Fixed-Priority Multiprocessor Scheduling
[RTAS 2010]

Joint work with
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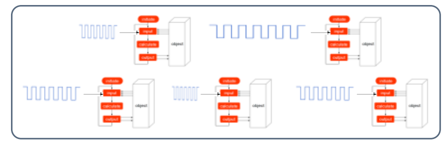
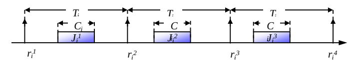
Real-time Systems



- N periodic tasks (of different rates/periods)



Utilization/workload: C_i/T_i

- How to schedule the jobs to avoid deadline miss?

On Single-processors

- Liu and Layland's Utilization Bound [1973]
  (the 19th most cited paper in computer science)

$$\sum_{\tau_i \in \tau} U_i \leq N(2^{1/N} - 1)$$

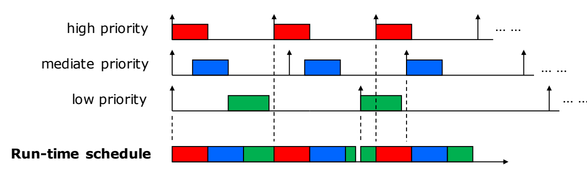
$\Rightarrow$ the task set is schedulable

number of tasks

- $N \rightarrow \infty, \quad N(2^{1/N} - 1) = 69.3\%$
- Scheduled by *RMS (Rate Monotonic Scheduling)*

Rate Monotonic Scheduling

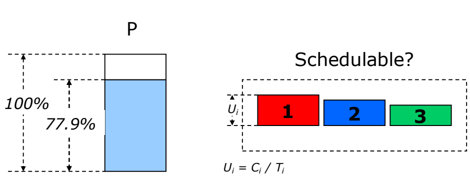
- Priority assignment: shorter period → higher prio.
- Run-time schedule: the highest priority first



- **How to check whether all deadlines are met?**

Liu and Layland's Utilization Bound

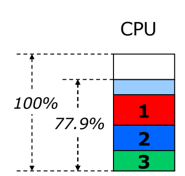
- Schedulability Analysis



Liu and Layland's bound:
$3 \times (2^{1/3} - 1) = 77.9\%$

Liu and Layland's Utilization Bound

- Schedulability Analysis



Yes, schedulable!

Liu and Layland's bound:
$3 \times (2^{1/3} - 1) = 77.9\%$

Multiprocessor (multicore) Scheduling

- Significantly more difficult:
  - Timing anomalies
  - Hard to identify the worst-case scenario
  - Bin-packing/NP-hard problems
  - Multiple resources e.g. caches, bandwidth
  - … …

Open Problem (since 1973)

- Find a multiprocessor scheduling algorithm that can achieve Liu and Layland's utilization bound

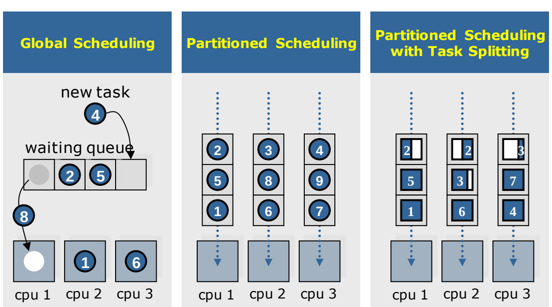
$$\frac{\sum C_i/T_i}{M} \leq N(2^{1/N}-1)$$

$\Rightarrow$ the task set is schedulable

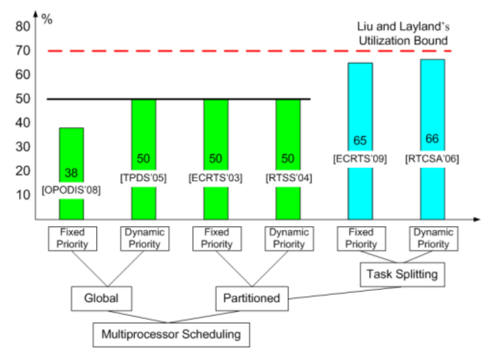
?

number of processors

Multiprocessor Scheduling



Best Known Results (before 2010)



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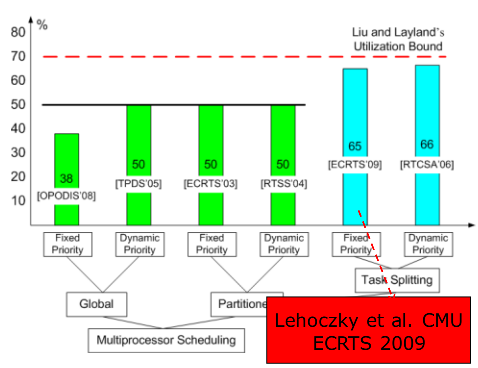

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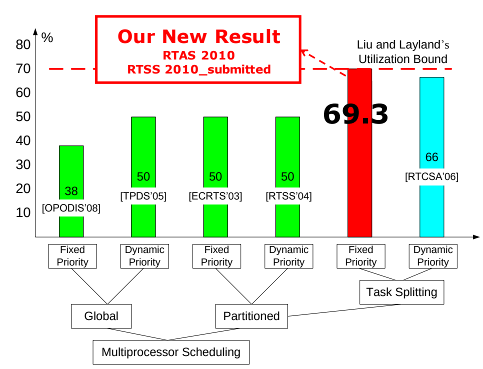
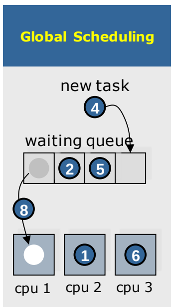
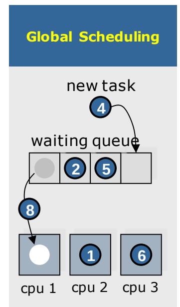


Multiprocessor Scheduling



Would fixed-priority scheduling e.g. "RMS" work?

