Model Checking of Fault-Tolerant Distributed Algorithms

Part IV: Parameterized Model Checking of Fault-tolerant Distributed Algorithms by Abstraction

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VTSA 2014, Luxembourg

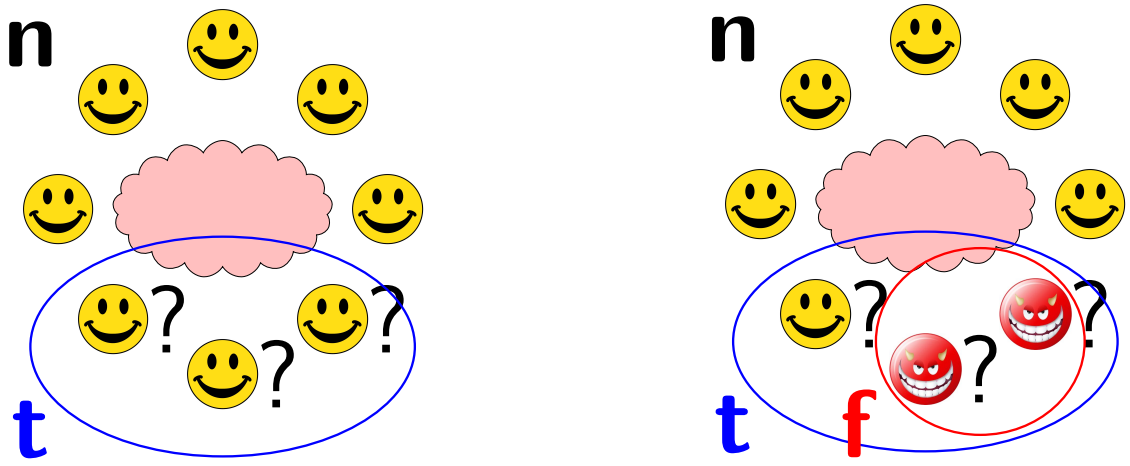
Fault-tolerant DAs: Model Checking Challenges

- unbounded data types
  counting how many messages have been received
- parameterization in multiple parameters
  among n processes $f \leq t$ are faulty with $n > 3t$
- contrast to concurrent programs
  fault tolerance against adverse environments
- degrees of concurrency
  many degrees of partial synchrony
- continuous time
  fault-tolerant clock synchronization

Model checking problem for fault-tolerant DA algorithms

Parameterized model checking problem:

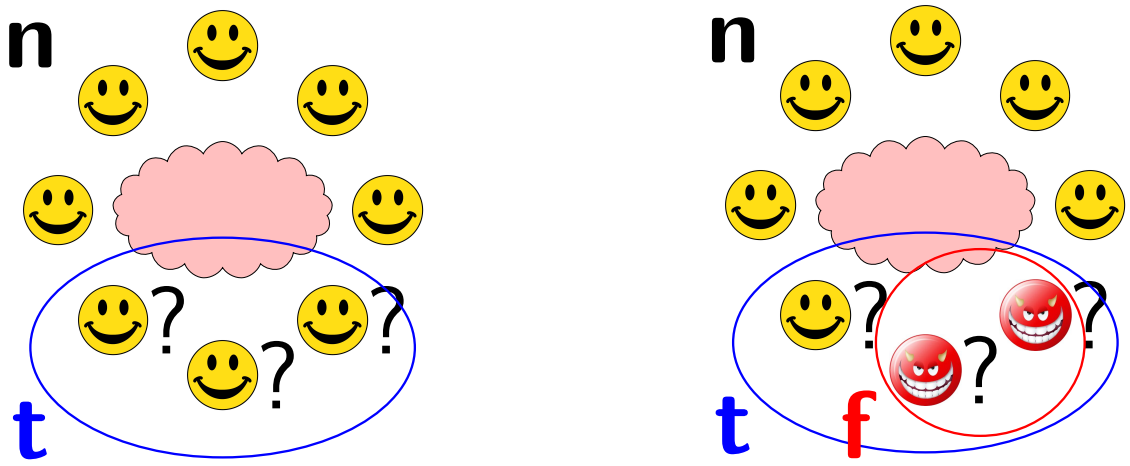
- given a distributed algorithm and spec. φ
- show for all n, t, and f satisfying $n > 3t \wedge t \geq f \geq 0$
  $M(n, t, f) \models \varphi$
- every $M(n, t, f)$ is a system of $n - f$ correct processes



Model checking problem for fault-tolerant DA algorithms

Parameterized model checking problem:

