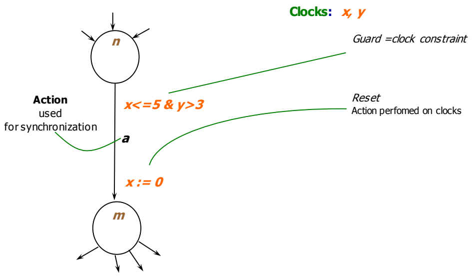
OUTLINE

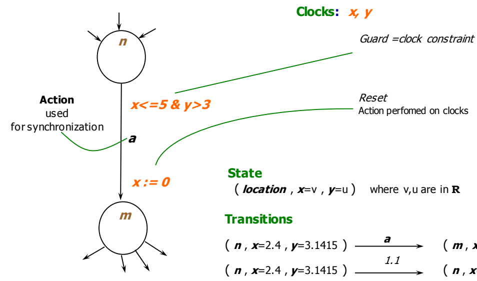
- Model Checking in a Nutshell
- Timed automata and TCTL
- A UPPAAL Tutorial
  - Data stuctures & central algorithms
  - UPPAAL input languages

Timed Automata, TCTL & Verification Problems

Timed Automata: Syntax



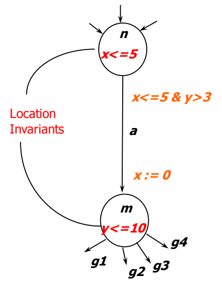
Timed Automata: Semantics



State
(*location* , *x*=v , *y*=u)   where v,u are in R

Transitions
(*n* , *x*=2.4 , *y*=3.1415) —*a*→ (*m* , *x*=0 , *y*=3.1415)
(*n* , *x*=2.4 , *y*=3.1415) —*1.1*→ (*n* , *x*=3.5 , *y*=4.2415)

Timed Automata with *Invariants*

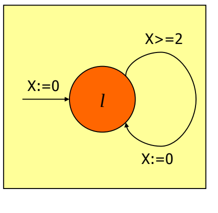


Clocks: *x, y*

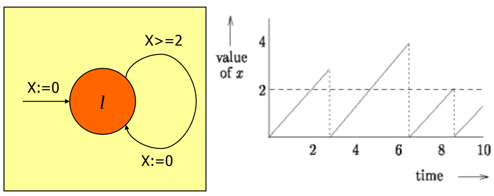
Transitions
(*n* , *x*=2.4 , *y*=3.1415) —~~3.2~~→
(*n* , *x*=2.4 , *y*=3.1415) —*1.1*→ (*n* , *x*=3.5 , *y*=4.2415)

Invariants insure progress!!

Timed Automata: Example



Timed Automata: Example



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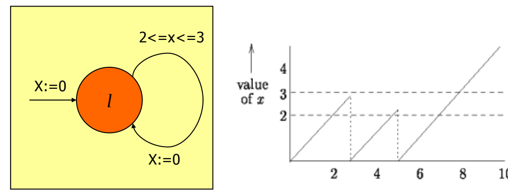
Timed Automata: Example



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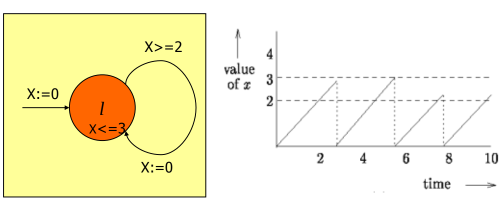
Timed Automata: Example



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Timed Automata

=

Finite Automata + Clock Constraints + Clock resets

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Clock Constraints

g ::= x ~ n | g & g

where

- x is a clock variable
- $\sim \in \{<, >, \leq, \geq\}$
- n is a natural number

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Semantics (definition)

- ***clock valuations***: $V(C) \quad v: C \rightarrow R_{\geq 0}$
- ***state***: (l, v) where $l \in L$ and $v \in V(C)$
- ***action transition*** $(l,v) \xrightarrow{a} (l',v')$ iff $l \xrightarrow{g\ a\ r} l'$ $g(v)$ and $v' = v[r]$ and $Inv(l')(v')$
- ***delay Transition*** $(l,v) \xrightarrow{d} (l, v+d)$ iff $Inv(l)(v+d')$ whenever $d' \leq d \in R_{\geq 0}$