Multiprocessor Scheduling

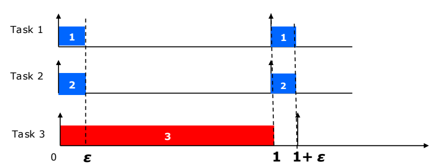


Would fixed-priority scheduling e.g. "RMS" work?

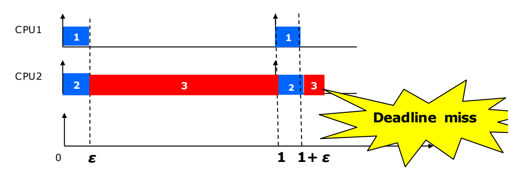
Unfortunately "RMS" suffers from the Dhall's anomali

Utilization may be "0%"

Dhall's anomali



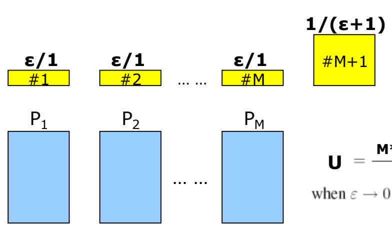
Dhall's anomali



Schedule the 3 tasks on 2 CPUs using "RMS

Dhall's anomali

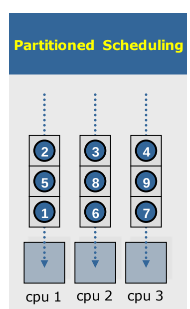
(M+1 tasks and M processors)



$$\mathbf{U} = \frac{M*\varepsilon + 1/(1+\varepsilon)}{M} \rightarrow \mathbf{0}$$

when $\varepsilon \rightarrow 0$ and $M \rightarrow +\infty$

Multiprocessor Scheduling



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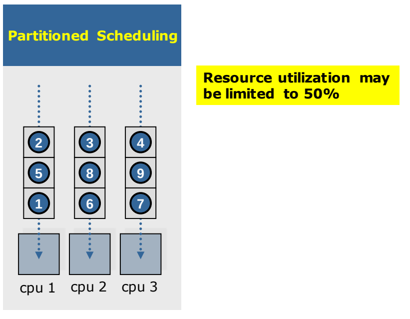
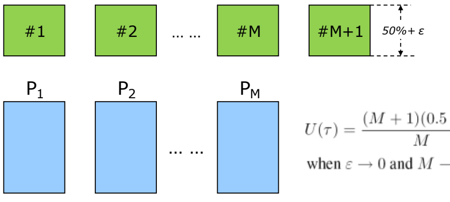


Partitioned Scheduling

- The Partitioning Problem is similar to Bin-packing Problem (NP-hard)
- Limited Resource Usage, 50%

$\sum C_i/T_i \le 1$ necessary condition to guarantee schedulability



$$U(\tau) = \frac{(M+1)(0.5+\varepsilon)}{M} \to 0.5$$

when $\varepsilon \to 0$ and $M \to +\infty$

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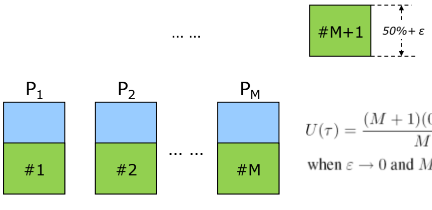

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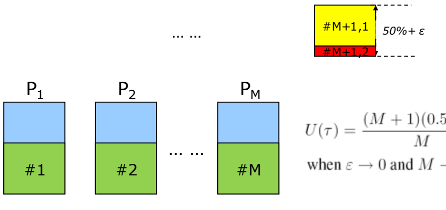

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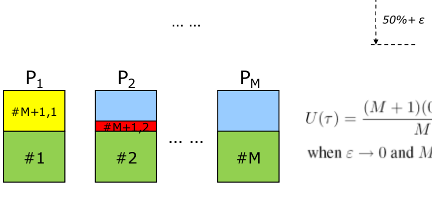

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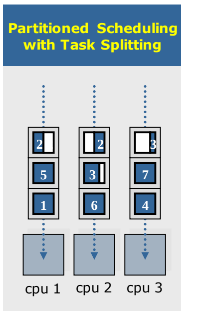
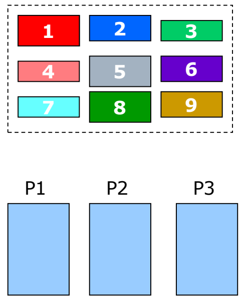
Multiprocessor Scheduling



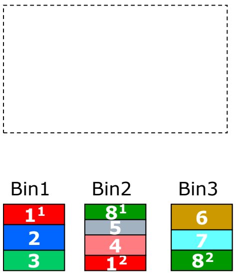
Partitioned Scheduling

- Partitioning



Bin-Packing with Item Splitting

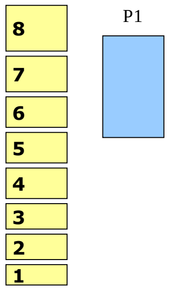
- Resource can be “fully” (better) utilized