- given a distributed algorithm and spec. φ
- show for all n, t, and f satisfying *resilience condition*
  $M(n, t, f) \models \varphi$
- every $M(n, t, f)$ is a system of $N(n, f)$ correct processes



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Properties in Linear Temporal Logic

Unforgeability (U). If $v_i = 0$ for all correct processes i, then for all correct processes j, accept_j remains 0 forever.

$$\mathbf{G}\left(\left(\bigwedge_{i=1}^{n-f} v_i = 0\right) \rightarrow \mathbf{G}\left(\bigwedge_{j=1}^{n-f} accept_j = 0\right)\right)$$

Completeness (C). If $v_i = 1$ for all correct processes i, then there is a correct process j that eventually sets accept_j to 1.

$$\mathbf{G}\left(\left(\bigwedge_{i=1}^{n-f} v_i = 1\right) \rightarrow \mathbf{F}\left(\bigvee_{j=1}^{n-f} accept_j = 1\right)\right)$$

Relay (R). If a correct process i sets accept_i to 1, then eventually all correct processes j set accept_j to 1.

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Threshold-guarded fault-tolerant distributed algorithms

Threshold-guarded FTDAs

Fault-free construct: quantified guards (t=f=0)

- Existential Guard
  `if received` *m* `from` *some* `process then ...`
- Universal Guard
  `if received` *m* `from` *all* `processes then ...`

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Fault-Tolerant Algorithms: n processes, at most t are Byzantine

- Threshold Guard
  `if received` m `from` $n - t$ `processes then ...`
- (the processes cannot refer to **f**!)

Control Flow Automata

Variables of process i

v_i : {0, 1} **init with** 0 **or** 1
$accept_i$: {0, 1} **init with** 0

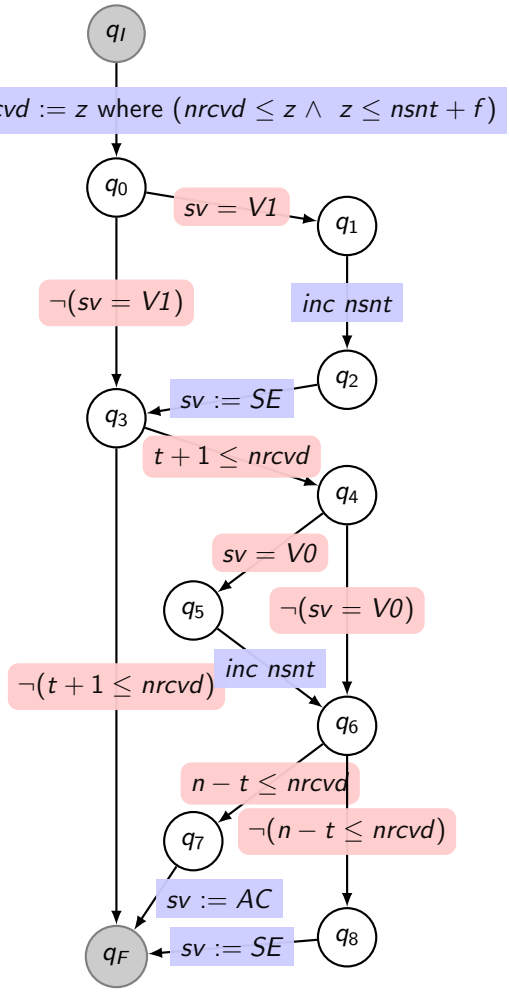
An indivisible step:

if $v_i = 1$
then send (echo) to all;