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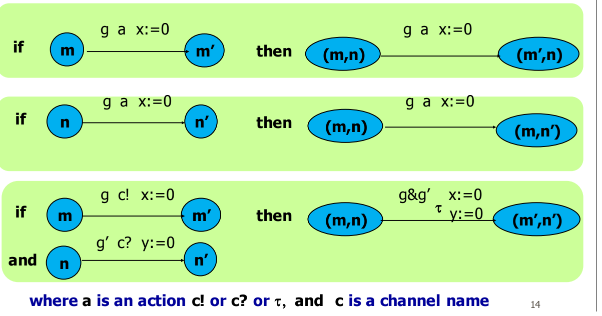
Modeling Concurrency

- Products of automata
- CCS Parallel composition
  - implemented in UPPAAL

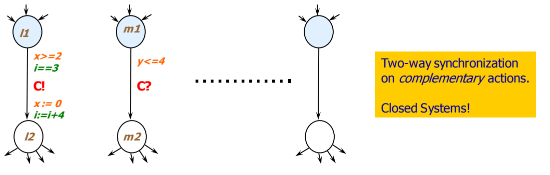
CCS Parallel Composition (implemented in UPPAAL)



where a is an action c! or c? or τ, and c is a channel name

The UPPAAL Model

= Networks of Timed Automata + Integer Variables +….



Example transitions

(l1, m1,………, x=2, y=3.5, i=3,…..) $\xrightarrow{\tau}$ *(l2,m2,……..,x=0, y=3.5, i=7,…..)*

Verification Problems

Location Reachability (def.)

n is reachable from m if there is a sequence of transitions:

$$(m, u) \longrightarrow^* (n, v)$$

(Timed) Language Inclusion, L(A) ⊆ L(B)

$(a_0, t_0)\ (a_1, t_1)\ \ldots\ldots\ (a_n, t_n) \in L(A)$

If

"A can perform a_0 at t_0, $a_1$at t_1 … … a_n at t_n"

$(l_0, u_0) \xrightarrow{t_0} (l_0, u_0+t_0) \xrightarrow{a_0} (l_1, u_1) \ldots\ldots$

Verification Problems

- Timed Language Equivalence & Inclusion ☹
  - 1-clock, finite traces, decidable [Ouaknine & Worrell 04]
  - 1-clock, infinite traces & Buchi-conditions, undecidable [Abdulla et al 05]
- Universality ☹
- Untimed Language Inclusion ☺
- (Un)Timed (Bi)simulation ☺
- Reachability Analysis/Emptiness ☺
- Optimal Reachability (synthesis problem) ☺
  - If a location is reachable, what is the minimal delay before reaching the location?

Timed CTL = CTL + clock constraints

Note that the semantics of TA defines a transition system where each state has a Computation Tree

Computation Tree Logic, CTL

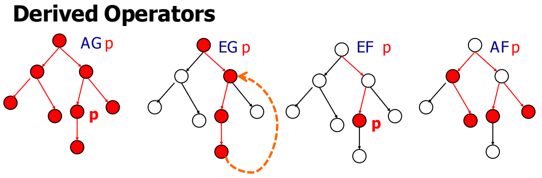
Clarke & Emerson 1980

Syntax

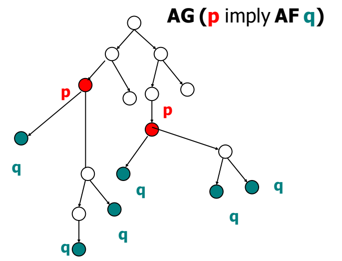
ϕ :: = P | ¬ ϕ | $\phi \vee \phi$ | EX ϕ | E[ϕ U ϕ] | A[ϕ U ϕ]

where **P** ∈ AP (atomic propositions)

Derived Operators



Liveness: p - -> q *"p leads to q"*



Timed CTL (a simplified version)

Syntax

ϕ :: = p | ¬ ϕ | $\phi \vee \phi$ | EX ϕ | E[ϕ U ϕ] | A[ϕ U ϕ]

where **p** ∈ AP (atomic propositions) **or Clock constraint**

Timed CTL (a simplified version)

