A Learning Approach to Proof Step Size in Mathematics Tutoring

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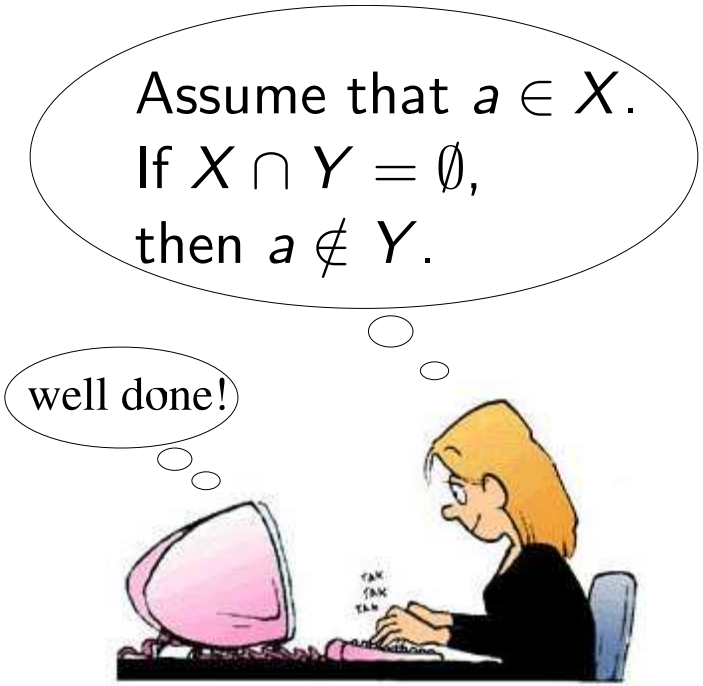
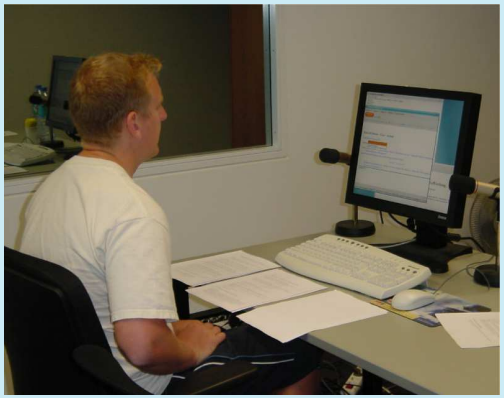
Background



- DIALOG project: Domain reasoning techniques for proof tutoring. Empirical evaluations (e.g. [Benzmüller et al., 2006]).
- One topic in DIALOG : evaluate proof steps w.r.t. correctness, granularity, relevance using MAS ΩMEGA.
- ΩMEGA-Tutor [Dietrich and Buckley, 2007]: Proof reconstruction using ΩMEGA.

Empirical Studies on Tutorial Dialog (Wizard-of-Oz)



Tutor: Let R and S be relations in an arbitrary set M. It holds: $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$. Do the proof interactively with the system!

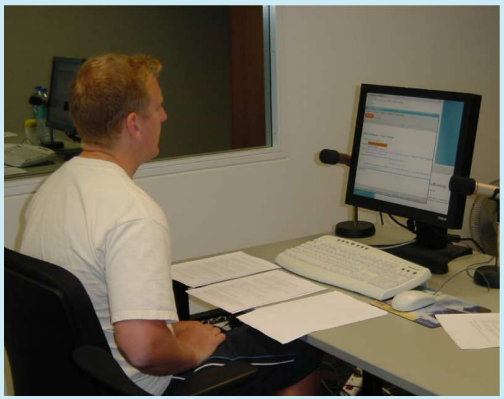
Student: $(x, y) \in (R \circ S)^{-1}$

Tutor: Correct! Good start!

Student: Then $(y, x) \in R \circ S$.

Tutor: Correct!

Granularity of Proof Steps



⋮

Student: Let $(x, y) \in (R \circ S)^{-1}$

Tutor: Now try to draw conclusions from this!

Student: Then $(x, y) \in S^{-1} \circ R^{-1}$

This cannot be concluded directly. You need some intermediate steps!