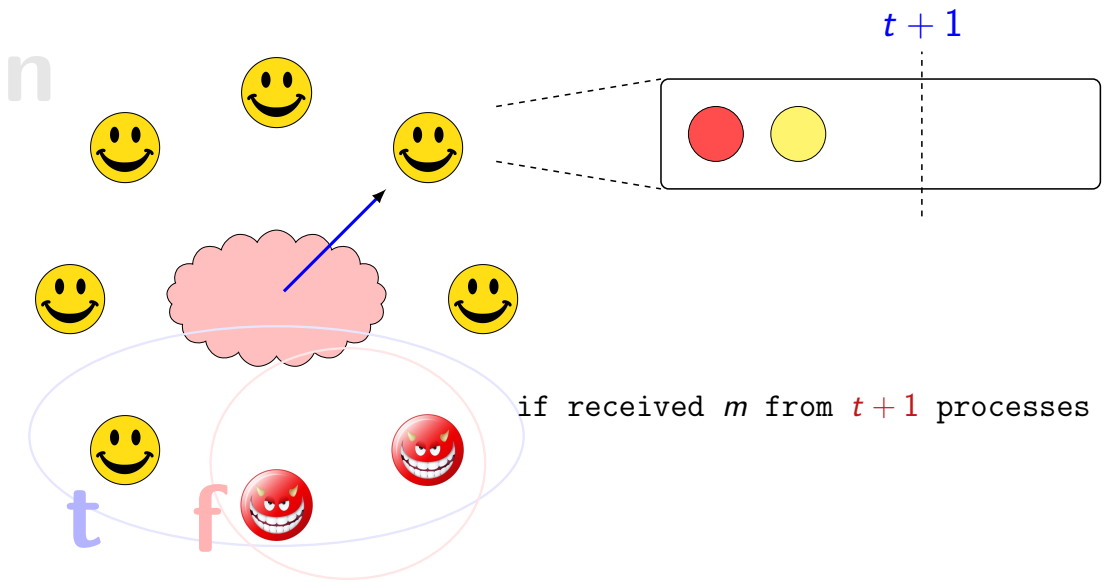
if received (echo) from at least
t + 1 distinct processes
and not sent (echo) before
then send (echo) to all;

if received (echo) from at least
n - t distinct processes
then $accept_i$:= 1;

$n - f$ copies of the process



Counting argument in threshold-guarded algorithms



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Correct processes count distinct incoming messages

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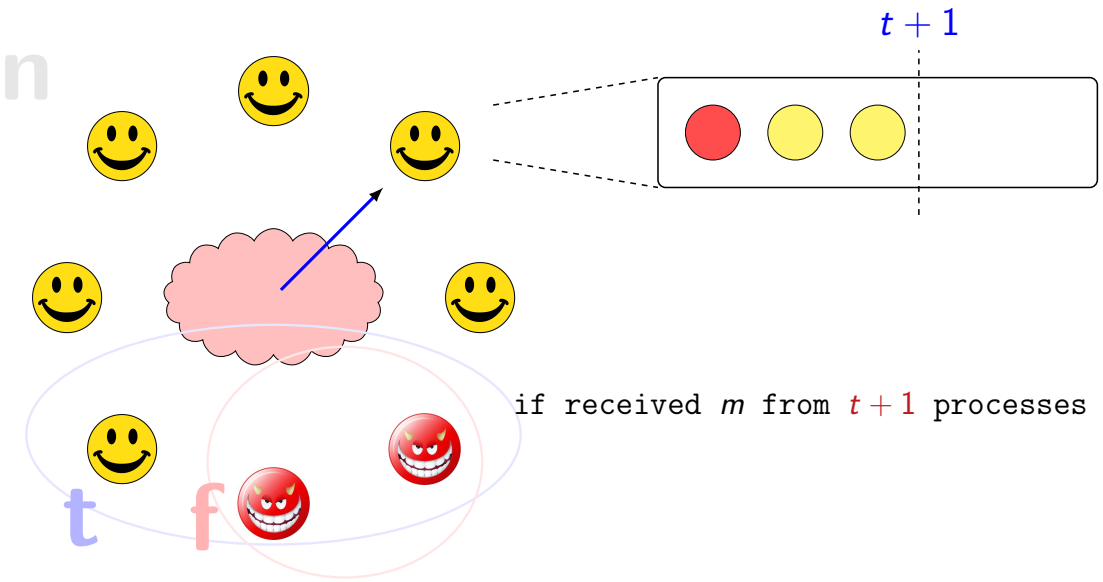

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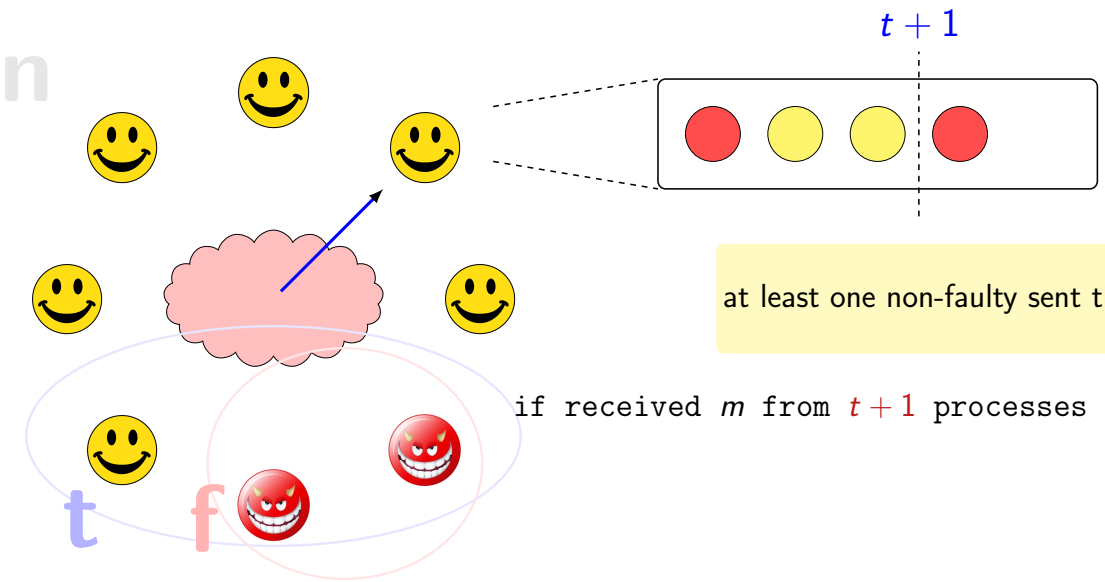

