

OBDD-based Tbox Reasoning in \mathcal{SHIQ}

or:

Playing Dominoes

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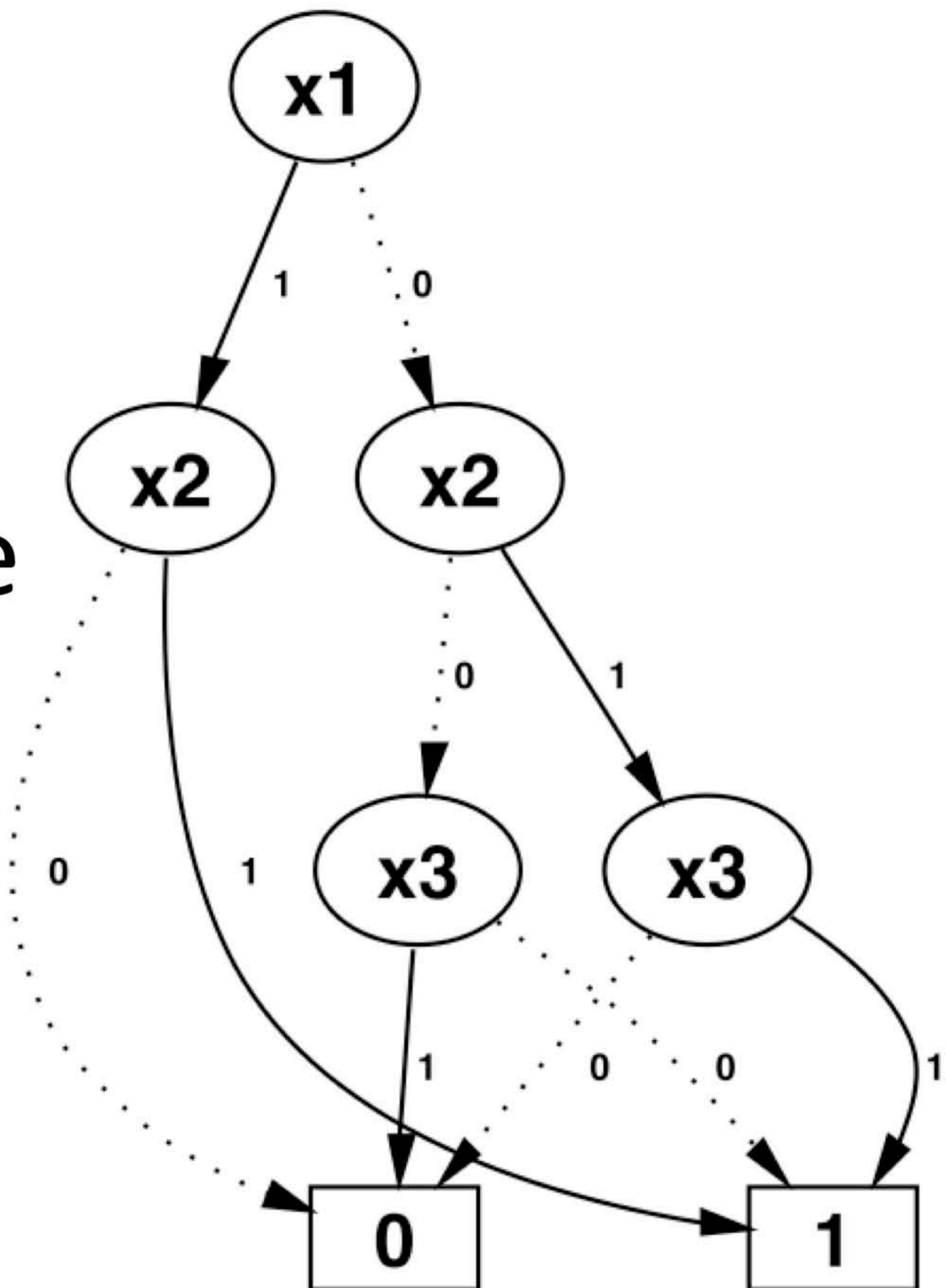
joint work with

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Ordered Binary Decision Diagrams

- short: OBDD, datastructure to represent boolean functions
- used for model checking wrt. temporal logics (e.g. in software and hardware verification)
- Can represent large models (like e.g. state spaces) in a compressed way
- Natural question: use OBDDs for DL reasoning tasks?




