Sine Qua Non for Large Theory Reasoning

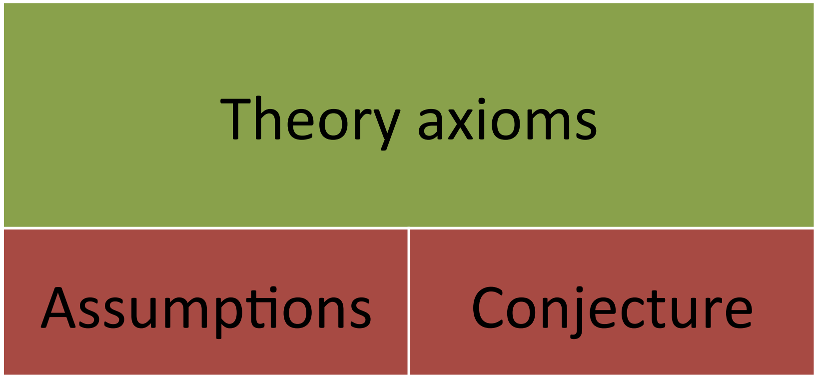
Kryštof Hoder

Andrei Voronkov

Large Theory Reasoning in First-Order Logic

- Traditional FO problems
  - Not too many axioms
  - Axiomatizations of algebras, set theory
- Large theory problems
  - Many axioms, but most of them are irrelevant to the conjecture
- Axiom selection
  - attempts to remove the irrelevant and keep the important

Structure of a First-Order problem



Sources of Large Theory Problems

- Ontology reasoning
  - SUMO, YAGO, CyC
  - Up to 10m axioms
  - Proofs involve few axioms, almost no equalities
- Mathematical libraries
  - Mizar Mathematical Library
  - Tens of thousands axioms
  - More complex proofs, equalities

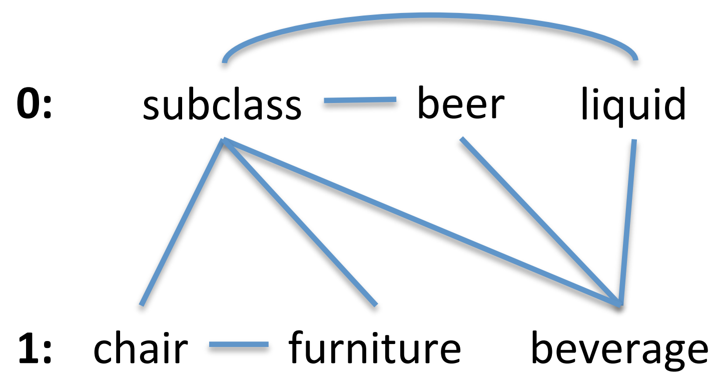
Idea: Simple Relevance

- Based on mutual occurrences of symbols in axioms
- Symbol *s* is 0-relevant if it occurs in the goal
- If s is *d*-relevant and appears in A, A and its symbols are (*d*+1)-relevant
- Select axioms *d*-relevant to the conjecture (and assumptions)

  $d \in \{1,\ldots,\infty\}$

subclass(beverage, liquid)
subclass(beer, beverage)
subclass(chair, furniture)

? subclass(beer, liquid)

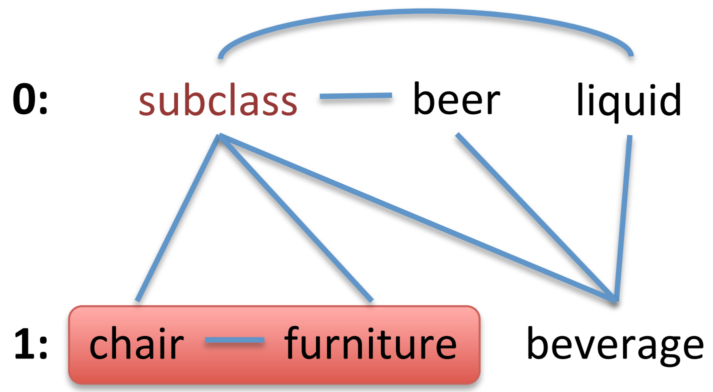


Problem: Common Symbols

- With this notion of relevance almost all axioms are usually selected
- Common symbols (such as ‘subclass’ or ‘subsumes’) make relevant otherwise unrelated symbols

subclass(beverage, liquid)
subclass(beer, beverage)
subclass(chair, furniture)