at least one non-faulty sent the message

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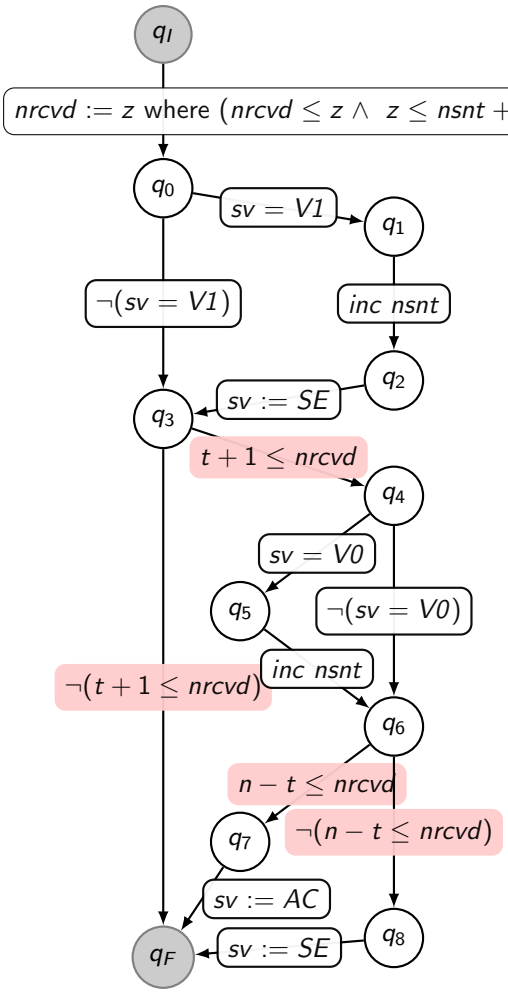

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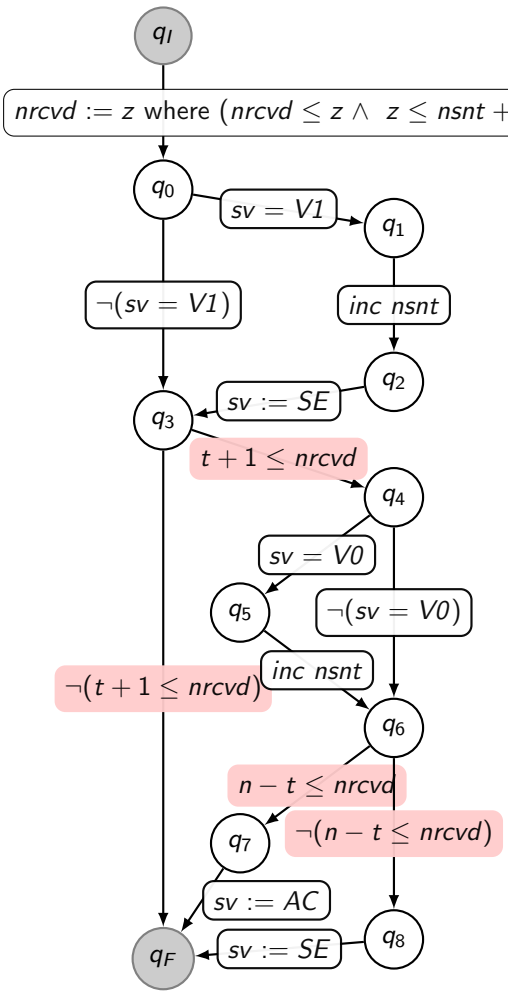