SHIQ

- Description Logic, prominent fragment of OWL, completely supported by most reasoners (KAON2, Racer, FaCT++, Pellet)
- Logically: derivative of multi-modal logic K_m , enriched by cardinalities, inverses, hierarchies and transitivity on roles.

\mathcal{ALCIb}

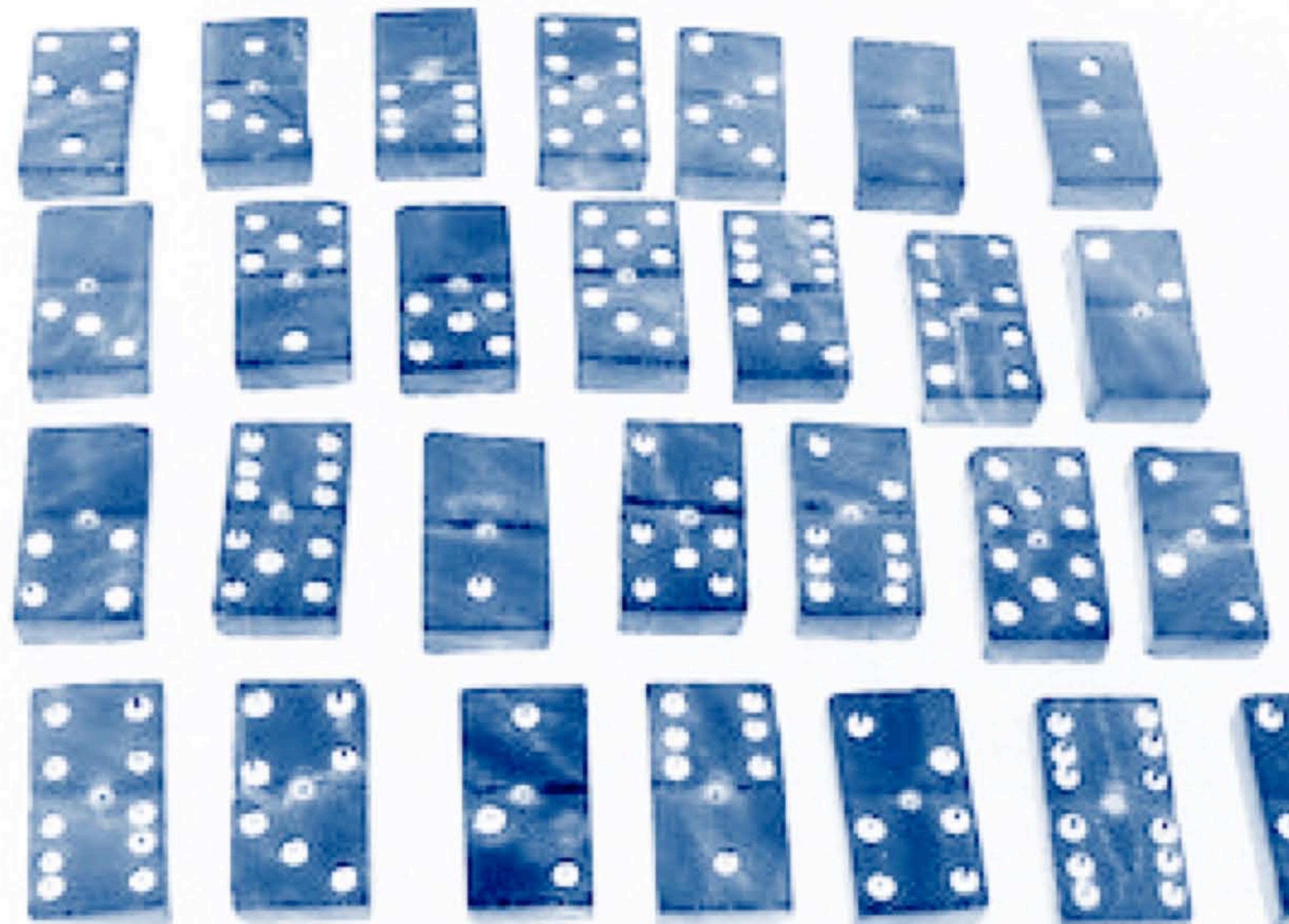
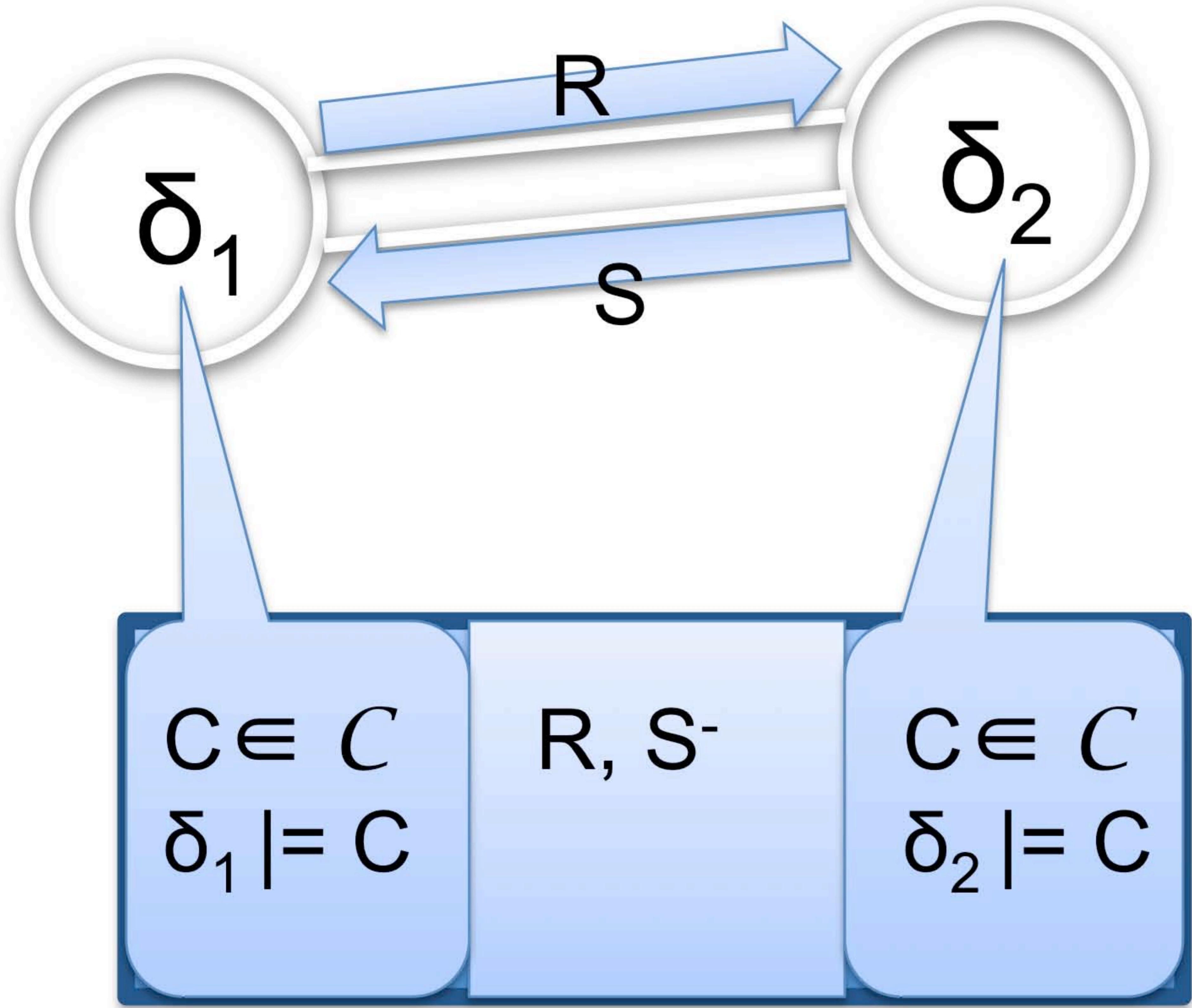
- \mathcal{SHIQ} without
 - transitivity and hierarchies on roles
 - cardinality constraints
- ...but with safe boolean constructors on roles, i.e. we can say: „A selective person’s friends which are not his relatives are all nice.“

Selective $\sqsubseteq \forall(\text{hasFriend} \sqcap \neg \text{hasRelative}).\text{Nice}$



Definition Domino Set

- given interpretation, fix „interesting“ concepts C
- generate one domino for every pair of individuals



Why $\mathcal{ALCI}b$ is nice...

