

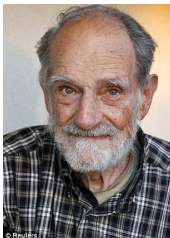
The Stable Marriage Problem

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Stable Marriage Problem



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JANUARY

1962

D. Gale and L.S. Shapley: *College Admissions and the Stability of Marriage*, American Mathematical Monthly 69, 9-14, 1962.

The Problem

*“A certain community consists of n men and n women. Each person ranks those of the opposite sex in accordance with his or her preferences for a partner. We seek a satisfactory way of marrying off all members of the community. We call a marriage **unstable** if there are a man and woman who are not married to each other but prefer each other to their actual mates.”*

Instance for $n = 4$

	1	2	3	4
Ann	Y	W	X	Z
Beth	W	Z	Y	X
Cora	X	Z	Y	W
Dee	Z	X	W	Y

	1	2	3	4
Will	A	D	C	B
Xavier	A	B	C	D
Yohan	B	D	C	A
Zack	C	A	B	D

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There are $4! = 24$ marriages in total

Stable Marriages



Stable Marriages



Do stable marriages always exist?
If so, can they be found efficiently?

The Proposal Algorithm

“Men propose, women dispose . . .”

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When we're done, marry off all engaged couples.

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Beth	W	Z	Y	X
Cora	X	Z	Y	W
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- 1 The algorithm always terminates.
- 2 The algorithm always produces a stable marriage.
- 3 The output does not depend on the proposal order, is the best possible stable marriage for each man, and the worst possible for each woman.
- 4 A “female-optimal” marriage can be generated by having the women propose instead.

- 1 If same-sex unions are allowed then stable marriages do not always exist.

Variations

- 1 If same-sex unions are allowed then stable marriages do not always exist.
- 2 College admission problem and couples version.