Assertion Level Proofs as Basis for Granularity Analysis

1. Tutor: Show $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$!

`Exercise:` $\vdash (R \circ S)^{-1} = S^{-1} \circ R^{-1}$

Assertion Level Proofs as Basis for Granularity Analysis

1. Tutor: Show $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$!
2. Student: Let $(x, y) \in (R \circ S)^{-1}$.

$$\dfrac{\dfrac{\dfrac{}{\texttt{s1:}\quad (x,y) \in (R \circ S)^{-1} \vdash (x,y) \in S^{-1} \circ R^{-1}}\,\Delta_1}{\vdash (R \circ S)^{-1} \subseteq S^{-1} \circ R^{-1}}\,\textbf{Def.}\subseteq \qquad \vdash S^{-1} \circ R^{-1} \subseteq (R \circ S)^{-1}}{\texttt{Exercise:}\quad \vdash (R \circ S)^{-1} = S^{-1} \circ R^{-1}}\,\textbf{Def.}=$$

Assertion Level Proofs as Basis for Granularity Analysis

1. Tutor: Show $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$!
2. Student: Let $(x, y) \in (R \circ S)^{-1}$.
3. Student: Hence $(y, x) \in (R \circ S)$.

$$\dfrac{\dfrac{\dfrac{}{\texttt{s2:}\quad (y,x) \in (R \circ S) \vdash \text{"—"}}\ \Delta_2}{\texttt{s1:}\quad (x,y) \in (R \circ S)^{-1} \vdash (x,y) \in S^{-1} \circ R^{-1}}\ \textbf{Def.}^{-1} \qquad \Delta_1}{\dfrac{\vdash (R \circ S)^{-1} \subseteq S^{-1} \circ R^{-1} \qquad \vdash S^{-1} \circ R^{-1} \subseteq (R \circ S)^{-1}}{\texttt{Exercise:}\quad \vdash (R \circ S)^{-1} = S^{-1} \circ R^{-1}}\ \textbf{Def.}=}\ \textbf{Def.}\subseteq$$

Assertion Level Proofs as Basis for Granularity Analysis

1. Tutor: Show $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$!
2. Student: Let $(x, y) \in (R \circ S)^{-1}$.
3. Student: Hence $(y, x) \in (R \circ S)$.
4. Student: Hence $(y, z) \in R \wedge (z, x) \in S$.

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\texttt{s3:}\quad (y,z) \in R \wedge (z,x) \in S \vdash \text{"—"}}}{\texttt{s2:}\quad (y,x) \in (R \circ S) \vdash \text{"—"}}\,\text{Def.}\circ}{\texttt{s1:}\quad (x,y) \in (R \circ S)^{-1} \vdash (x,y) \in S^{-1} \circ R^{-1}}\,\text{Def.}^{-1} \qquad \Delta_1}{\dfrac{\vdash (R \circ S)^{-1} \subseteq S^{-1} \circ R^{-1} \qquad \vdash S^{-1} \circ R^{-1} \subseteq (R \circ S)^{-1}}{\texttt{Exercise:}\quad \vdash (R \circ S)^{-1} = S^{-1} \circ R^{-1}}\,\text{Def.}=}\,\text{Def.}\subseteq$$

Δ_3 Δ_2

Assertion Level Proofs as Basis for Granularity Analysis

1. Tutor: Show $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$!
2. Student: Let $(x, y) \in (R \circ S)^{-1}$.
3. Student: Hence $(y, x) \in (R \circ S)$.
4. Student: Hence $(y, z) \in R \wedge (z, x) \in S$.
5. Student: Hence $(z, y) \in R^{-1} \wedge (x, z) \in S^{-1}$.

$$\vdots$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\texttt{s4:}\quad (z,y) \in R^{-1} \wedge (x,z) \in S^{-1} \vdash \text{"}\_\text{"}}}{(y,z) \in R \wedge (x,z) \in S^{-1} \vdash \text{"}\_\text{"}}\,\mathbf{Def}^{-1}\ \Delta_4}{\texttt{s3:}\quad (y,z) \in R \wedge (z,x) \in S \vdash \text{"}\_\text{"}}\,\mathbf{Def}.^{-1}\ \Delta_3}{\texttt{s2:}\quad (y,x) \in (R \circ S) \vdash \text{"}\_\text{"}}\,\mathbf{Def}.\circ\ \Delta_2}{\texttt{s1:}\quad (x,y) \in (R \circ S)^{-1} \vdash (x,y) \in S^{-1} \circ R^{-1}}\,\mathbf{Def}.^{-1}}{\vdash (R \circ S)^{-1} \subseteq S^{-1} \circ R^{-1} \qquad \vdash S^{-1} \circ R^{-1} \subseteq (R \circ S)^{-1}}\,\mathbf{Def}.\subseteq\ \Delta_1$$

$$\dfrac{\vdash (R \circ S)^{-1} \subseteq S^{-1} \circ R^{-1} \qquad \vdash S^{-1} \circ R^{-1} \subseteq (R \circ S)^{-1}}{\texttt{Exercise:}\quad \vdash (R \circ S)^{-1} = S^{-1} \circ R^{-1}}\,\mathbf{Def}.=$$

How can Students' Proof Steps Differ in Complexity?

