First-Order Deduction for Large Knowledge Bases

Stephan Schulz
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Interest in Large Theories

2007 ESARLT at CADE-21 in Bremen

- Ontologies (SUMO)
- Common Sense Reasoning (CYC)
- Mizar

2008 First CASC LTB at IJCAR in Sydney

- MaLARea, SInE, Vampire LTB

2009 CASC LTB at CADE-22 in Montreal

2010 CASC LTB in IJCAR in Edinburgh

- Rules change every year!

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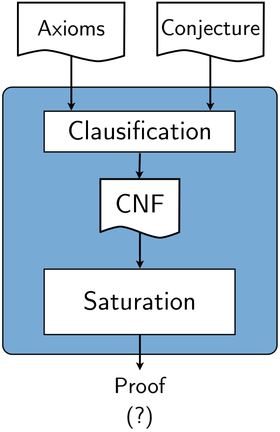
Saturation-Based Theorem Proving

Leading approach for first-order reasoning

- Proof by refutation
- Conjecture and axioms converted to flat CNF
- Saturation with destructive simplification
- Goal: Empty clause
- Powerful calculi
- Highly advanced data structures
- Decent search heuristics



A Classical Example: Reasoning in Rings

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RNG009-5: Peterson/Stickel, 1981

A ring with $x^3 = x$ is commutative

- 9 formulas
- 3 KB of text (with comments)

Proof search

- ~200000 steps
- ~50 MB of text

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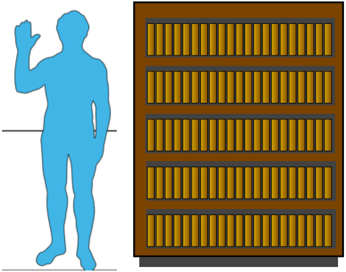

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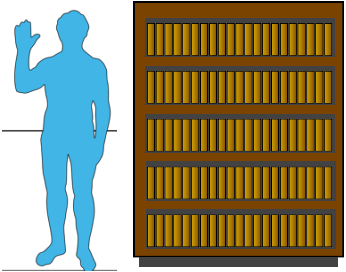
CSR066-6: Smith et al, 2007

OpenCYC "Common sense" reasoning

- 3341990 formulas
- 480 MB of text (with few comments)



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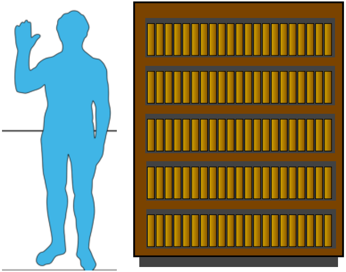

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Bad Ideas for Mega-Axiom Problems

“First simplify the whole specification”

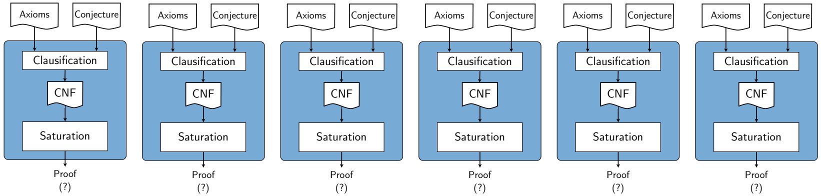
“First consider *all* axioms”