- With the right choice of „interesting concepts“, we can „reconstruct“ models from domino sets...



Reasoning with dominoes...

- ergo: for satisfiability check of theory, we only need to consider domino sets
- given $\mathcal{ALCI}b$ theory, we construct *canonical domino set*: iff this is empty, the theory is unsatisfiable
- greatest fixpoint construction: start with all dominoes, sucessively delete unsupported ones
- OBDDs used to represent intermediate domino sets and carry out pruning steps

But what about $S\mathcal{H}\mathcal{I}\mathcal{Q}$?

- Idea: stepwise equisatisfiable reduction of $S\mathcal{H}\mathcal{I}\mathcal{Q}$ knowledge base to $\mathcal{ALCI}b$:
- $S\mathcal{H}\mathcal{I}\mathcal{Q} \rightarrow \mathcal{ALCHI}Q \rightarrow \mathcal{ALCHI}b^{\leq} \rightarrow \mathcal{ALCI}b^{\leq} \rightarrow \mathcal{ALCI}f b \rightarrow \mathcal{ALCI}b$



$S\mathcal{H}\mathcal{I}\mathcal{Q} \rightarrow \mathcal{ALCHI}Q \rightarrow \mathcal{ALCHI}b^{\leq} \rightarrow \mathcal{ALCI}b^{\leq} \rightarrow \mathcal{ALCI}f b \rightarrow \mathcal{ALCI}b$



$SHIQ \rightarrow ALCHIQ$

- Well-known transformation (aka „box-pushing“) used e.g. by Tobies, Motik
- Essentially: whenever role S implies role R and R is stated to be transitive, add

$$\forall R.C \sqsubseteq \forall S.\forall S.C$$

for all „interesting“ concept expressions C.



$\mathcal{ALC}\mathcal{H}\mathcal{I}\mathcal{Q} \rightarrow \mathcal{ALC}\mathcal{H}\mathcal{I}\mathcal{b}^{\leq}$

- Remove expressions of the form: $\geq n R.C$
- Achieved by introducing n auxiliary roles R_1, \dots, R_n , which are stated
 - to imply R : $R_i \sqsubseteq R$ and
 - to be mutually disjoint: $T \sqsubseteq \forall(\neg R_i \sqcap R_j). \perp$



$$\mathcal{ALCHIb}^{\leq} \rightarrow \mathcal{ALCIb}^{\leq}$$

- Remove role hierarchy axioms.
- No big deal:
 $R \sqsubseteq S$ is equivalent to the disjointness of $\neg R$ and S and can hence be expressed by

$$T \sqsubseteq \forall(\neg R \sqcap S). \perp .$$



$$\mathcal{ALCI}b^{\leq} \rightarrow \mathcal{ALCI}f b$$

- Remove expressions of the form: $\leq n R.C$
- Somewhat „dual“ to removal of \geq -expressions
- Achieved by introducing n auxiliary roles R_1, \dots, R_n , which are stated
 - to „cover“ R : $T \sqsubseteq \forall(\neg R_1 \sqcap \dots \sqcap \neg R_n \sqcap R). \perp$,
 - to have C as domain: $T \sqsubseteq \forall R_i.C$, and
 - to be functional.



$$\mathcal{ALCI}f\!b \rightarrow \mathcal{ALCI}b$$

- Remove functionality constraints on roles.
- It can be proven that it suffices to require „undistinguishability“ of the role targets
- For any role R. stated to be functional:
 - require the same concept-memberships of role targets: $\exists R.C \sqsubseteq \forall R.C$ (for all relevant concepts C)
 - require the same role-memberships:
 $\exists(R \sqcap S).T \sqsubseteq \forall(\neg R \sqcap S).\perp$



Conclusion

- novel paradigm for DL reasoning
- implementation of prototype underway
- future work
 - evaluation & comparison with existent reasoners
 - extension to Aboxes
 - extension to nominals (for coverage of full OWL DL)