Previous Algorithms

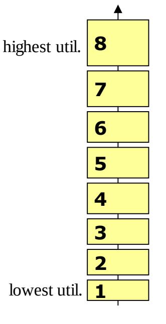
[Kato et al. IPDPS’08] [Kato et al. RTAS’09] [Lakshmanan et al. ECRTS’09]

- Sort the tasks in some order e.g. utilization or priority order
- Select a processor, and assign as many tasks as possible



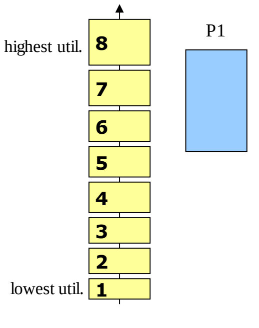
Lakshmanan’s Algorithm [ECRTS’09]

- Sort all tasks in decreasing order of utilization



Lakshmanan’s Algorithm [ECRTS’09]

- Pick up one processor, and assign as many tasks as possible



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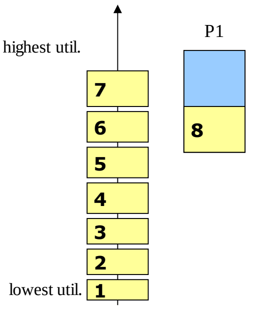

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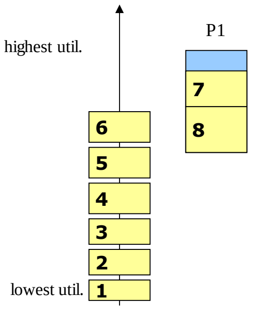

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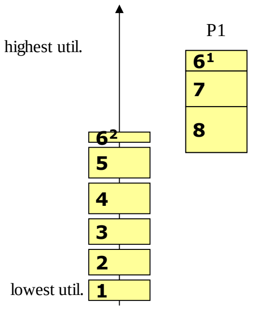

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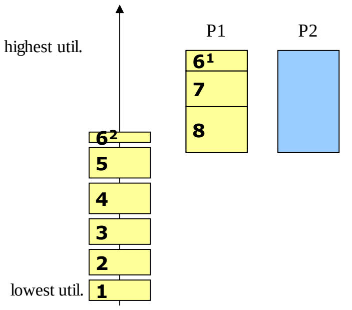

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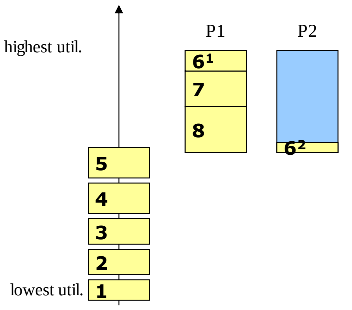

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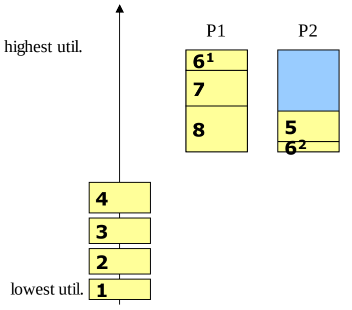

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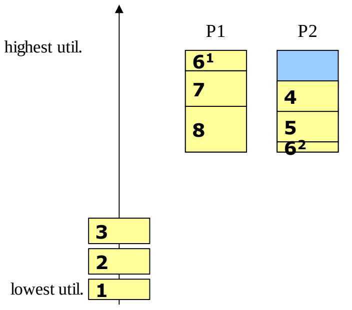

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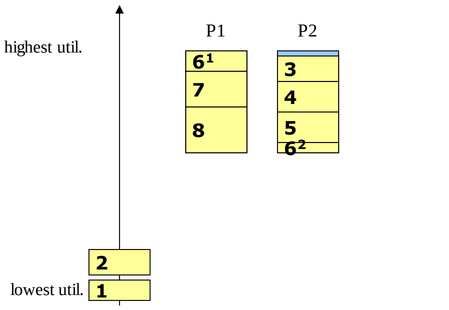

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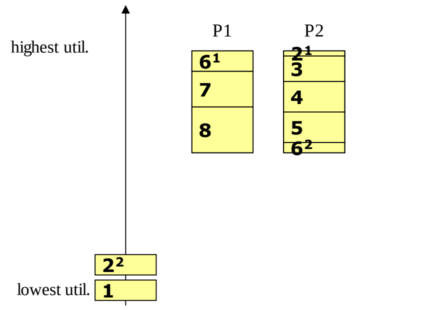

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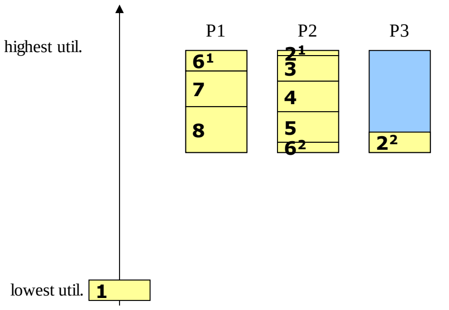

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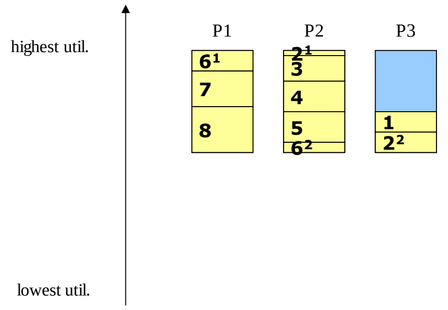

