Undecidability of Parameterized Model Checking

Part III: Undecidability of Parameterized Model Checking

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Classic Model Checking Problems

Consider P^N as interleaving of N processes of type P.

Finite-state MC

Input:

- a process template P (finite-state),
- an LTL formula φ,
- the number of processes $N \geq 1$.

Problem: check, whether $P^N \models \varphi$.

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Parameterized MC

Input:

- a process template P (finite-state),
- an (indexed) LTL formula ϕ,

Problem: check, whether $\forall N \geq 1.\ P^N \models \phi(N)$.

Decidable?

The First Result: Undecidability of PMC

Apt & Kozen 1986:

One process $P(n)$ with a parameterized loop bound n,

$P(n)$ simulates n steps of a Turing machine

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flag := false
for i := 1 to n do
  simulate one step of T
if T has not halted then
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As non-halting is undecidable,

so is PMC for n non-communicating processes $P^n(n)$.

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Note: This proof is about sequential and unbounded programs.

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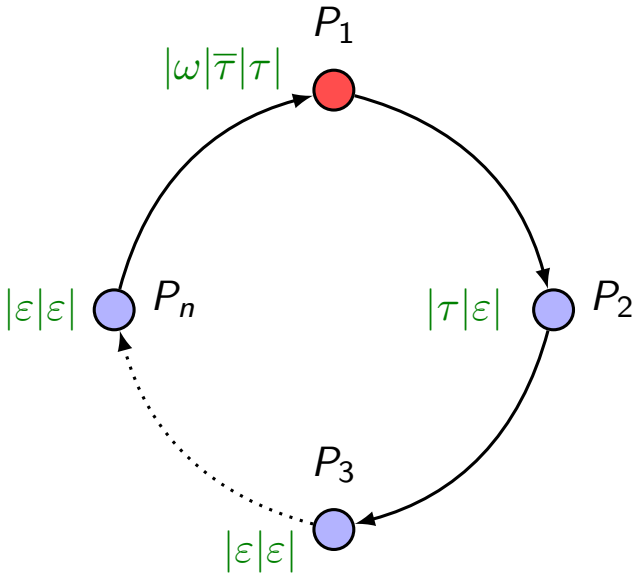
P^n simulates a Turing machine that writes not more than in $2n+1$ cells.

- Process P_1 serves as a head and stores three letters.
- Each process P_i, $i > 1$, stores two letters.
- The head moves to the right
  $\Rightarrow$ the tape is cyclically shifted.



Helmut Veith (www.forsyte.at) Checking Fault-Tolerant Distributed Algos VTSA, October 2014 4 / 1

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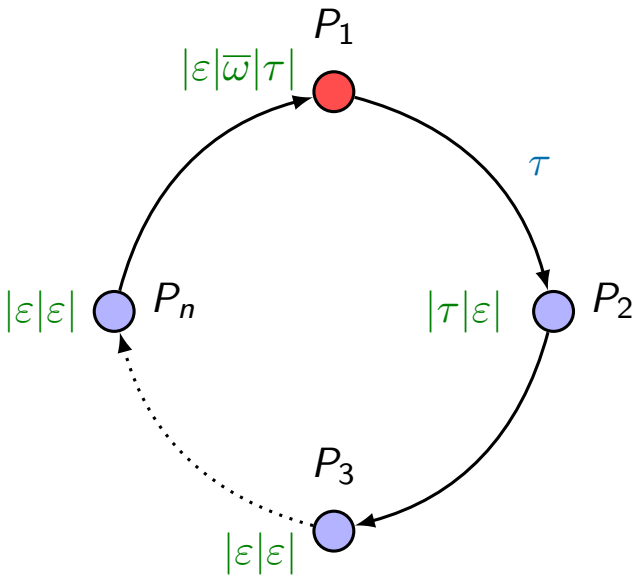