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- intervals with symbolic boundaries:
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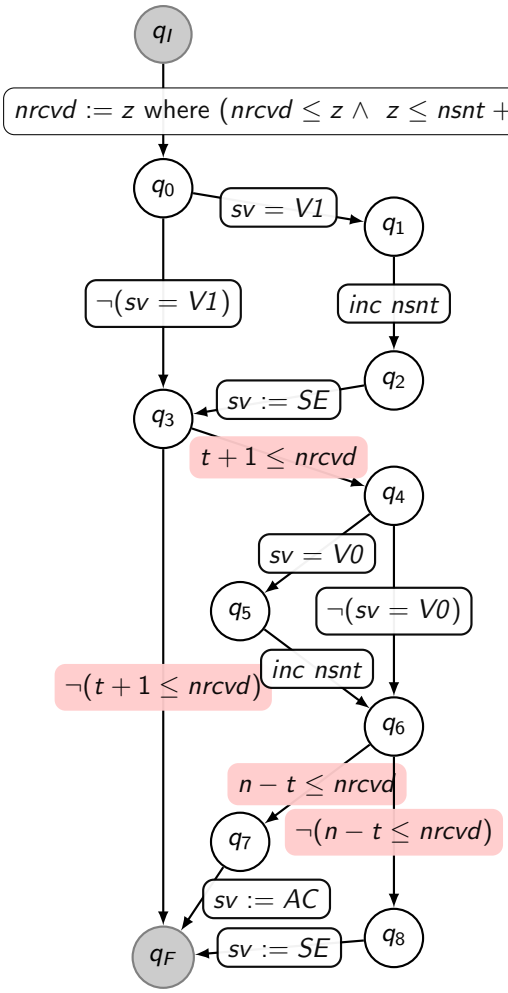

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- Parameteric Interval Abstraction (PIA)
- Similar to interval abstraction:
  $[t+1, n-t)$ rather than $[4, 10)$.
- Total order: $0 < 1 < t+1 < n-t$ for all parameters satisfying RC:
  $n > 3t, t \geq f \geq 0$.

Technical challenges

We have to reduce the verification of an infinite number of instances where

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2. the number of processes is parameterized

to one finite state model checking instance

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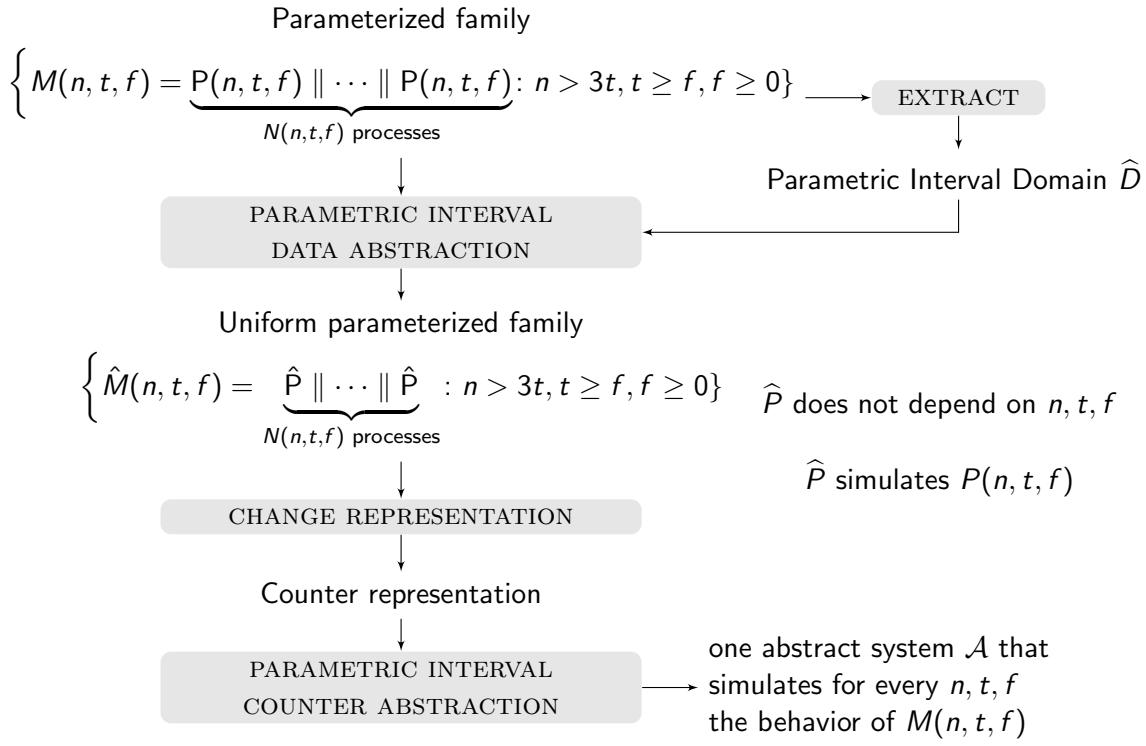
abstraction is an over approximation $\Rightarrow$ possible abstract behavior that does not correspond to a concrete behavior.