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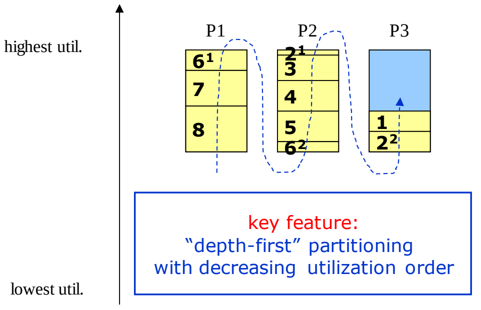

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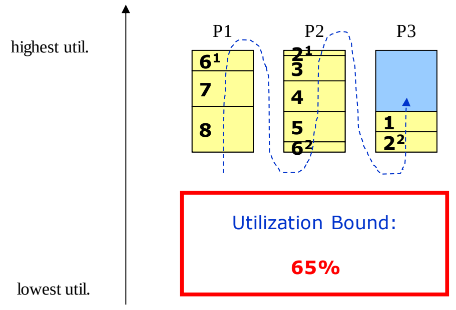
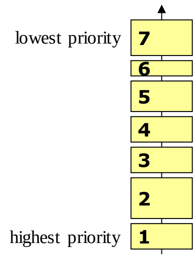


Our Algorithm
[RTAS10]

"width-first" partitioning
with increasing priority order

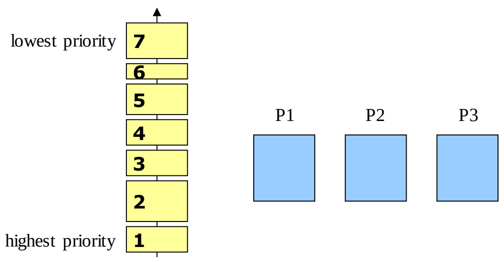
Our Algorithm

- Sort all tasks in increasing priority order



Our Algorithm

- Select the processor on which the assigned utilization is the lowest



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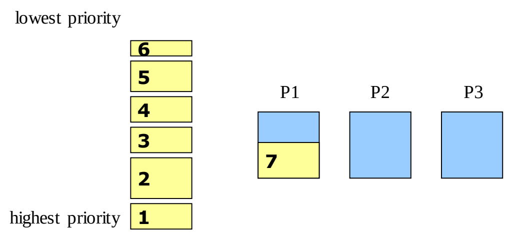

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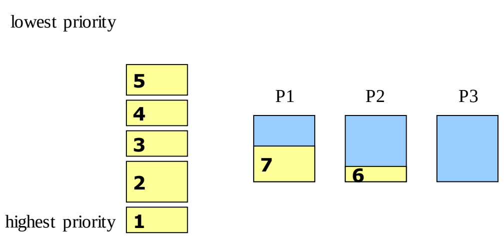

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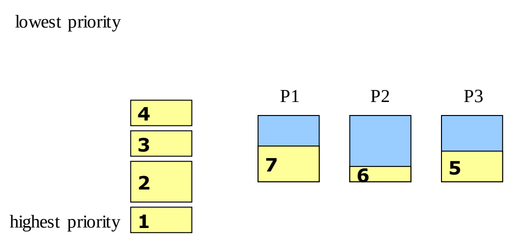

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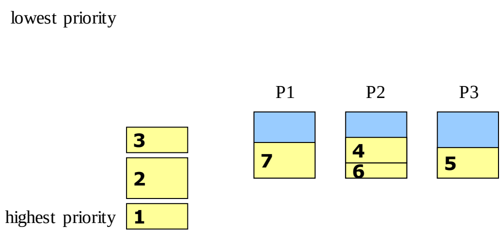

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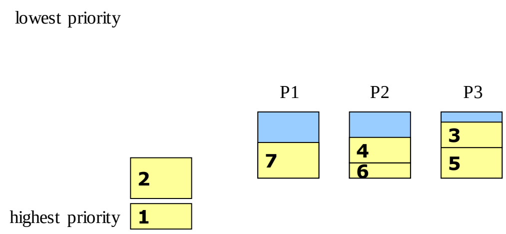

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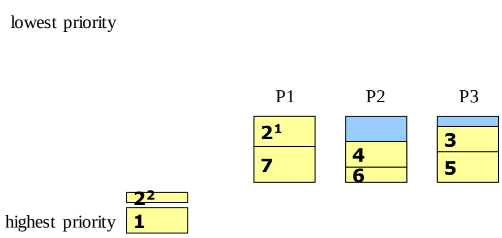

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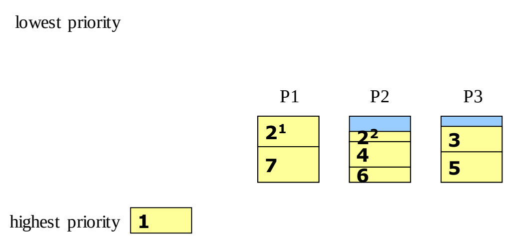

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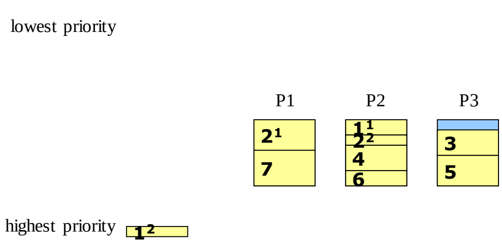