Quadratic or cubic algorithms

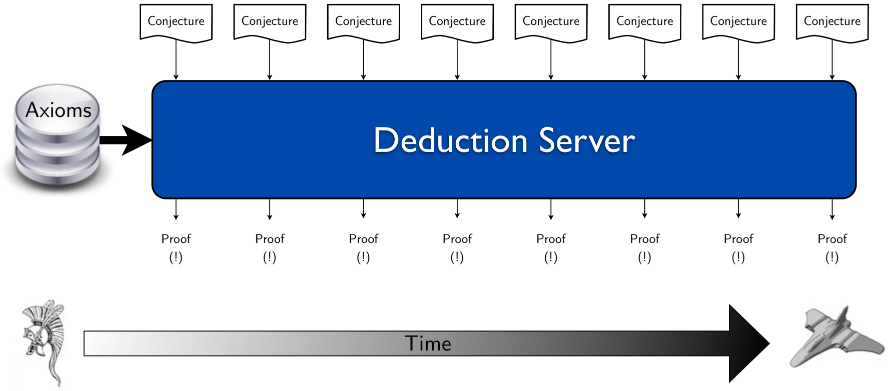
Undirected saturation

Reload the specification for each new conjecture

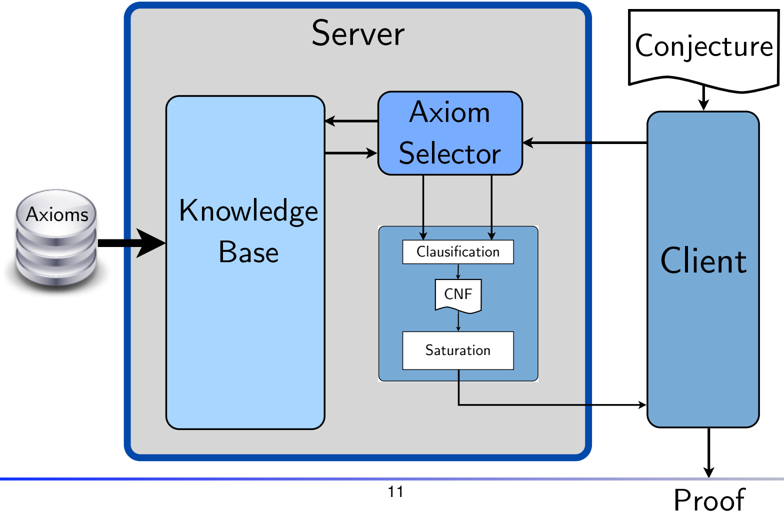
Multiple Queries the Hard Way



Deduction as a Service



Deduction Server Architecture



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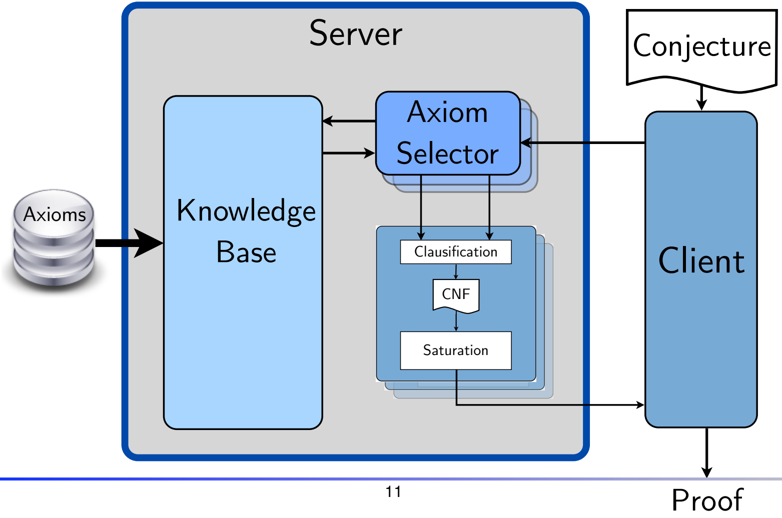
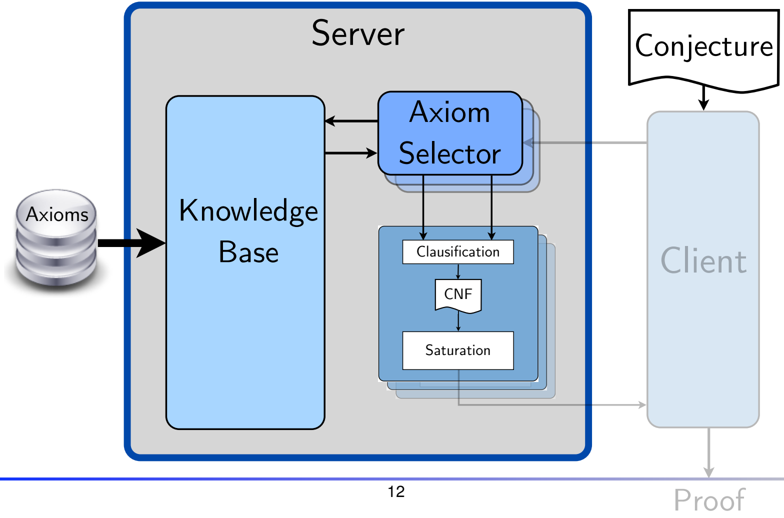
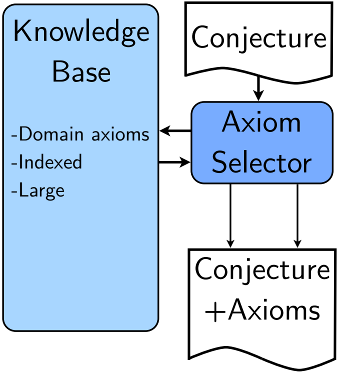