Syntax

ϕ :: = p | ¬ ϕ | $\phi \vee \phi$ | EX ϕ | E[ϕ U ϕ] | A[ϕ U ϕ]

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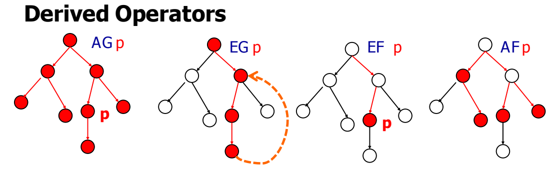
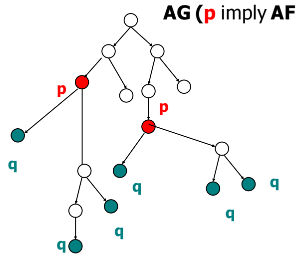
Derived Operators



A[] P in UPPAAL | **E[] P in UPPAAL** | **E<> P in UPPAAL** | **A<> P in UPPAAL**

Derived Operators (cont.)

AG (p imply **AF q)**



p - -> q in **UPPAAL**

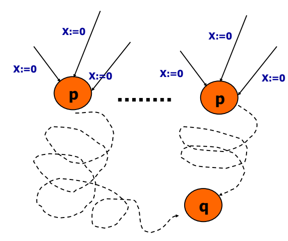
Bounded Liveness

[TACAS 98]

Verify: **"whenever p is true, q should be true within 10 sec**

P - - > **(**q and x<10**)**

Use extra clock x
Add x:=0 on all edges leading to P



Bounded Liveness/Responsiveness

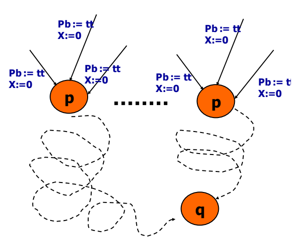
(reachability analysis, more efficient?)

[TACAS 98]

Verify: **"whenever p is true, q should be true within 10 sec**

AG ((P_b and x>10) imply q)

Use extra clock x and boolean P_b
Add P_b := tt and x:=0 on all edges leading to location P



Bounded Liveness/Responsiveness

(reachability analysis, more efficient?)

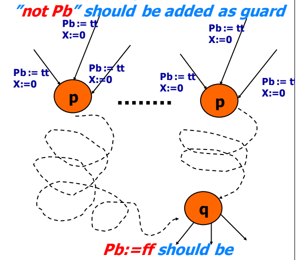
[TACAS 98]

This is not really correct; "not Pb" should be added as guard

Verify: **"whenever p is true, q should be true within 10 sec**

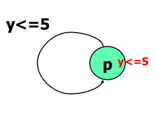
AG ((P_b and x>10) imply q)

Use extra clock x and boolean P_b
Add P_b := tt and x:=0 on all edges leading to location P

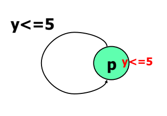


Pb:=ff should be On all eadges leaving q

Problem with Zenoness/Time-stop

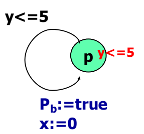


EXAMPLE



We want to specify "whenever P is true, Q should be true within 10 time units

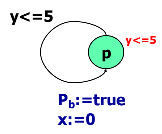
EXAMPLE



We want to specify **"whenever P is true, Q should be true within 10 time units**

AG ((P$_b$ and x>10) imply Q)

EXAMPLE



We want to specify **"whenever P is true, Q should be true within 10 time units**

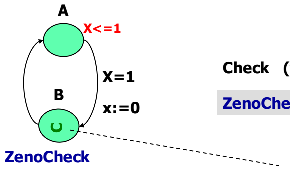
AG ((P$_b$ and x>10) imply q)

is satisfied !!!

Solution with UPPAAL

Check Zeno-freeness by an extra observer

System || ZenoCheck



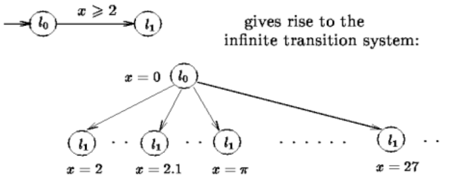
Check (yes means "no zeno loops")

ZenoCheck.A - - > ZenoCheck.B

Committed location!