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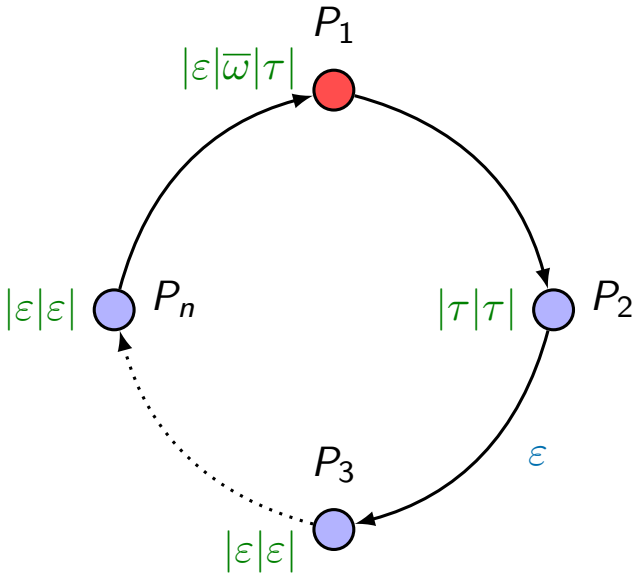

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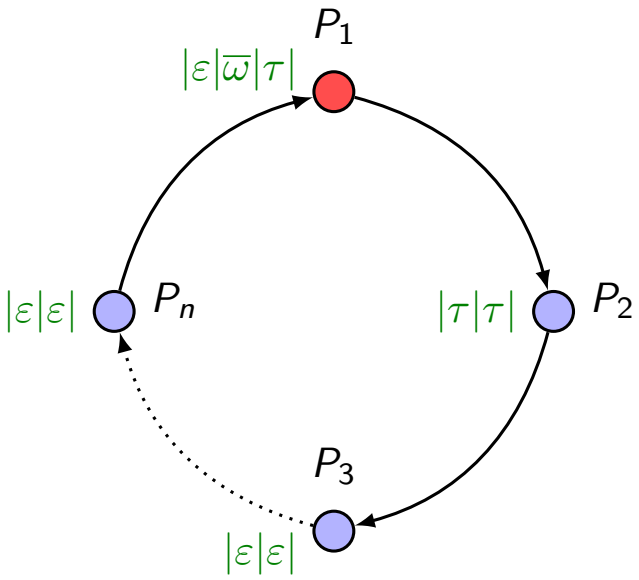
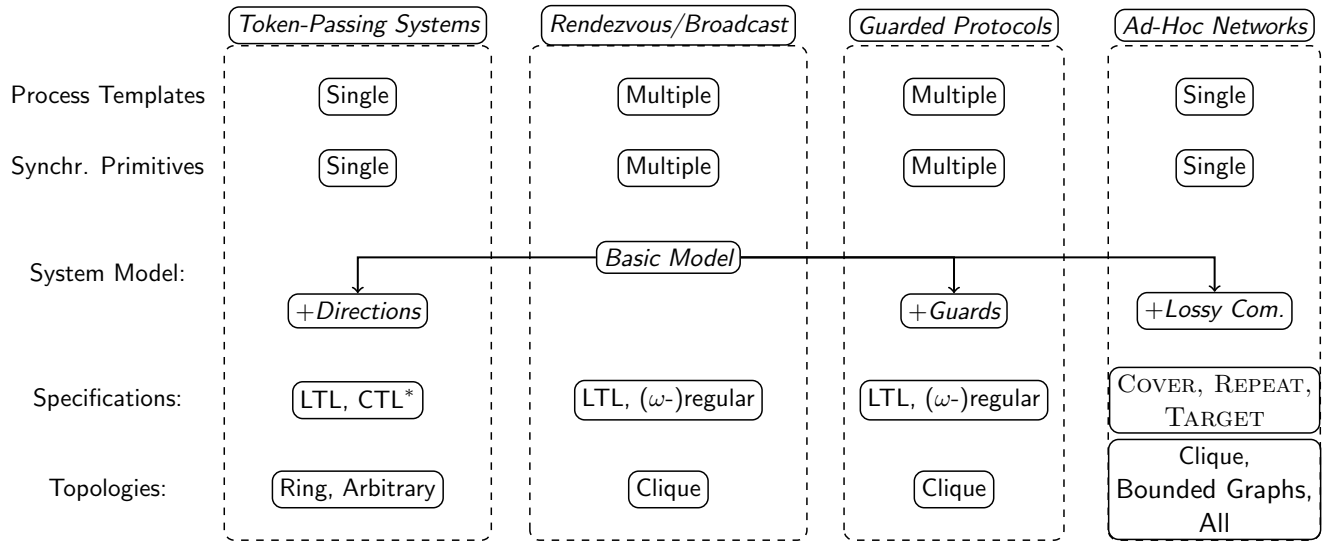
- Process P_1 serves as a head and stores three letters.
- Each process P_i, $i > 1$, stores two letters.
- The head moves to the right $\Rightarrow$ the tape is cyclically shifted.
- The proof is very technical.
- A simpler proof is given by Emerson & Namjoshi, 1995.