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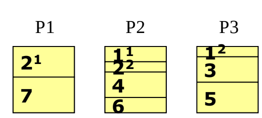
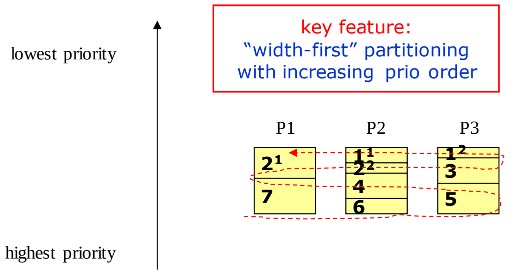


Our Algorithm

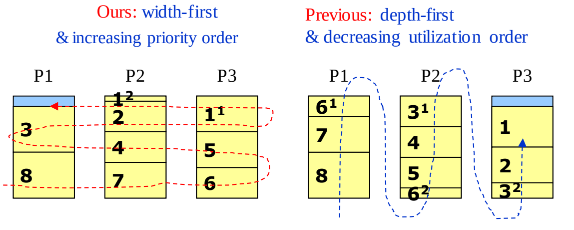
- Select the processor on which the assigned utilization is the lowest

key feature:
"width-first" partitioning
with increasing prio order



Comparison

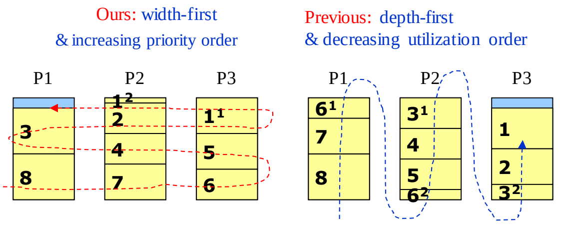
Why is our algorithm better?



Comparison

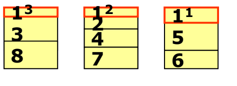
Why is our algorithm better?

By our algorithm split tasks generally have higher priorities



Split Task

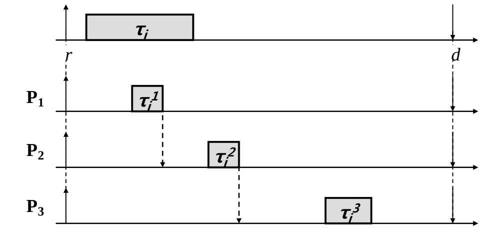
- Consider an extreme scenario:
  - suppose each subtask has the highest priority
  - schedulable anyway, we do not need to worry about their deadlines



- The difficult case is when the tail task is not on the top
  - the key point is to ensure the tail task is schedulable

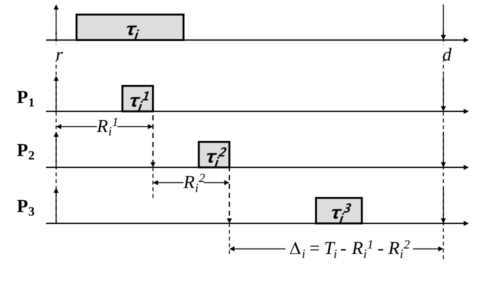
Split Task

- Subtasks should execute in the correct order



Split Task

- Subtasks get "shorter deadlines"



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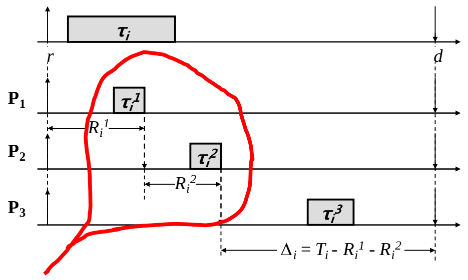
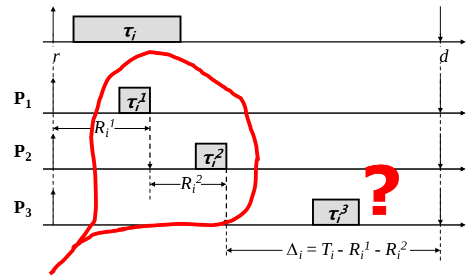


These two are on the top: no problem with schedulability

Split Task

- Subtasks should execute in the correct order

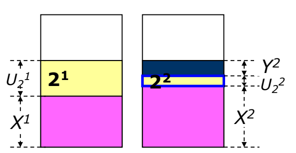


These two are on the top: no problem with schedulability

Why the tail task is schedulable?

The typical case: two CPUs and task 2 is split to two sub-tasks

As we always select the CPU with the lowest load assigned, we know



$$Y^2 + U_2^2 <= U_2^1$$

$$\Downarrow$$

$$Y^2 <= U_2^1 - U_2^2$$

That is, the "blocking factor" for the tail task is bounded.

Theorem

For a task set in which each task τ_i satisfies

$$U_i \le \frac{\Theta(N)}{1+\Theta(N)}$$

we have

$$\frac{\sum C_i/T_i}{M} \le N(2^{1/N}-1)$$

$\Rightarrow$ the task set is schedulable

$$\Theta(N) = N(2^{\frac{1}{N}} - 1) \qquad N \to \infty, \quad \frac{\Theta(N)}{1+\Theta(N)} \doteq 0.41$$