REACHABILITY ANALYSIS
using Regions

Infinite State Space!

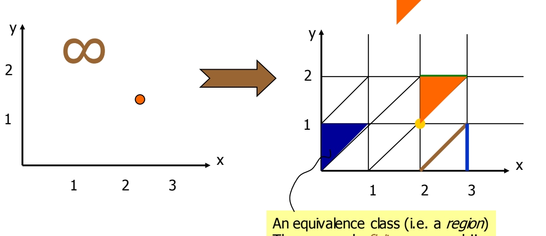


However, the reachability problem is decidable ☺ Alur&Dill 1991

Region: From infinite to finite

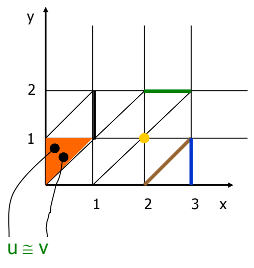
Concrete State
(n, x=2.2, y=1.5)

Symbolic state (region)
(n,)



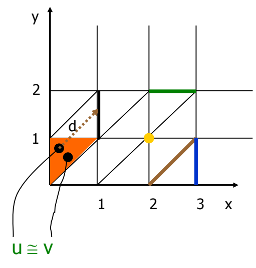
An equivalence class (i.e. a *region*)
There are only *finite* many such!!

Region equivalence (Intuition)



u ≅ v iff (l,u) and (l,v) may reach the same set of eqivalence classes

Region equivalence (Intuition)



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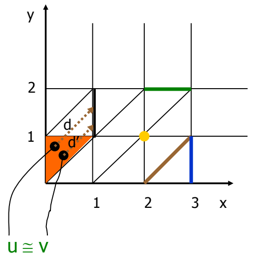
Region equivalence (Intuition)

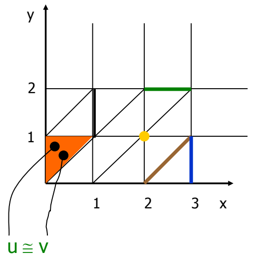


u ≅ v iff (l,u) and (l,v) may reach the same set of eqivalence classes

Region equivalence *[Alur and Dill 1990]*

- u,v are clock assignments
- u≈v iff
  - For all clocks x,
    either (1) u(x)>Cx and v(x)>Cx
    or (2) ⌊u(x)⌋=⌊v(x)⌋
  - For all clocks x, if u(x)<=Cx,
    {u(x)}=0 iff {v(x)}=0
  - For all clocks x, y, if u(x)<=Cx and u(y)<=Cy
    {u(x)}<= {u(y)} iff {v(x)}<= {v(y)}

Region equivalence (alternatively)

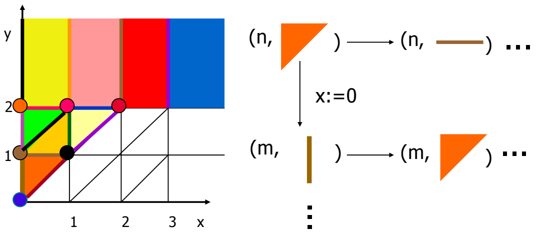


u ≅ v iff u and v satisfy exactly the same set of constraints in the form of
xi ~ m and xi-xj ~ n
where ~ is in {<,>,≤,≥}
and m,n < MAX

This is not quite correct; we need to consider the MAX more carefully

Region Graph

Finite-State Transition System!!