Many more results on Parameterized Model Checking

We have recently finished a book on:
"Decidability of Parameterized Verification".

To appear in Synthesis Lectures on Distributed Computing Theory.



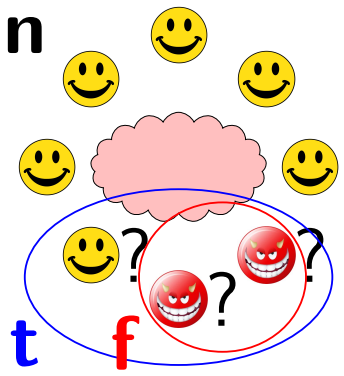
Fault-tolerant distributed algorithms and parameterized model checking

Recall our parameterized problem:

- given a process of a distributed algorithm and spec. φ
- show for all n, t, and f satisfying $n > 3t \wedge t \geq f \geq 0$
  $$P(n, t, f) \parallel \cdots \parallel P(n, t, f) \models \varphi$$
- every $M(n, t, f)$ is a system of $n - f$ correct processes

The problem statement is different!

- Processes are parameterized in n, t, and f.
- Processes communicate by sending messages to everybody.
- Processes count received messages.
- LTL specifications over $[\forall i.\ f(i)]$ and $[\exists i.\ f(i)]$.



Is our parameterized model checking problem decidable?

Undecidability of PMC for FTDA

Given a two-counter machine $\mathcal{M}$,

we construct a process P and an LTL formula φ.

Then the non-halting of $\mathcal{M}$ is formulated as: $\forall n \geq 2.\ P^n \models \varphi$.

The idea of the proof:

- Construct a finite-state process P
- P has only finitely many locations
- P does not communicate at all
- LTL formulas with **G** and **F** over atomic propositions like $[\exists i.pc_i = Z]$.

Counter Machines

A machine is composed of the following commands

$$\begin{aligned} \ell_i &: \textbf{inc } \mathcal{C}(\ell_i);\ \textbf{goto } \ell_j \\ \ell_i &: \textbf{if } \mathcal{C}(\ell_i) = 0 \textbf{ then goto } \ell_j \\ &\quad\ \textbf{else dec } \mathcal{C}(\ell_i);\ \textbf{goto } \ell_k \\ \ell_m &: \textbf{halt} \end{aligned}$$

over a few counters, e.g., $\mathcal{C}(\ell_i) \in \{B, C\}$.

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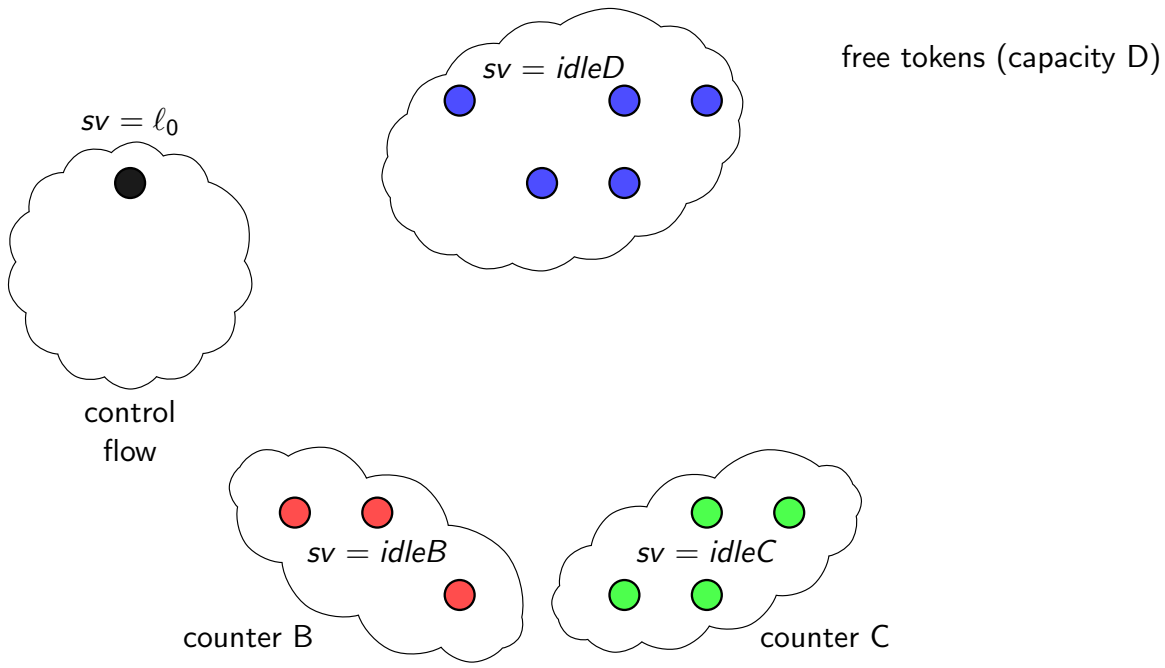
Many undecidability results are proven by reduction to halting of a 2CM.