? subclass(beer, liquid)



Solution: Trigger-Based Selection

- We had

  If s is *d*-relevant and <u>appears in</u> A, A and its symbols are (*d*+1)-relevant

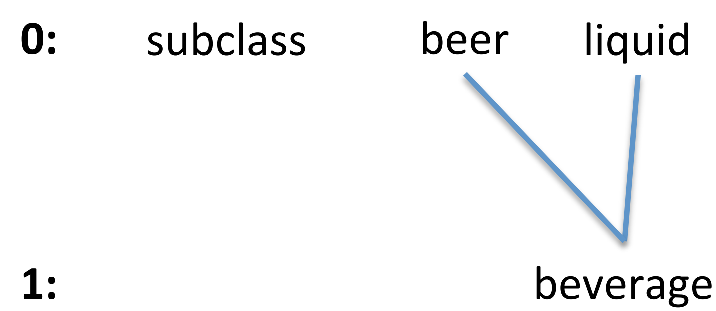
- Assuming a 'triggers' relation between symbols and axioms, we can write

  If s is *d*-relevant and <u>triggers</u> A, A and its symbols are (*d*+1)-relevant

subclass(beverage, liquid)
subclass(beer, beverage)
subclass(chair, furniture)

? subclass(beer, liquid)

We want:



What Is a Common Symbol?

- There is no a priori information on symbol commonness
- We approximate it by number of occurrences
  - more common symbols appear in more axioms

subclass(beverage, liquid)
subclass(beer, beverage)
subclass(chair, furniture)

Occ.	Symbols
3	subclass
2	beverage
1	liquid, beer, chair, furniture

‘Triggers’ relation

Occ.	Symbols
3	subclass
2	beverage
1	liquid, beer, chair, furniture

- Should penalize common symbols
- But not ignore them completely

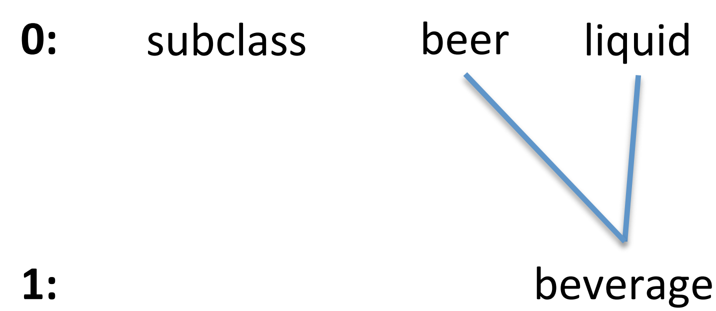
  subclass(x, y) ∧ subclass(y, z)
  → subclass(x, z)

- Our solution:

Only the least common symbols trigger an axiom.

subclass(beverage, liquid)
subclass(beer, beverage)
subclass(chair, furniture)

? subclass(beer, liquid)



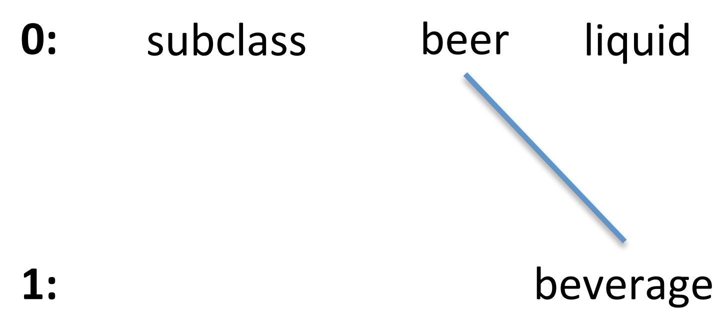
Incompleteness, Unstability

- Small difference in number of occurrences can lead to loss of important axioms

1: subclass(X,Y) ∧ subclass(Y,Z) → subclass(X,Z)
subclass(petrol,liquid)
¬subclass(stone,liquid)
2: subclass(beverage,liquid)
1: subclass(beer,beverage)
subclass(guinness,beer)

