

#### **Information Theory**

- MDL + patterns
- Entropy + correlation
- MaxEnt + iterative DM

#### **Mixed Grill**

- Influence Propagation

- Redescription Mining
  - <special request>

### Service Announcement #1



# Who are the Culprits?

B. Aditya Prakash VT Jilles Vreeken

Christos Faloutsos



#### 4 July 2014 (TADA)







### First question of the day



How can we find the number *and* location of starting points for epidemics in graphs?

– *or* –

#### Who are the culprits?

## **Virus Propagation**

Susceptible-Infected (SI) Model



Diseases over contact networks



CDC data: Visualization of the first 35 tuberculosis (TB) patients and their 1039 contacts

# **Culprits: Problem definition**



### Question: Who started it?

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# **Culprits: Problem definition**

2d grid



### Question: Who started it?



Prior work: [Lappas et al. 2010, Shah et al. 2011]

### **Culprits: Exoneration**



### **Culprits: Exoneration**



# Who are the culprits

Two-step solution
1) use MDL for *number* of seeds
2) for a given number:

exoneration =
centrality + penalty



Running time linear! (in edges and nodes)

 $O(k^*(E_I + E_F + V_I))$ 



# Modeling using MDL

#### Minimum Description Length principle Induction by Compression Related to Bayesian approaches

MDL = Model + Data

Cost of a Model: scoring the seed-set

$$\mathcal{L}(\mathcal{S}) = \mathcal{L}_{\mathbb{N}}(|\mathcal{S}|) + \log \binom{N}{|\mathcal{S}|}$$

Number of *possible* |S|-sized sets

Encoding *integer* |*S*|

# Modeling using MDL

Encoding the Data: Propagation Ripples





### Total MDL cost

 $\mathcal{L}(G_I, \mathcal{S}, R) = \mathcal{L}(\mathcal{S}) + \mathcal{L}(R \mid \mathcal{S})$ 

### How to optimize the score?

#### Two-step process

- Given *k* quickly identify high-quality set *S*
- Given set S, optimize the ripple R

# Optimizing the score

- High-quality k-seed-set
- exoneration



### Best single seed:

- smallest eigenvector of Laplacian sub-matrix
- analyze a *Constrained* SI epidemic

#### Exonerate neighbors

Repeat

# Optimizing the score

Optimizing R

Get the MLE ripple!



Finally use MDL score to tell us the best set

NETSLEUTH: Linear running time in nodes and edges

$$O(k^*(E_I + E_F + V_I))$$

Evaluation functions:MDL based

$$Q_{\text{MDL}} = \frac{\mathcal{L}(G_I, \mathcal{S}, R)}{\mathcal{L}(G_I, \mathcal{S}^*, R^*)}$$





Overlap based

$$Q_{\rm JD} = \frac{\mathbb{E}[JD_{\mathcal{S}}(\mathcal{V}_I)]}{\mathbb{E}[JD_{\mathcal{S}^*}(\mathcal{V}_I)]}$$

(JD = Jaccard distance)

Closer to 1 the better

### Experiments: # of Seeds



# Experiments: Quality (MDL and JD)



Prakash, Vreeken, Faloutsos 2012

### Experiments: Quality (Jaccard Scores)



### **Experiments: Scalability**



### Conclusion

#### **Given**: Graph and Infections **Find**: Best 'Culprits'

#### Two-step solution

- use MDL for number of seeds
- for a given number:

**exoneration** = centrality + penalty

#### NetSleuth:

Linear running time in nodes and edges  $O(k^*(E_I+E_F+V_I))$ 



# **Connection Pathways**



Leman Akoglu



**Jilles Vreeken** 





Polo Chau



Nikolaj Tatti



#### **Christos Faloutsos**

(Akoglu et al. SDM'13)

### Question at hand

# How can we use a graph to **explain** a few **selected nodes**?



### Given a 'list' of authors...

#### What can we say?

let's use relational information



### Given a 'list' of authors...

What can we say?

let's use relational information



# Using the co-authorship graph...

#### Any structure?

too cluttered Bonnie E. John Scott E. Hudson Shumin Zhai Christos Faloutsos Brad\_A.\_Myers Abigail\_Sellen H.\_V.\_Jagadish William\_Buxton Steve\_Benford David J. DeWitt James A. Landay Rakesh Agrawal Ravin Balakrishnan Jeffrey\_F.\_Naughton Hiroshi Ishii Surajit Chaudhuri Michael\_J.\_Carey Hector Garcia-Molina Raghu\_Ramakrishnan Gerhard Weikum

# The Problem

#### Given

- a large graph G
- a handful of nodes S

marked by an external process

#### What can we say about **S**?

- are they **close by**?
- are they **segregated**?
- do they form groups?

#### Can we connect them?

- with simple paths?
- maybe using a few connectors?