For instance, [Esparza 1997]:
checking, whether a Petri net satisfies a linear μ-calculus formula simulates halting of a two-counter machine.

How a configuration looks like?

The process states are partitioned into four classes:



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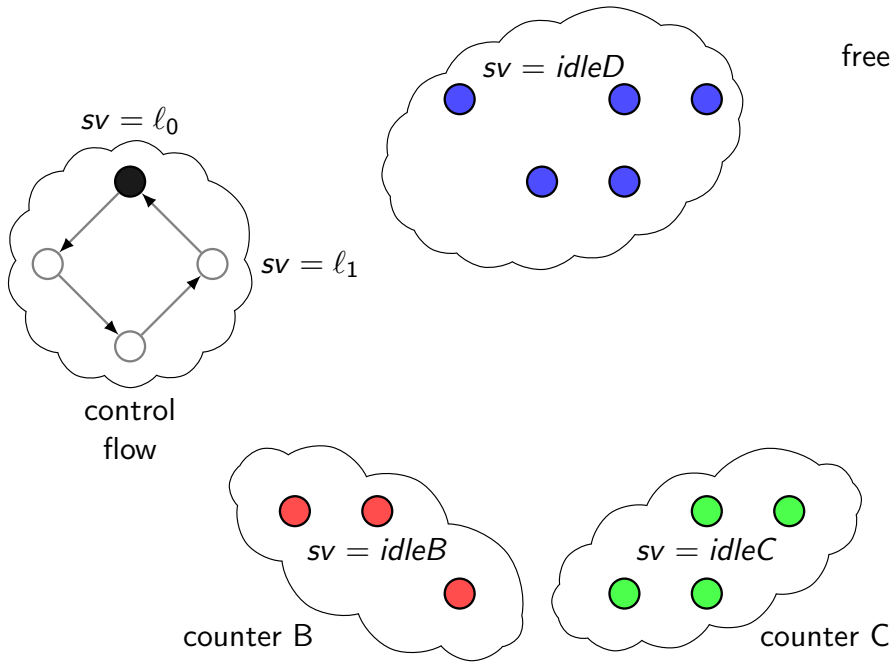

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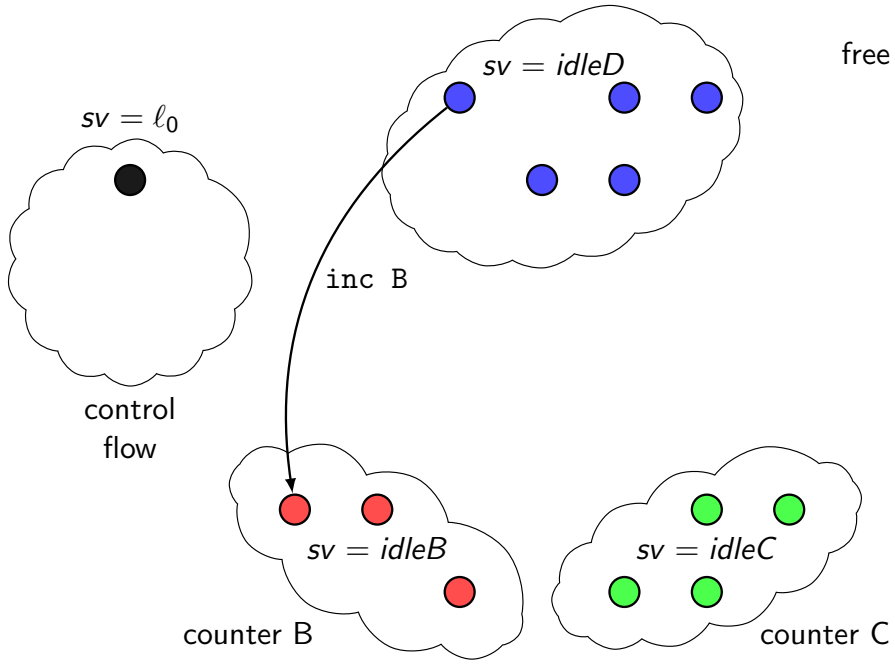

