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Integrating Answer Set Programming and Satisfiability Modulo Theories

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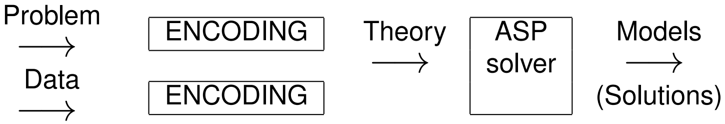
Answer Set Programming (ASP)

- Term coined by Vladimir Lifschitz in the late 1990s
- An approach to modeling and solving **knowledge intensive search problems** with defaults, exceptions, definitions:
  planning, configuration, model checking, network management, linguistics, bioinformatics, combinatorics, ...
- Solving a problem in ASP:
  Encode the problem as a logic program such that **solutions** to the problem are given by stable models (**answer sets**) of the program.

Problem $\longrightarrow$ $\boxed{\text{ENCODING}}$
Data $\longrightarrow$ $\boxed{\text{ENCODING}}$
Program $\longrightarrow$ $\boxed{\text{ASP solver}}$ Models $\longrightarrow$ (Solutions)

ASP

- Expressive KR language for problem encoding
- Uniform encoding:
  separate problem specification and problem instance data
- Compact, easily maintainable representation
- Program development, reusable modules, debugging, testing, optimization
- Integrating KR, DDB, and search techniques
- Handling dynamic, knowledge intensive applications:
  data, frame axioms, exceptions, defaults, definitions



Coloring Problem

Uniform Encoding: the problem specification is a fixed program working for every input graph given as a set of facts.

```
% Problem specification
1 { colored(V,C):color(C) } 1 :- vtx(V).
:- edge(V,U), colored(V,C), colored(U,C).

% Data
vtx(a). ...
edge(a,b). ...
color(r). color(g). ...
```

☞ Legal colorings of the graph given as data and stable models of the problem encoding and data correspond: a vertex v colored with a color c iff `colored`(v, c) holds in a stable model.

ASP Solver Technology

- ASP solvers need to handle two challenging tasks: complex data and search
- Current systems employ a two level architecture with two steps:
- **Grounding** step handles complex data:
  - Given program *P* with variables, generate a set of ground instances of the rules preserving stable models.
  - LP and DDB techniques employed
- **Model search** for ground programs

☞ **ASP = KR + DDB + Search**

Integrating ASP and SMT

- Solvers for the **propositional satisfiability problem (SAT)** are used widely as model search engines.
- Extensions of SAT emerging:
  Satisfiability Modulo Theories (SMT)
- Efficient SMT solvers for expressive theories (integers, reals, uninterpreted function with equality, bit vectors, arrays, ...) are becoming available
- Is it possible to integrate ASP and SMT to exploit the strengths of both approaches?

Integrating ASP and SMT

Two interrelated lines of work:

- Using **SMT solvers as model search engines** for ground programs
- **Combining** ASP and SMT modelling **languages**

Outline

- Stable models and propositional satisfiability
- Stable models and linear constraints
- Satisfiability Modulo Theories
- Translating LPs to SMT
- Integrating ASP and SMT

Stable models

- For a logic program consisting of rules of the form

$$a \leftarrow b_1, \ldots, b_m, \text{not } c_1, \ldots, \text{not } c_n.$$

  a **stable model** is a set of atoms (i) satisfying the rules where (ii) each atom is **justified** by the rules (negation by default).

- **Example.** P: $b \leftarrow .$
  $f \leftarrow b, \text{not } eb.$
  $eb \leftarrow p.$

- $\{b, eb\}$ is not a stable model of P but
  $\{b, f\}$ is the (unique) stable model of P.

- For rules with variables **Herbrand interpretation** used (UNA, DCA):
  stable models of a set of rules are defined to be those of the Herbrand instantiation of the rules.

Stable Models and SAT

- LPs with stable models are closely related to SAT through program completion.
  Example.

P :	Completion $\mathrm{CC}(P)$:
$a \leftarrow b, \mathrm{not}\ c$	$(a \leftrightarrow ((b \wedge \neg c) \vee (\neg b \wedge d))) \wedge$
$a \leftarrow \mathrm{not}\ b, d$	$\neg b \wedge \neg c \wedge \neg d$

- For **tight programs** (no positive recursion) **models of the completion and stable models** coincide (Fages 1994).
- SAT solvers provide an interesting platform for implementing ASP solvers.