OBS: there are only Finite many regions

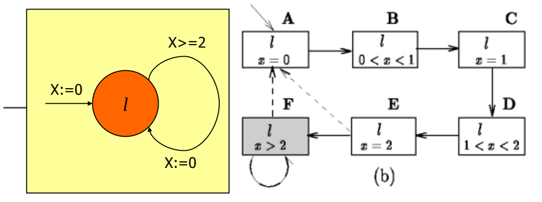
(m, [u]) ⟶ (n, [v]) if (m, u) ⟶ (n,v)

Theorem

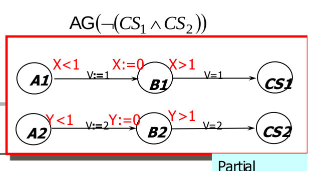
u≈v implies
- u(x:=0) ≈ v(x:=0)
- u+n ≈ v+n for all natural number n
- for all d<1: u+d ≈ v+d' for some d'<1

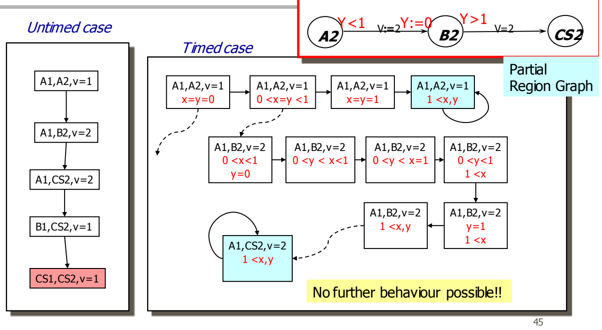
"Region equivalence' is preserved by "addition" and reset.
(also preserved by "subtraction" if clock values are "bounded")

Region graph of a simple timed automata



Fischers again





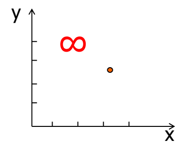
Problems with Region Construction

- Too many 'regions'
  - Sensitive to the maximal constants
  - e.g. x>1,000,000, y>1,000,000 as guards in TA
- The number of regions is highly exponential in the number of clocks and the maximal constants.

REACHABILITY ANALYSIS using ZONES

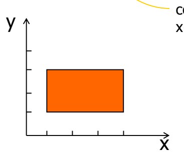
Zones: From infinite to finite

State
(n, x=3.2, y=2.5)

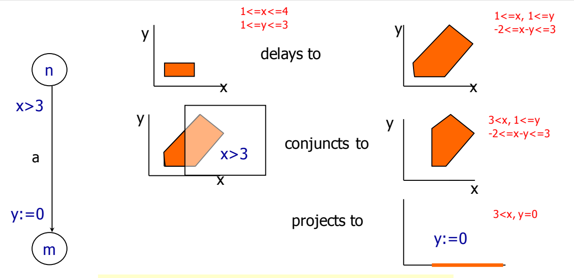


Symbolic state (zone)
(n, 1≤x≤4,1≤y≤3)

Zone:
conjunction of
x-y~n, x~n

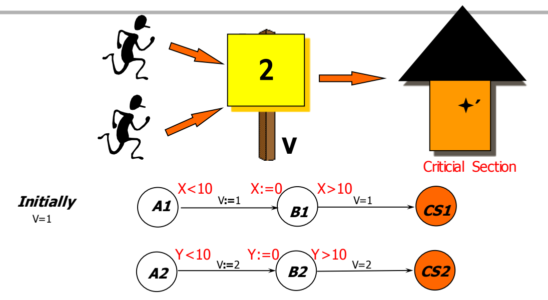


Symbolic Transitions

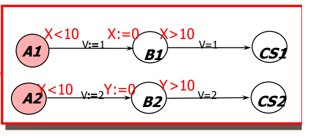


Thus (n, 1<=x<=4,1<=y<=3) =a=> (m, 3<x, y=0)

Fischer's Protocol
analysis using zones



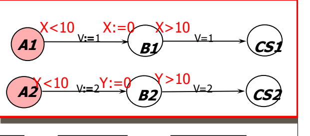
Fischers cont.



Untimed case

A1,A2,v=1 → A1,B2,v=2 → A1,CS2,v=2 → B1,CS2,v=1 → CS1,CS2,v=1

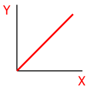
Fischers cont.



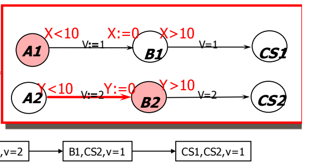
Untimed case

A1,A2,v=1 → A1,B2,v=2 → A1,CS2,v=2 → B1,CS2,v=1 → CS1,CS2,v=1

Taking time into account



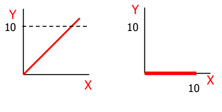
Fischers cont.