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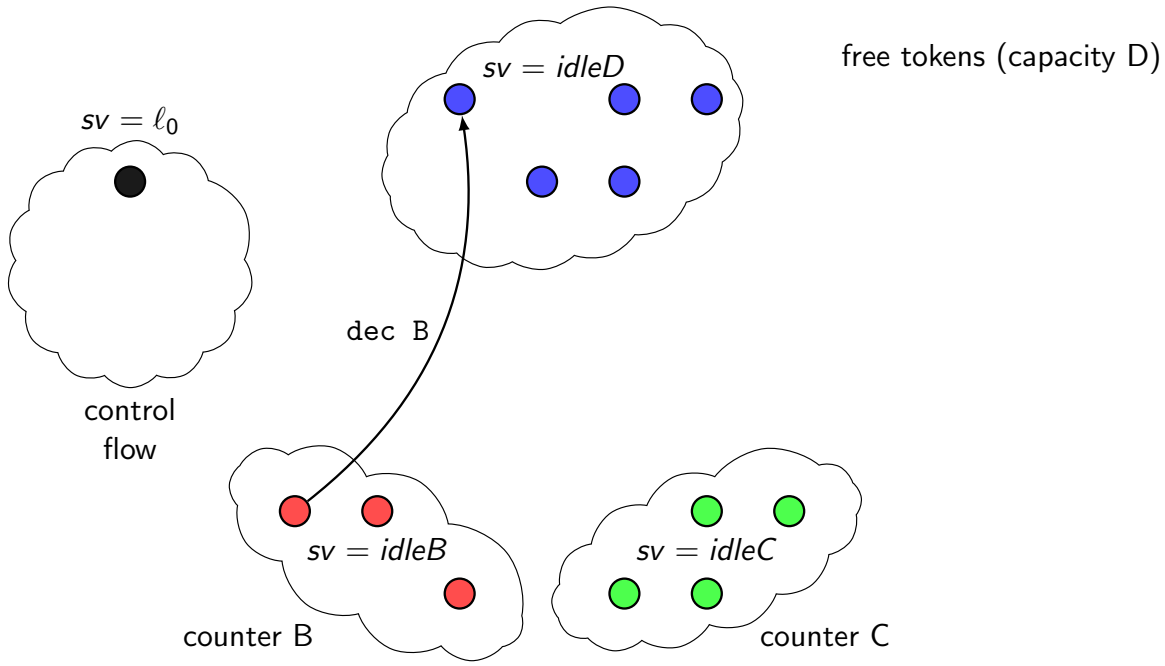