Stable Models and SAT

- However, translating general (non-tight) LPs to SAT is challenging
  - Modular translation not possible (I.N. 1999)
  - Without new atoms exponential blow-up (Lifschitz & Razborov 2006)
- There are one pass translations:
  - Polynomial size (Ben-Eliyahu & Dechter 1994; Lin & Zhao 2003)
  - $O(\|P\| \times \log|At(P)|)$ size (Janhunen 2004)
- Also incremental translations have been developed extending the completion dynamically with **loop formulas** (Lin & Zhao 2002)

  ☞ ASSAT and CMODELS ASP solvers

Stable Models and SAT

- Question: what needs to be added to SAT to allow a compact linear size translation of LPs to SAT?
- A possibility: stable models can be characterized using **orderings** (Elkan 1990; Fages 1994).
- Such an ordering can be captured with a restricted set of linear constraints on integers using **level rankings** (I.N. AMAI 2008)
- A suitable simple extension of propositional logic with such restricted linear constraints called **difference logic** is supported by most SMT solvers.

Stable Models and Linear Constraints

- A **level ranking** of a model M is a function assigning positive integers to atoms such that for each atom $a \in M$ there is supporting rule with (i) a as the head, (ii) body true in M and (iii) for each **positive body** atom the **level ranking is smaller** than that of a.
- **Example.** Consider a program P

$p_1 \leftarrow .$
$p_2 \leftarrow p_1.$
$p_3 \leftarrow p_1. \quad p_3 \leftarrow p_4.$
$p_4 \leftarrow p_2. \quad p_4 \leftarrow p_3.$

Function $\mathrm{lr}_1(p_i) = i$ is a level ranking of $M = \{p_1, p_2, p_3, p_4\}$

Theorem (I.N, AMAI 2008)

Let M be a model of the completion of a ground program P. Then M is a stable model of P iff there is a level ranking of M for P.

Satisfiability Modulo Theories

- **Satisfiability Modulo Theories (SMT) problem**: a first-order theory T is given and the problem is to determine whether a formula **F is T-satisfiable** (whether $T \wedge F$ is satisfiable in the usual first-order sense).
- Some restrictions are typically assumed:
  - F is a **ground** (quantifier-free) formula that can contain **free constants** not in the signature of T but all other predicate and function symbols are in the signature of T.
  - **T-satisfiability of a conjunction** of such **ground literals** is **decidable**.

Example: Difference Logic

- T is the theory of integers
- F is limited to contain only linear difference constraints of the form

$$x_i + k \geq x_j \text{ (or equivalently } x_j - x_i \leq k\text{)}$$

  where k is an arbitrary integer constant and $x_i, x_j \in \mathcal{X}$ are free constants (which can be seen as integer valued variables).
- **Difference logic = propositional logic + linear difference constraint**
- For example,

$$(x_1 + 2 \geq x_2) \leftrightarrow (p_1 \rightarrow \neg(x_2 - 3 \geq x_1))$$

  is a formula in difference logic where $2, 3$ are integer constants, x_1, x_2 **free function constants**, and p_1 **a free predicate constant**.

Translating LPs to Difference Logic

- Mapping $T_{\mathrm{diff}}(P)$ of a logic program P to difference logic consists of two parts:
  - completion $\mathrm{CC}(P)$ of P and
  - **ranking constraints** $\mathrm{R}(P)$.
- The completion $\mathrm{CC}(P)$ contains for each atom a having k rules in P, the formula

$$a \leftrightarrow bd_a^1 \vee \cdots \vee bd_a^k$$

and for each such rule a formula

$$bd_a^i \leftrightarrow b_1 \wedge \cdots \wedge b_m \wedge \neg c_1 \wedge \cdots \wedge \neg c_n$$

Ranking Constraints

- The ranking constraints R(P) contain for each atom a which has $k \geq 1$ rules in P, a formula in difference logic

$$a \rightarrow \bigvee_{i=1}^{k} (bd_a^i \wedge (x_a - 1 \geq x_{b_1}) \wedge \cdots \wedge (x_a - 1 \geq x_{b_m}))$$

  where x_a, x_{b_i} are free function constants denoting the rankings of atoms a, b_i.

Example.

P:	CC(P):	R(P):
$p \leftarrow q, \text{not } r.$	$\neg r$	$p \rightarrow (bd_p^1 \wedge (x_p - 1 \geq x_q))$
$q \leftarrow p, \text{not } r.$	$p \leftrightarrow bd_p^1$	$q \rightarrow (bd_q^1 \wedge (x_q - 1 \geq x_p)).$
	$bd_p^1 \leftrightarrow q \wedge \neg r$	
	$q \leftrightarrow bd_q^1$	
	$bd_q^1 \leftrightarrow p \wedge \neg r$	

Difference Logic Captures Stable Models

Theorem (I.N., AMAI 2008)

- *If a set of atoms M is a stable model of a finite normal program P, then there is a satisfying valuation τ of $\mathrm{T}_{\mathrm{diff}}(P)$ such that $M = \{a \in \mathrm{At}(P) \mid \tau(a) = \top\}$.*
- *If there is a satisfying valuation τ of $\mathrm{T}_{\mathrm{diff}}(P)$, then $M = \{a \in \mathrm{At}(P) \mid \tau(a) = \top\}$ is a stable model of P.*

☞ A solver for difference logic can be used for computing stable models.