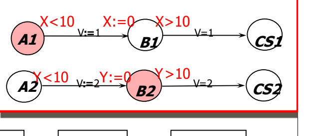
Untimed case

A1,A2,v=1 → A1,B2,v=2 → A1,CS2,v=2 → B1,CS2,v=1 → CS1,CS2,v=1

Taking time into account



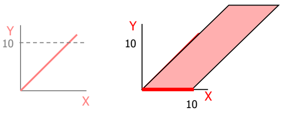
Fischers cont.



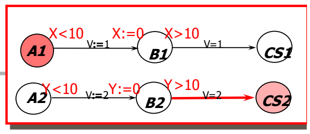
Untimed case

A1,A2,v=1 → A1,B2,v=2 → A1,CS2,v=2 → B1,CS2,v=1 → CS1,CS2,v=1

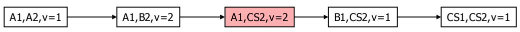
Taking time into account



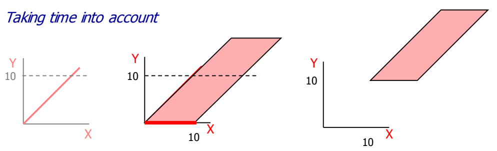
Fischers cont.



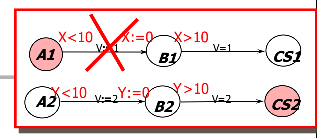
Untimed case



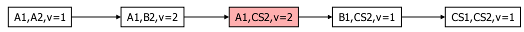
Taking time into account



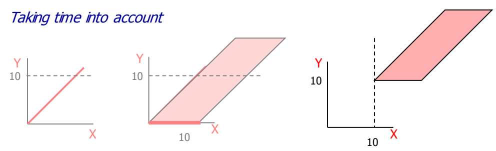
Fischers cont.



Untimed case



Taking time into account



Zones = Conjuctive constraints

- A zone Z is a conjunctive formula:
  g_1 & g_2 & ... & g_n
  where g_i may be $x_i \sim b_i$ or $x_i - x_j \sim b_{ij}$
- Use a zero-clock x_0 (constant 0), we have
  $\{x_i - x_j \sim b_{ij} \mid \sim \text{ is } < \text{ or } \leq, i,j \leq n\}$
- This can be represented as a MATRIX, DBM
  (Difference Bound Matrices)

Solution set as semantics

- Let Z be a zone (a set of constraints)
- Let [Z]={u | u is a solution of Z}

(We shall simply write Z instead [Z])

Operations on Zones

- Post-condition (Delay): SP(Z) or Z↑
  - [Z↑] = {u+d| d ∈ R, u∈[Z]}
- Pre-condition: WP(Z) or Z↓ (the dual of Z↑)
  - [Z↓] = {u| u+d∈[Z] for some d∈R}
- Reset: {x}Z or Z(x:=0)
  - [{x}Z] = {u[0/x] | u ∈[Z]}
- Conjunction
  - [Z&g]= [Z]∩[g]

Two more operations on Zones

- Inclusion checking: $Z_1 \subseteq Z_2$
  - solution sets
- Emptiness checking: Z = Ø
  - no solution

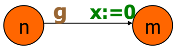
Theorem on Zones

The set of zones is closed under all zone operations

- That is, the result of the operations on a zone is a zone
- Thus, there will be a zone to represent the sets: [Z↑], [Z↓], [{x}Z]

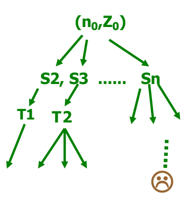
One-step reachability: Si ⁓ Sj

- Delay: (n,Z) → (n,Z') where Z'= Z↑ ∧ inv(n)
- Action: (n,Z) → (m,Z') where Z'= {x}(Z ∧**g**)

if 

- **Reach**: (n,Z) ⁓ (m,Z') if (n,Z) →→(m,Z')
- **Successors**(n,Z)={(m,Z') | (n,Z) ⁓(m,Z'), Z'≠Ø}

Now, we have a search problem



EF ☹