Current State



Axiom Selection



Input

- Conjecture
- Optionally hypotheses

Output

- "Relevant" axioms
- Conjecture

Constraints

- Backtrackable
- Fast (enough)

Relevancy Analysis

Algorithm:

- Start with the conjecture
- All symbols in the conjecture are relevant
- All formulas containing relevant symbols are relevant
- Iterate for n steps or up to fixpoint

Relevancy Analysis

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- All symbols in the conjecture are relevant
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- Iterate for n steps or up to fixpoint

Result: Unsatisfactory

- Some successes
- But: Selection to coarse
- Typically 5-7 iterations to fixpoint
- Often fixpoint is whole specification

Generalized SInE

Basic d-relation:

- Generality measure $g : sig \rightarrow \mathbb{N}$
- Formula C is in d-relation to f if f is $g-$minimal among all symbols in C

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Generalization:

- Different generality measures
- Different relaxation criteria for d-relations
- Limit iteration
- Limit number of formulas selected

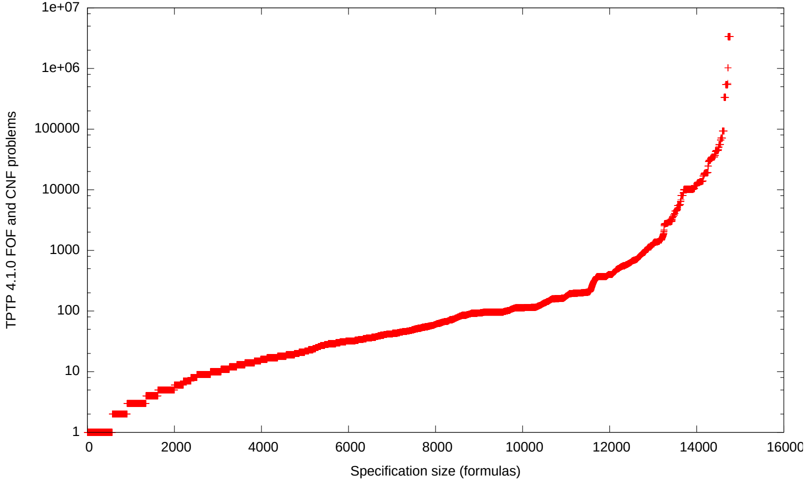
First Results

Platform:

- 800 Mhz Quad-Core AMD Opteron
- 8GBytes RAM

CSR066+6	Plain E	Server Mode
Parsing Problem	109s	Amortised
Axiom Selection	-	35 s
Clausification	240s	0.9 s
Proof	Timeout ($>$1200 s)	348 s