Some Criteria

Homogeneity: Are different facts applied in one single student step, or is the same fact repeated?

Verbal Explanation: Does the student name the facts and techniques he uses?

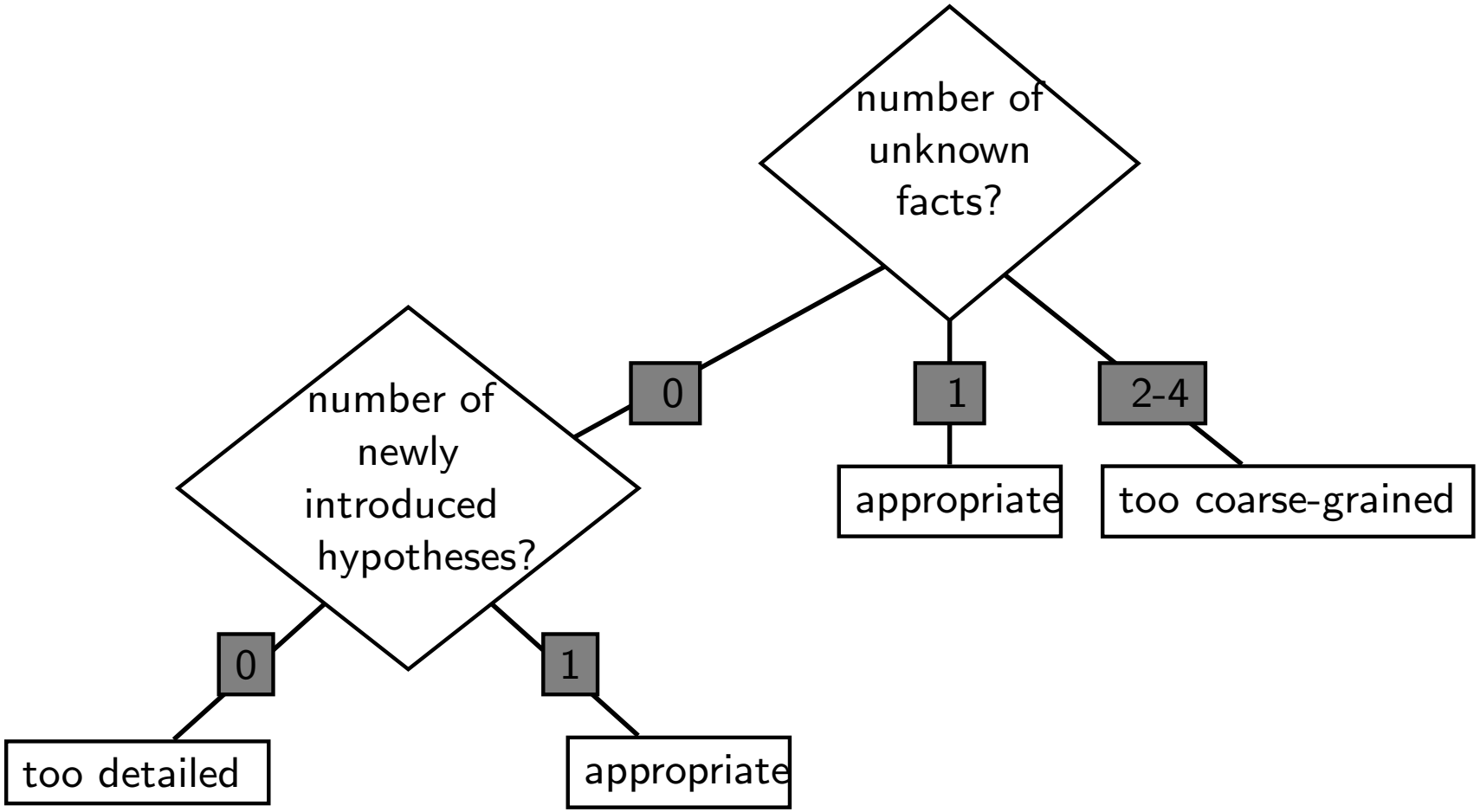
Introduction of Hypotheses or Subgoals

Learning Progress: Does the student master the concepts involved in the step?

Question

How are the different criteria related to the question of appropriate granularity?

Relating Granularity Criteria to Judgments – An Example



The Presented Approach

A Granularity Analysis Module

- ...using assertion-level proof reconstructions
- ...performing an analysis w.r.t. granularity criteria
- ...adaptive via machine-learning techniques

Prototypical Intergration into E-Learning Environment

The tutoring of proof exercises for the ActiveMath E-Learning Environment [Melis and Siekmann, 2004] is facilitated by ΩMEGA's proof step analysis.

See you at the poster!

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