3. Refining spurious counter-examples

Abstraction overview



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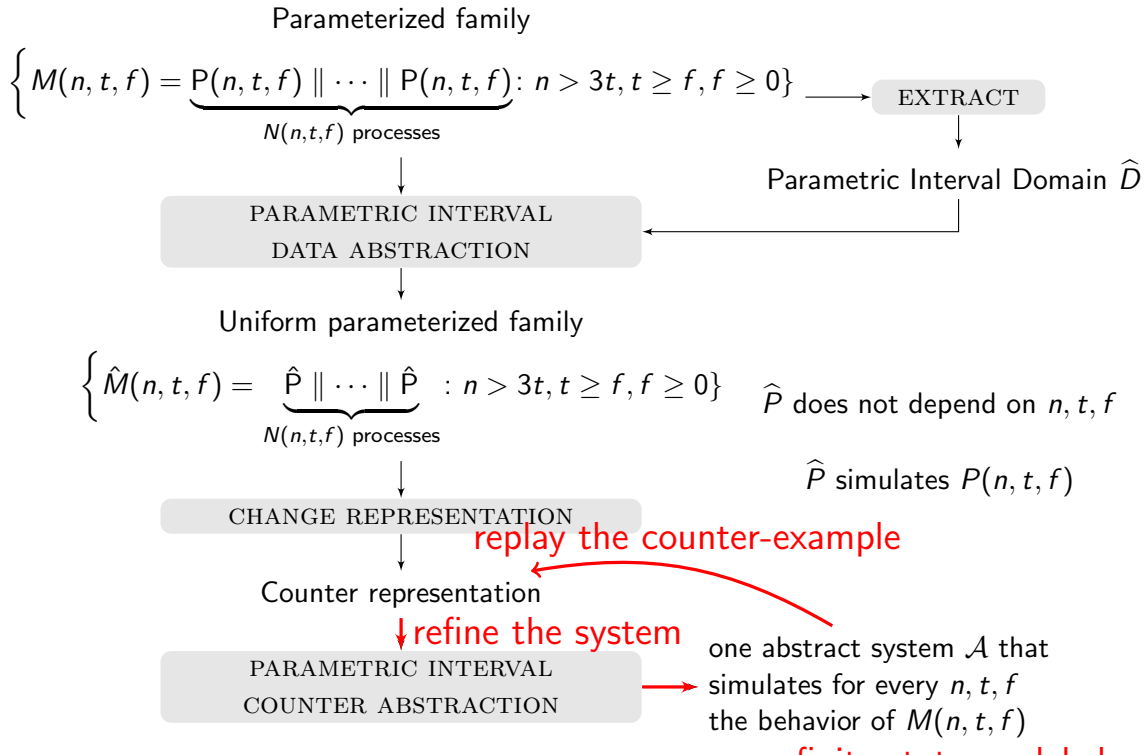
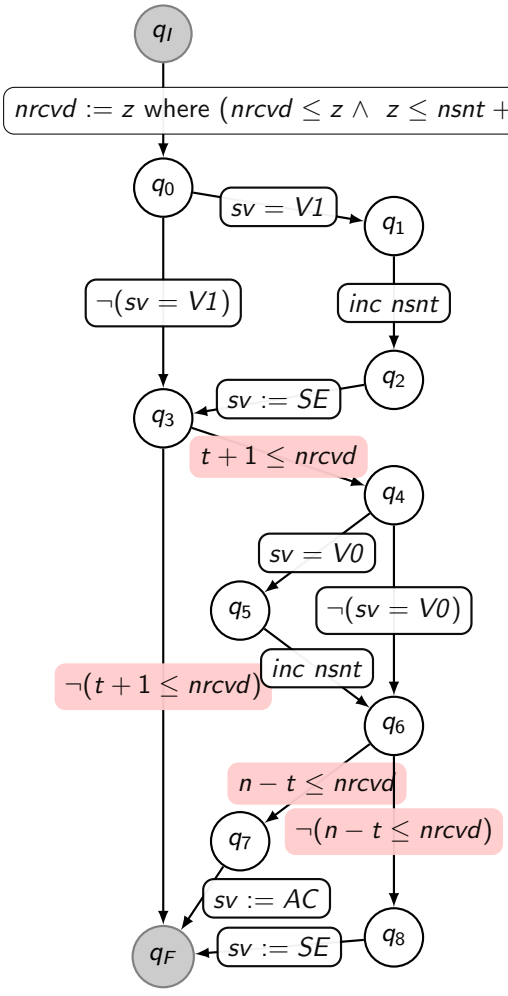