Evaluation: Large TPTP Problems



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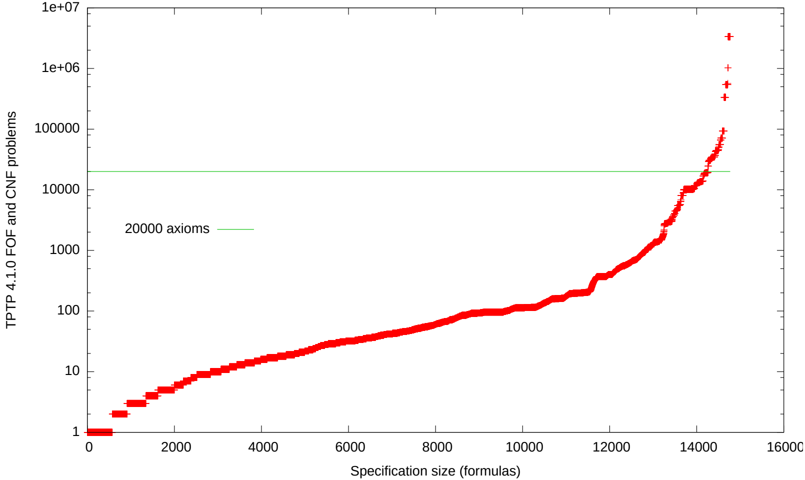


Selected Results

TPTP 4.1.0

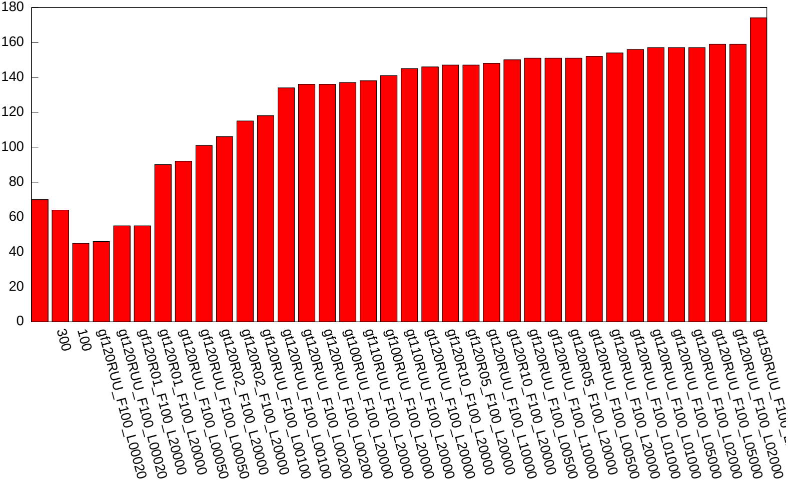
512 problems with $\geq$2000 axioms

Platform:

- 2.2 GHz Intel Xeon
- 16 GBytes

Strategy	Solutions
Full, 300 s	70
Full, 100 s	64
GSInE(f120,RUU,F100,L00020)	45
GSInE(t120,RUU,F100,L00050)	90
GSInE(f120,RUU,F100,L00100)	115
GSInE(t100,RUU,F100,L20000)	136
GSInE(t120,RUU,F100,L20000)	145
GSInE(t120,R10,F100,L20000)	148
GSInE(t120,RUU,F100,L00500)	151
GSInE(f120,RUU,F100,L01000)	154
GSInE(t120,RUU,F100,L05000)	157
GSInE(t150,RUU,F100,L20000)	159

All Results


180
160
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Future Work

Complete server

- TCP connection
- Configurability
- Failure-tolerant parser

Implement client

- Telnet and netcat are sufficient but inconvenient

Evaluate different SInE variants

- Which paramters work for which problem clases?
- Which other generality measures are useful?

OpenCYC "Common sense" reasoning

- 3341990 formulas
- 480 MB of text (with few comments)

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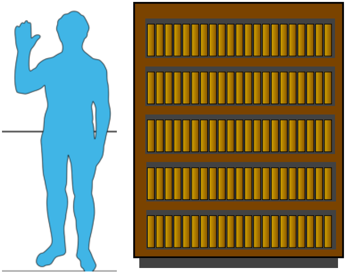

CSR066-6: Smith et all, 2007

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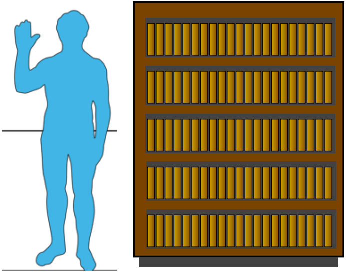

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