Theorem

For a task set in which each task τ_i satisfies

$$U_i \le \frac{\Theta(N)}{1+\Theta(N)}$$ get rid of this constraint

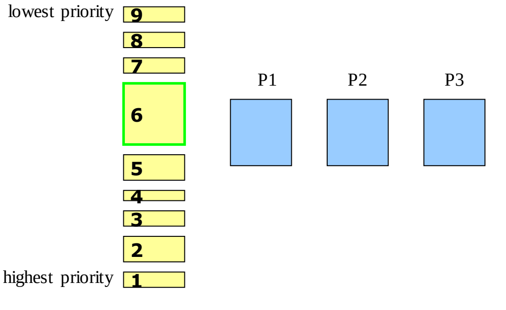
we have

$$\frac{\sum C_i/T_i}{M} \le N(2^{1/N}-1)$$

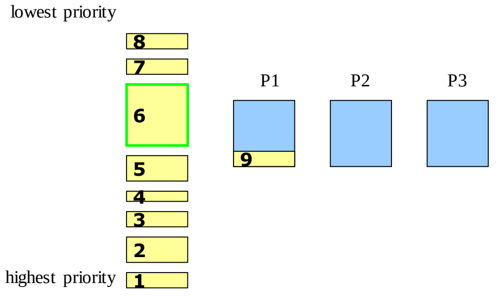
$\Rightarrow$ the task set is schedulable

$$\Theta(N) = N(2^{\frac{1}{N}} - 1) \qquad N \to \infty, \quad \frac{\Theta(N)}{1+\Theta(N)} \doteq 0.41$$

Problem of Heavy Tasks



Problem of Heavy Tasks



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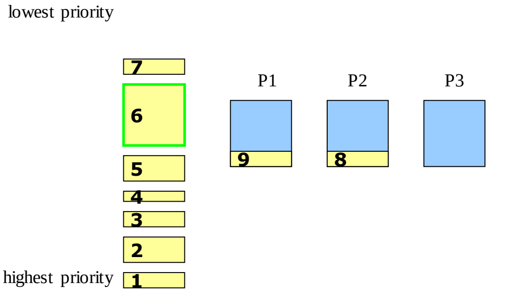

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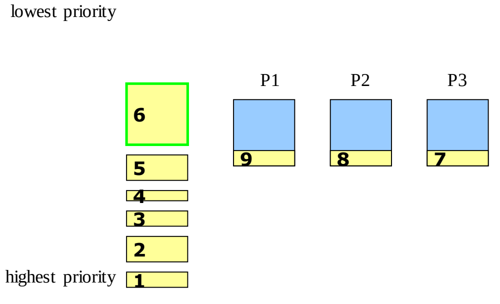

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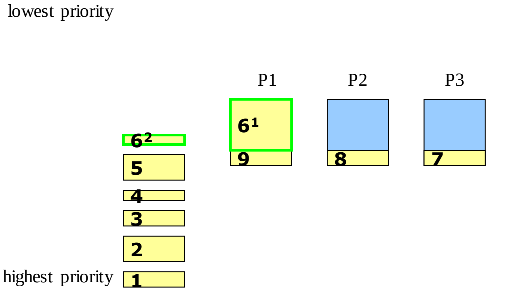

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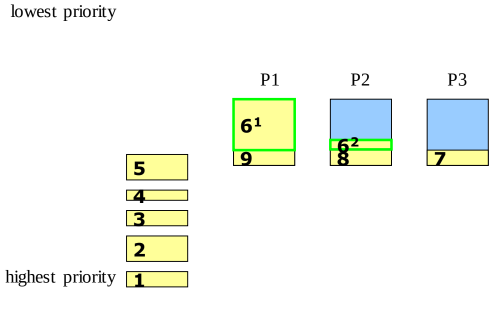

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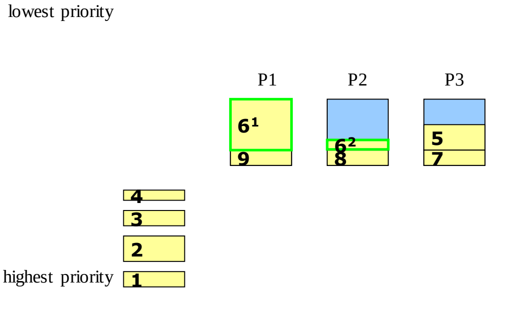

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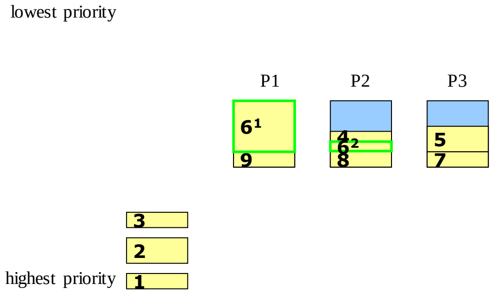

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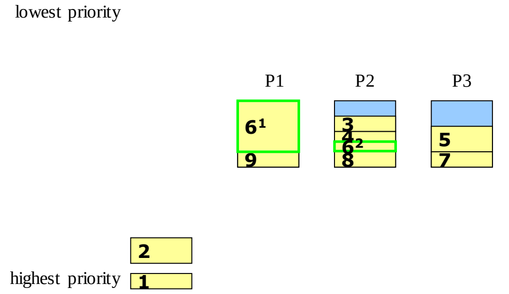

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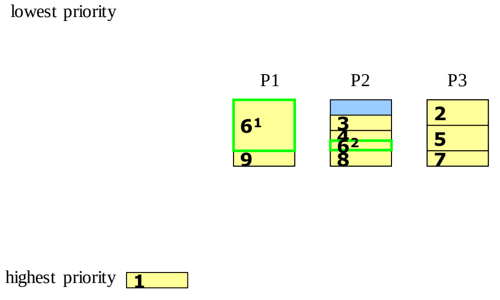
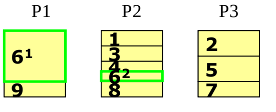
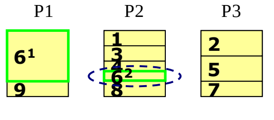