Data abstraction



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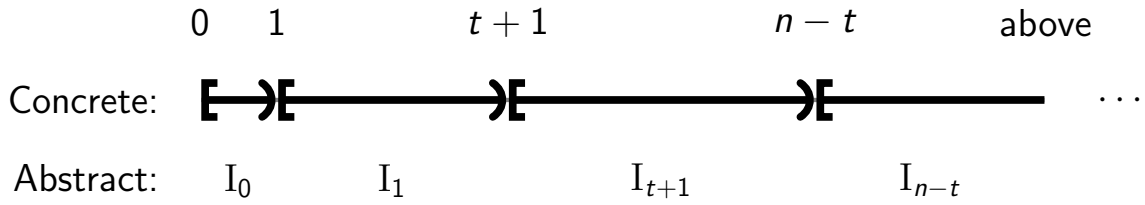
Abstract operations



Concrete $t + 1 \leq x$

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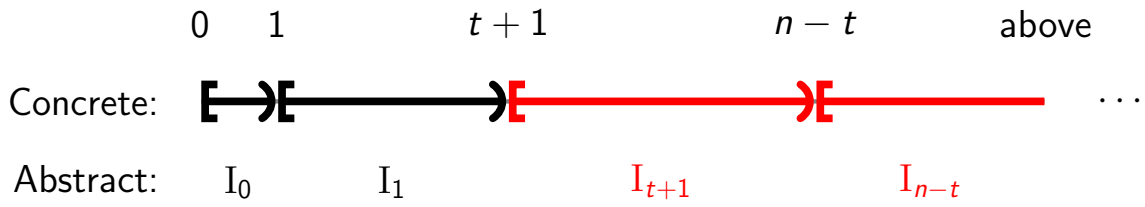

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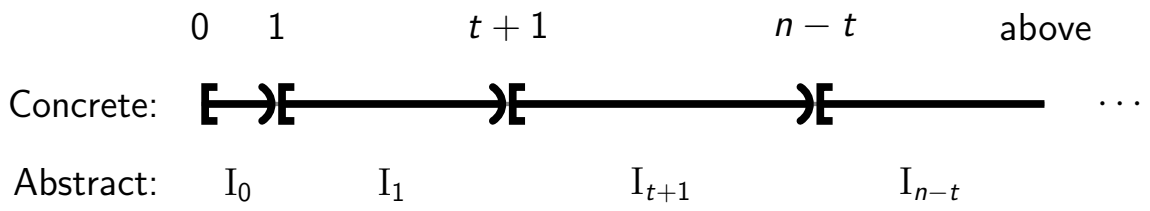

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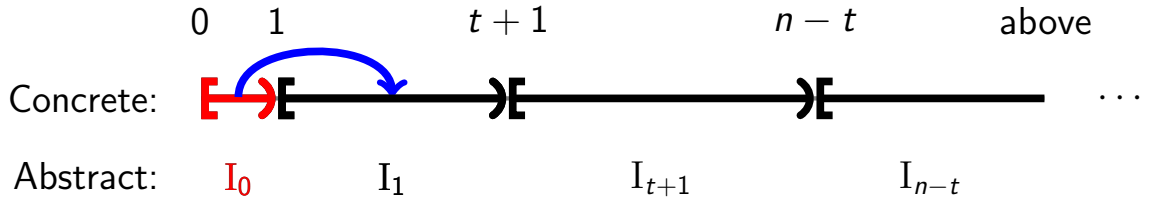

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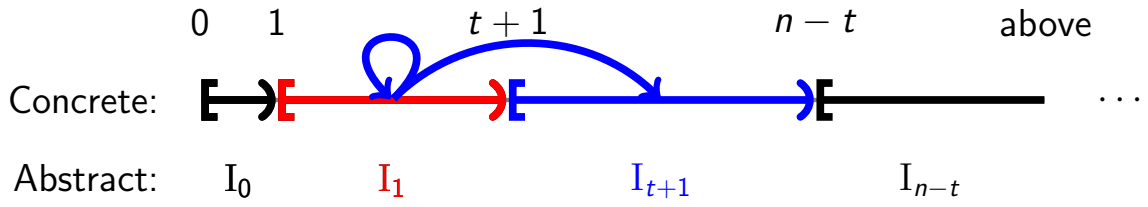