Occ.	Symbols
7	subclass
3	liquid
2	beer, beverage
1	petrol, stone, guinness

? subclass(beer,liquid)



Incompleteness, Unstability

- Small difference in number of occurrences can lead to loss of important axioms
- Or simply too little axioms may be selected
- We need a possibility to extend the 'triggers' relation

1: subclass(X,Y) ∧ subclass(Y,Z) → subclass(X,Z)
subclass(petrol,liquid)
¬subclass(stone,liquid)
subclass(beverage,liquid)
1: subclass(beer,beverage)
subclass(guinness,beer)
subclass(pilsner,beer)

Occ.	Symbols
7	subclass
3	liquid, beer
2	beverage
1	petrol, stone, guinness, pilsner

? subclass(beer,liquid)

0: subclass beer liquid

Tolerance

- We had

  Only the least common symbols trigger an axiom

- Having tolerance parameter t
  - Only symbols with *t* times more occurrences than the least common symbol trigger an axiom
- For t=∞ the selection degrades to the simple relevance

t=1.5:

1: subclass(X,Y) ∧ subclass(Y,Z) → subclass(X,Z)
subclass(petrol,liquid)
¬subclass(stone,liquid)
2: subclass(beverage,liquid)
1: subclass(beer,beverage)
subclass(guinness,beer)
subclass(pilsner,beer)

Occ.	Symbols
7	subclass
3	liquid, beer
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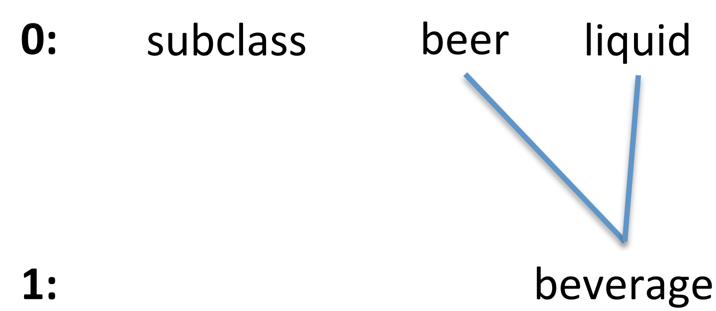

Experiments

Two parameters:

t Tolerance

d Depth limit (selects *d*-relevant axioms)

Problem sizes

problems	average size (axioms)	average size (atoms)
SUMO	298,420	323,170
CYC	3,341,990	5,328,216
Mizar	44,925	332,143

Numbers of selected axioms

SUMO:

$d \backslash t$	1.0	1.2	1.5	2.0	3.0	5.0
1	12	13	14	16	21	28
2	70	82	115	158	272	654
3	188	230	372	762	1950	5980
4	316	470	942	3021	8720	23440
5	540	979	2417	8179	22644	52241
7	1027	2708	8517	24445	54958	97481
∞	1116	8361	26959	57322	82379	107926

CYC:

$d \backslash t$	1.0	1.2	1.5	2.0	3.0	5.0
1	29	35	41	47	60	72
2	142	287	442	607	1027	1476
3	505	937	1451	2484	5311	10482
4	1784	3232	5716	11603	29963	69015
5	4432	8870	16806	37599	110186	249192
7	10698	25607	56337	150277	431875	832935
∞	36356	495360	1310965	1562064	1822427	2057597

Mizar:

$d \backslash t$	1.0	1.2	1.5	2.0	3.0	5.0
1	4903	4911	4921	4936	4973	5038
2	5296	5395	5553	5823	6427	7743
3	6118	6451	7068	8280	10841	16337
4	6893	7556	9001	12176	18300	28878
5	7432	8517	11165	16945	26842	37284
7	7897	9991	15788	26203	36507	41443
∞	8047	15987	28353	35345	39389	41762

Experiments

Solved problems

atoms	only with Sine	only without Sine	together
10,000	243	64	721
20,000	217	10	542
40,000	208	7	464
80,000	187	3	373
160,000	138	1	243
320,000	80	1	168
640,000	50	0	100
1,280,000	50	0	50
rating 1	232	25	402

Implemented in Vampire (http://vprover.org)

```
vampire --mode axiom_selection --sine_selection axioms
        --sine_tolerance t --sine_depth d
```