A Learning Approach to Proof Step Size in Mathematics Tutoring

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Assume that $a \in X$. If $X \cap Y = \emptyset$, then $a \notin Y$.

- **DIALOG** project: Domain reasoning techniques for proof tutoring. Empirical evaluations (e.g. [Benzmüller et al., 2006]).

- One topic in **DIALOG**: evaluate proof steps w.r.t. correctness, granularity, relevance using MAS ΩMEGA.

- ΩMEGA-Tutor [Dietrich and Buckley, 2007]: Proof reconstruction using ΩMEGA.
Empirical Studies on Tutorial Dialog (Wizard-of-Oz)

Tutor: Let $R$ and $S$ be relations in an arbitrary set $M$. It holds: $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$. Do the proof interactively with the system!

Student: $(x, y) \in (R \circ S)^{-1}$

Tutor: Correct! Good start!

Student: Then $(y, x) \in R \circ S$.

Tutor: Correct!
Granularity of Proof Steps

Student: Let \((x, y) \in (R \circ S)^{-1}\)

Tutor: Now try to draw conclusions from this!

Student: Then \((x, y) \in S^{-1} \circ R^{-1}\)

This cannot be concluded directly. You need some intermediate steps!
Assertion Level Proofs as Basis for Granularity Analysis

1. Tutor: Show \((R \circ S)^{-1} = S^{-1} \circ R^{-1}\)!

Exercise: \(\vdash (R \circ S)^{-1} = S^{-1} \circ R^{-1}\)
Assertion Level Proofs as Basis for Granularity Analysis

1. Tutor: Show \((R \circ S)^{-1} = S^{-1} \circ R^{-1}\)!
2. Student: Let \((x, y) \in (R \circ S)^{-1}\).

\[
\begin{align*}
\text{s1: } (x, y) \in (R \circ S)^{-1} & \implies (x, y) \in S^{-1} \circ R^{-1} \\
\implies (R \circ S)^{-1} \subseteq S^{-1} \circ R^{-1} & \quad \text{Def.} \subseteq \implies S^{-1} \circ R^{-1} \subseteq (R \circ S)^{-1} \\
\text{Exercise: } & \implies (R \circ S)^{-1} = S^{-1} \circ R^{-1} \\
\end{align*}
\]
Assertion Level Proofs as Basis for Granularity Analysis

1. Tutor: Show \((R \circ S)^{-1} = S^{-1} \circ R^{-1}\)!
2. Student: Let \((x, y) \in (R \circ S)^{-1}\).
3. Student: Hence \((y, x) \in (R \circ S)\).

\[\Delta_1\]

Exercise: \(\vdash (R \circ S)^{-1} = S^{-1} \circ R^{-1}\)
1. Tutor: Show \((R \circ S)^{-1} = S^{-1} \circ R^{-1}\)!
2. Student: Let \((x, y) \in (R \circ S)^{-1}\).
3. Student: Hence \((y, x) \in (R \circ S)\).
4. Student: Hence \((y, z) \in R \land (z, x) \in S\).
1. Tutor: Show \((R \circ S)^{-1} = S^{-1} \circ R^{-1}\)!
2. Student: Let \((x, y) \in (R \circ S)^{-1}\).
3. Student: Hence \((y, x) \in (R \circ S)\).
4. Student: Hence \((y, z) \in R \land (z, x) \in S\).
5. Student: Hence \((z, y) \in R^{-1} \land (x, z) \in S^{-1}\).

\[
\begin{align*}
\text{s1: } & (x, y) \in (R \circ S)^{-1} \vdash (x, y) \in S^{-1} \circ R^{-1} \\
& \downarrow \text{Def}^{-1} \\
& \vdash (R \circ S)^{-1} \subseteq S^{-1} \circ R^{-1} \\
& \downarrow \text{Def. } \subseteq \\
& \vdash S^{-1} \circ R^{-1} \subseteq (R \circ S)^{-1} \\
& \text{Exercise: } \vdash (R \circ S)^{-1} = S^{-1} \circ R^{-1}
\end{align*}
\]
Some Criteria

Homogeneity: Are different facts applied in one single student step, or is the same fact repeated?

Verbal Explanation: Does the student name the facts and techniques he uses?

Introduction of Hypotheses or Subgoals

Learning Progress: Does the student master the concepts involved in the step?

Question

How are the different criteria related to the question of appropriate granularity?
Relating Granularity Criteria to Judgments – An Example

- Too detailed
- Appropriate
- Too coarse-grained

- Number of unknown facts?
  - 0
    - Too detailed
  - 1
    - Appropriate
  - 2-4
    - Too coarse-grained

- Number of newly introduced hypotheses?
  - 0
  - 1
The Presented Approach

A Granularity Analysis Module

- using assertion-level proof reconstructions
- performing an analysis w.r.t. granularity criteria
- adaptive via machine-learning techniques

Prototypical Integration into E-Learning Environment

The tutoring of proof exercises for the ActiveMath E-Learning Environment [Melis and Siekmann, 2004] is facilitated by \( \Omega \text{MEGA}'s \) proof step analysis.

See you at the poster!
References

A corpus of tutorial dialogs on theorem proving; the influence of the presentation of the study-material.
In Proc. of LREC 2006, Genoa, Italy. ELDA.

Verification of Proof Steps for Tutoring Mathematical Proofs.
To appear.

Activemath: An intelligent tutoring system for mathematics.