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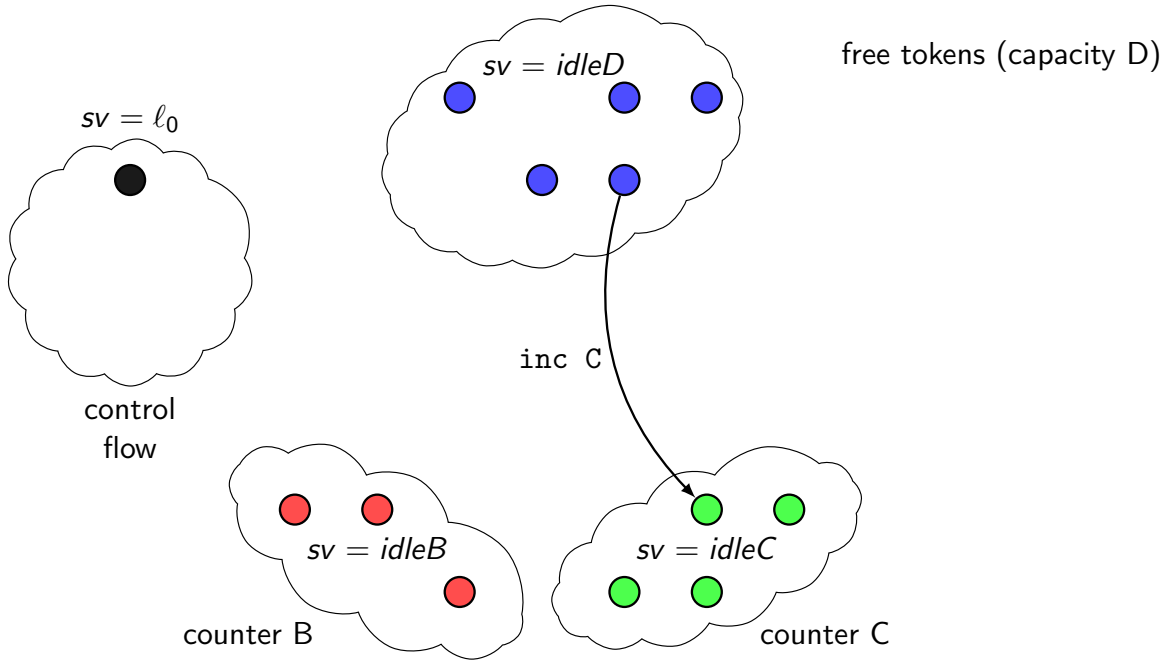

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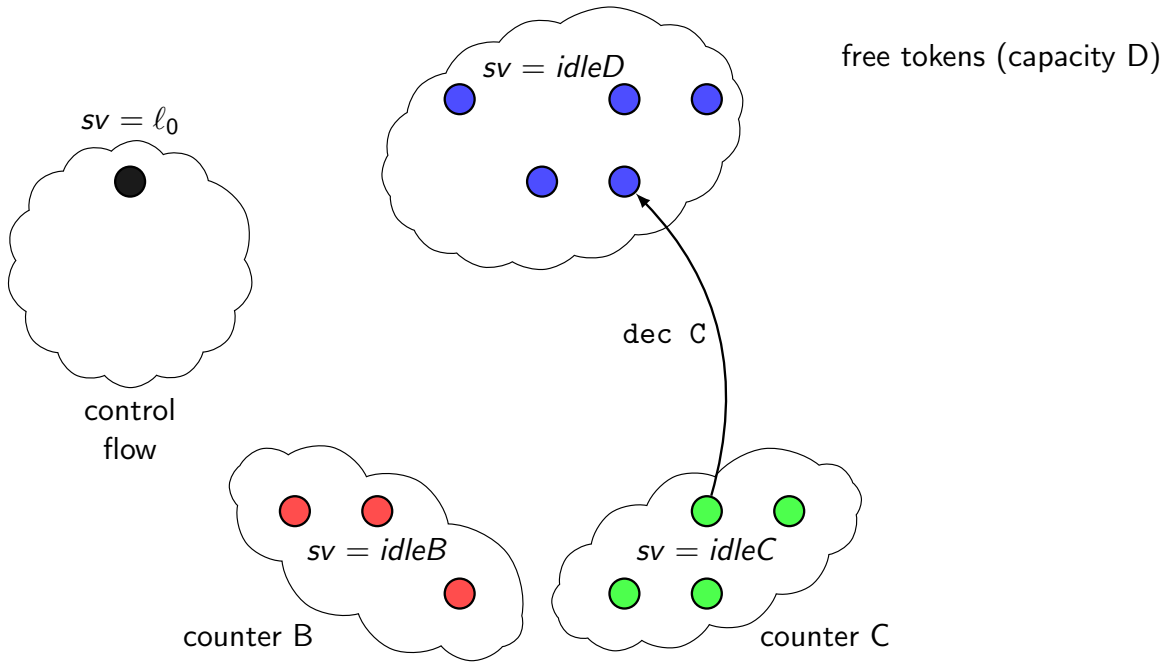

Conclusions

Parameterized model checking is undecidable for our problem.

We need either abstraction techniques, or semi-decision procedures.

In Part IV, we continue with abstraction techniques for parameterized model checking of fault-tolerant distributed algorithms.

Thank you!