Observations

- The translation is compact (of linear size).
- It uses a limited subset of difference logic:
  - Level rankings can be captured with constraints of the form $x_i - 1 \geq x_j$
- The translation can be made even more compact and the number of required linear constraints can be reduced dramatically in typical cases by exploiting **strongly connected components** given by the **positive dependency graph** of the program (I.N., AMAI 2008).
- The translation provides a **rich source of benchmarks** for difference logic solvers (wide range of ASP applications, for example, in the ASP competitions).

Experiments

- A translator from ground programs to difference logic which supports a number of variants of the translation available (Janhunen & I.N. & Sevalnev, LPNMR 2009). `http://www.tcs.hut.fi/Software/lp2diff/`
- Any state-of-the-art SMT solver supporting difference logic can be used without modification as the backend solver.
  ☞ The performance obtained by current SMT solvers (Z3, BARCELOGIC, YICES) surprisingly close to the best native ASP solvers (clasp).
  ☞ The same (or slightly better performance) is obtained by carefully bounding the integers, translating the linear constraints to CNF and then using state-of-the-art SAT solvers.

Integrating ASP and SMT

- Goal: combining KR and DDB modelling language features with rich theories offered by SMT solvers.
- A direct approach: rules of the form

$$a \leftarrow b_1, \ldots, b_m, \text{not } c_1, \ldots, \text{not } c_n, t_1, \ldots, t_l.$$

  where $t_1, \ldots, t_l$ are theory literals
- Semantics combines
  - Hebrand interpretation for rules with variables (UNA, DCA)
  - "classical" interpretation for theory atoms

Example: Routing with Real Time Constraints

```
% Data
vtx(a). ... edge(a,b,10.8). ... critical(c,122.5). ...

% Problem specification
{ route(X,Y) } :- edge(X,Y,T).
:- 2 { route(X,Y):edge(X,Y,T) }, vtx(X).
:- 2 { route(X,Y):edge(X,Y,T) }, vtx(Y).
:- route(X,Y), edge(X,Y,T), at(Y) - at(X) < T.
:- vtx(X), at(X) < 0.
r(start).
r(Y) :- r(X), route(X,Y).
missing_critical :- critical(X,T), not r(X).
missing_critical :- critical(X,T), r(X), at(X) > T.
:- missing_critical.
```

Integrating ASP and SMT

- The approach can be implemented by combining
  - ASP grounding techniques
  - the proposed translation to difference logic and
  - an SMT solver supporting difference logic as the model search engine
- For example,

  ```
  :- route(X,Y), edge(X,Y,T), at(Y) - at(X) < T.
  ```

  can be seen as a shorthand for a set of ground rules

  ```
  :- route(s1,s2), edge(s1,s2,s3), at(s2) - at(s1) < s3.
  ```

  where $s1, s2, s3$ range over Herbrand terms and $at(s1), at(s2), s_3$ are treated as **free constants** of the background theory.
- The grounder computes a sufficient set of such ground instances.

Integrating ASP and SMT

- Now the proposed translation can be used to map ground rules to SMT with the following extension in the completion:
- For a rule r of the form

$$a \leftarrow b_1, \ldots, b_m, \text{not } c_1, \ldots, \text{not } c_n, t_1, \ldots, t_l.$$

the formula capturing the satisfaction of the body is now

$$bd_a^i \leftrightarrow b_1 \wedge \cdots \wedge b_m \wedge \neg c_1 \wedge \cdots \wedge \neg c_n \wedge t_1 \wedge \cdots \wedge t_l$$

- Then any SMT solver supporting difference logic + the theory used in the theory literals $t_1, \ldots, t_l$ can be used as the **model search engine**.

Conclusions

- Difference logic allows for a compact translation of rules capturing stable models.
- The translation to difference logic opens up the possibility of using difference logic solvers as a computational platform for implementing ASP.
- The performance obtained by the translation and current SMT solvers is already surprisingly close to the best state-of-the-art ASP solvers.
- The translation provides a basis for combining ASP and SMT modelling languages

References

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