Problem of Heavy Tasks



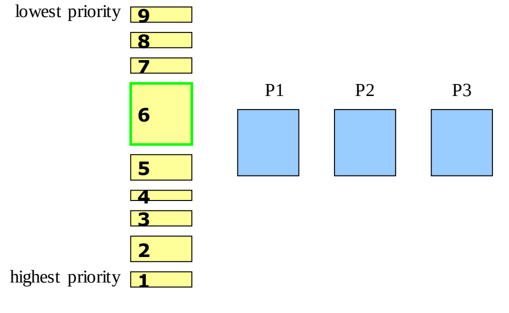
Problem of Heavy Tasks

the heavy tasks' tail task
may have too low priority level



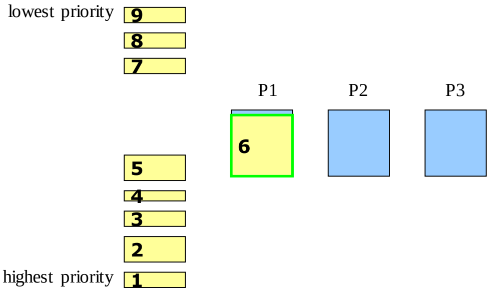
Solution for Heavy Tasks

- Pre-assigning the heavy tasks (that may have low priorities)



Solution for Heavy Tasks

- Pre-assigning the heavy tasks (that may have low priorities)



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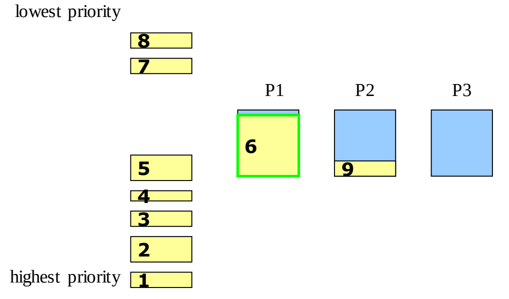

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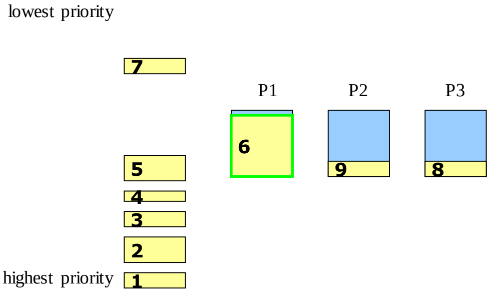

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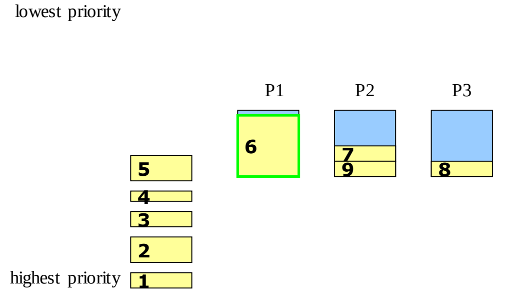

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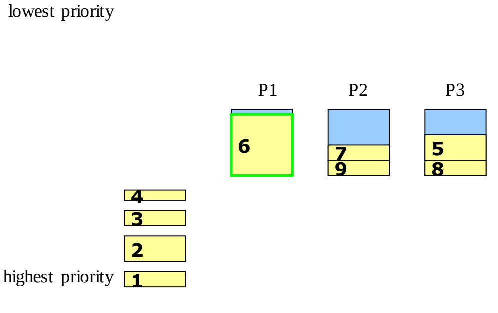

Solution for Heavy Tasks

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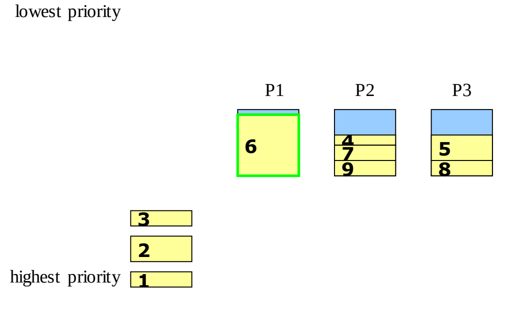

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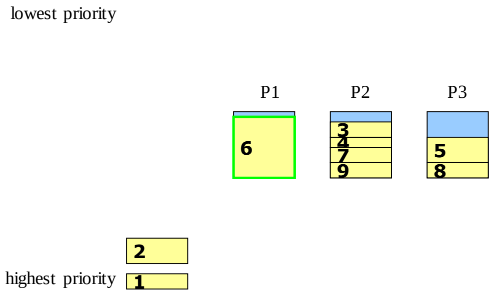

Solution for Heavy Tasks

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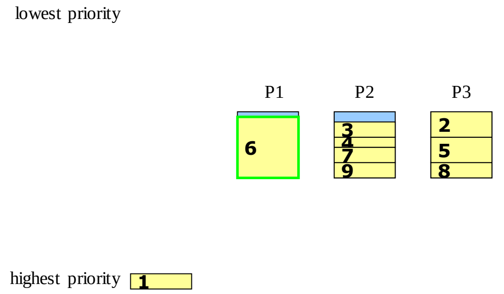