### Our approach

Use the network structure to explain S

Partition S into groups of nodes, such that

- "simple" paths in G connect the nodes in each group,
- nodes in different groups are "not easily reachable"



Use MDL to decide 'simple' and 'best' partitioning

# Example

#### Simple connection pathways

- good connectors
- better sensemaking



### 1. Graph anomaly description/summarization





Summarize top-k node anomalies by groups
 Find connections/connectors among groups



Summarize top-k query pages by groups
 Find connections/connectors among groups

### 3. Understanding dynamic events in graphs



Event spread within groups explained by the network
 Event spread between groups due to external influence

### 4. Understanding semantic coherence



Summarize words by semantically coherent groups
 Find connectors (other relevant words) per group

### 5. Understanding segregation (social science)

e.g. school-children friendship network



Summarize students by their social "circles"
 Study groups (and groups within groups)

**Problem:** Formally

#### **Problem Definition**

Given a graph G=(V,E) and a set of marked nodes M subseteq V

**Problem 1. Optimal partitioning** Find a **coherent** partitioning *P* of *M*. Find the optimal **number of partitions** |*P*|.

Problem 2. Optimal connection subgraphs Efficiently find the minimum cost set of subgraphs connecting the nodes in each part  $p_i \in P$ 

# **Objective: Informally**

Our key idea is to use information theory

Imagine a sender and a receiver.

- both sender and receiver know graph structure G,
- only the sender knows the set of marked nodes M
- goal: transmit *M* using as few bits as possible.

Why would this work?

- naïve: encode ID of each marked node with bits
- better: exploit "close-by" nodes, restart for fertiver nodes

 $\log |V| + \log d(u)$  vs.  $2 \log |V|$ 



## **Objective: Intuition**

#### We think of encoding as

- hopping from node to node to encode close-by nodes
- and flying to a new node to encode farther nodes
- until all marked nodes are identified

#### **Simplicity** of connection tree *T* is determined by:

- the amount of flights we make across the graph;
- ease of identifying the edges to follow next;
- ease of identifying the marked nodes in our tour;

**Objective: Formally**  
minimize  

$$P, T_i$$
  
 $L(P, M | G) = L(|P|) + \sum_i L(p_i)$ 

- encode #partitions  $L(|P|) = \log |V|$
- encode each part

#

$$L(p_i) = \log |V| + L(t) + \log |T| + \log {\binom{|T|}{||T||}}$$
root node spanning tree number of identities of t of  $p_i$  marked nodes in  $p_i$  marked nodes

encoding of tree per part

$$L(t) = L_{\mathbb{N}}(|t|+1) + \log \binom{d(v_t)}{|t|} + \sum_{j=1}^{|t|} L(b(t,j))$$
  
branches of node t  
identities of branch nodes  

$$recursively$$
  
encode all  
tree nodes

|+|

# Solution: Intuition

#### It's **NP**-hard.

Nikolaj

#### The problem is Marchard

Related steiner treeppoblem

Hence, we resort to heuristics...

#### The general idea:

- transform G into a directed weighted graph G'
- chop G' into sub-graphs
- find low-cost minimal spanning trees per sub-graph (we give 4 efficient algorithms)

# Solution: Preliminaries

#### Graph transformation

- given undirected unweighted G(V, E)
- we transform it into directed weighted G'(V, E, W)where  $w(u, v) = \log d(u)$  and  $w(v, u) = \log d(v)$

Given *G*', the problem becomes: find *the set of trees* with minimum total cost on the marked nodes.

#### Finding bounded-length paths

- (multiple) short paths of length up to  $\log |V|$  between marked nodes in G'
- employ BFS-like expansion

# Algorithms

#### 1) Connected components (CC)

- find induced subgraph(s) on marked nodes in G'
- find minimum cost directed tree(s)

#### 2) Minimum arborescence (ARB)

- construct transitive closure graph CG (with bounded paths)
- add universal node *u* with out-edges
- find minimum cost directed tree(s), remove u, re-expand paths

# Algorithms

#### 3) Level-1 trees (L1)

- find minimum cost depth-1 trees in CG
- expand paths



#### 4) Level-k trees (Lk)

- refine level-(*k*-1) trees by finding intermediate node *v*'s
- minimizing total cost, i.e. sum of cost to each v and subtrees



#### Synthetic examples

















Case studies on DBLP



DBLP: RECOMB vs. KDD



#### DBLP: NIPS vs. PODS

NicheWorks-interactive\_visualization\_...

Visual\_Analysis\_of\_Large\_Graphs

Topological\_fisheye\_views\_for\_visuali...

Multiscale\_visualization\_of\_small\_wor... 62

A\_System\_for\_Interactive\_Visual\_Analy...

Visualization\_of\_large\_graphs

A\_space\_efficient\_clustered\_visualiza...

ALVIN:\_a\_system\_for\_visualizing\_large...

Interactive\_visualization\_of\_large\_gr... 80

Visualizing\_very\_large\_graphs\_using\_c...

GScholar: 'large graphs' vs. 'visual'

### Intermediate Conclusions

#### Dot2Dot

- principled approach to describe sets of marked nodes using structure of the graph
- automatically finds good connectors
- automatically determines number of groups

New problem, but many **applications** in the wild

# Conclusions

### Graphs problems are often difficult

solutions are typically very ad hoc, very heuristic

### Information theory

offers a clean and principled way to define solutions

### **Identifying Infection Sources**

first to identify multiple sources – extensions currently underway

### **Explaining Node Sets**

first to define the problem – many applications in the wild

Thank you!

### Graphs problems are often difficult

solutions are typically very ad hoc, very heuristic

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