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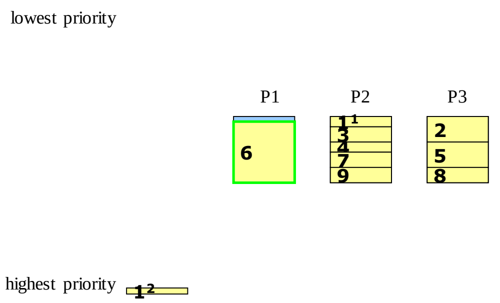
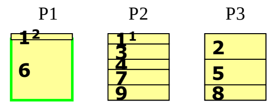


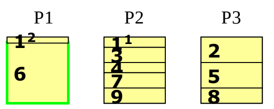
Solution for Heavy Tasks

- Pre-assigning the heavy tasks (that may have low priorities)



Solution for Heavy Tasks

- Pre-assigning the heavy tasks (that may have low priorities)



avoid to split heavy tasks
(that may have low priorities)

Theorem

- By introducing the pre-assignment mechanism, we have

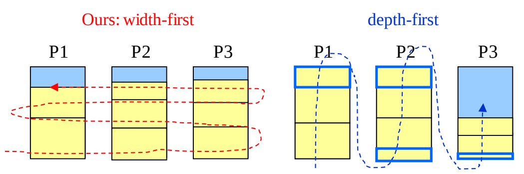
$$\frac{\sum C_i/T_i}{M} \leq N(2^{1/N}-1)$$

$$\Rightarrow \text{ the task set is schedulable}$$

Liu and Layland's utilization bound for all task sets!

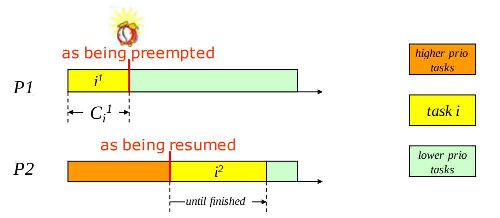
Overhead

- In both previous algorithms and ours
  - The number of task splitting is at most *M-1*
    - task splitting -> extra "migration/preemption"
  - Our algorithm on average has less task splitting

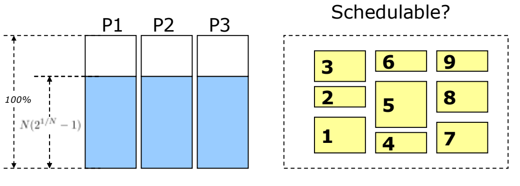


Implementation

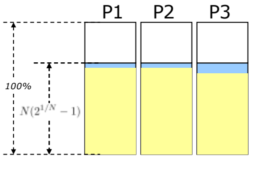
- Easy!
  - One timer for each split task
  - Implemented as "task migration"



Further Improvement



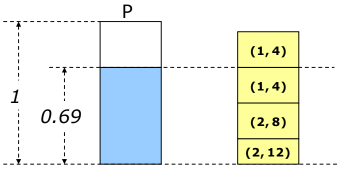
Uisng Liu and Layland's Utilization Bound



Yes, schedulable
by our algorithm

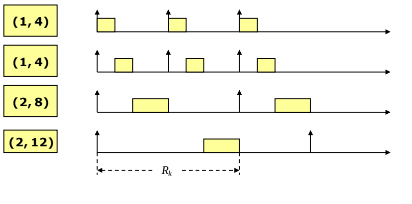
Utilization Bound is Pessimistic

- The Liu and Layland utilization bound is sufficient but not necessary
- many task sets are actually schedulable even if the total utilization is larger than the bound



Exact Analysis

- Exact Analysis: Response Time Analysis [Lehoczky_89]
  - pseudo-polynomial



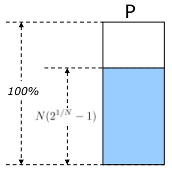
$$R_k = \sum_{T_i < T_k} \left\lceil \frac{R_k}{T_i} \right\rceil C_i + C_k$$

task k is schedulable iff
$R_k <= T_k$

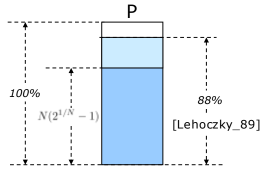
Utilization Bound v.s. Exact Analysis

- On single processors

Utilization bound Test
for RMS



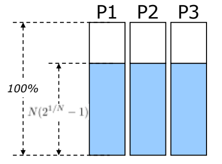
Exact Analysis
for RMS

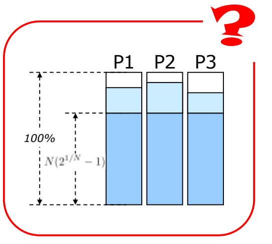


On Multiprocessors

- Can we do something similar on multiprocessors?

Utilization bound Test
the algorithm introduced above





Beyond Layland & Liu's Bound [RTSS 2010, rejected!]

- Our RTAS10 algorithm:
  - Increasing RMS priority order & worst-fit partitioning
  - Utilization test to determine the maximal load for each processor
  - The maximal load for each processor bounded by 69.3% $N(2^{\frac{1}{N}}-1)$
- Improved algorithm:
  - Employ Response Time Analysis to determine the maximal workload on each processor
  - more flexible behavior (more difficult to prove ...)
  - Same utilization bound for the worst case, but
  - Much better average performance (by simulation)

I believe this is "the best algorithm" one can hope for "fixed-prioritiy multiprocessor scheduling"

Conclusions

- The (multicore) Timing Problem is challenging
  - Difficult to guarantee Real-Time
  - and Difficult to analyze/predict
- Solutions: Partition & Isolation
  - Shared caches: coloring/partition
  - Memory bus/bandwidth: TDMA, ?
  - Processor cores: partition-based scheduling

Thanks!