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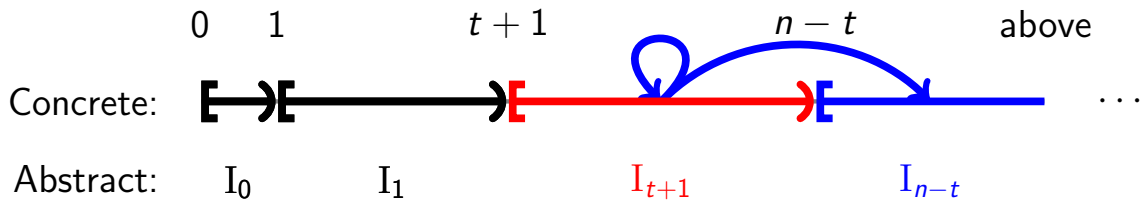

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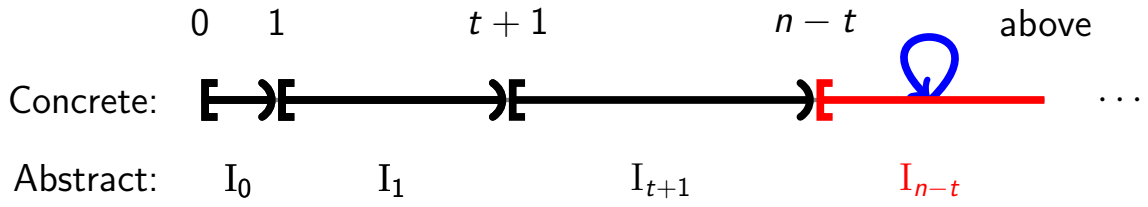

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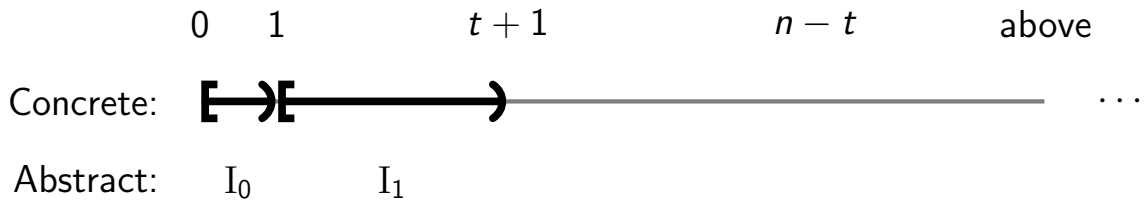

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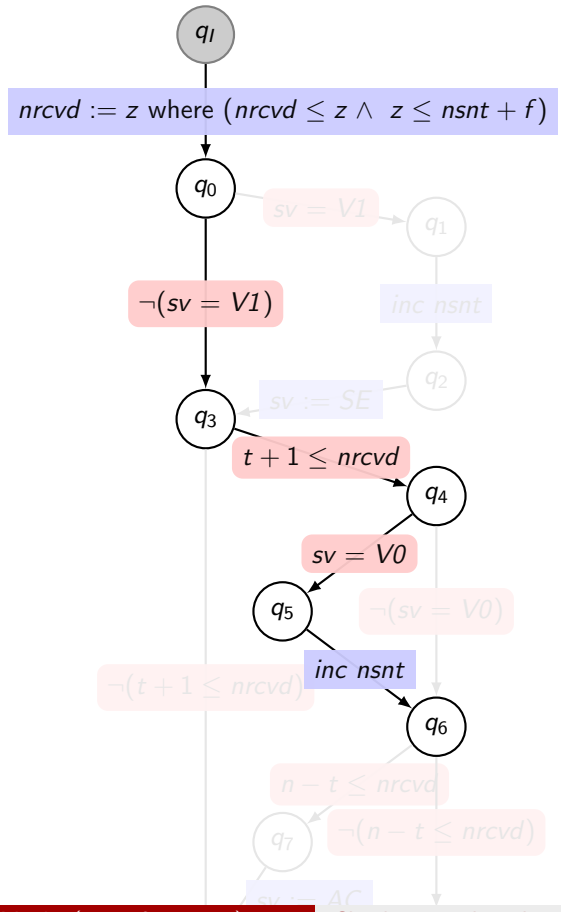
abstract increase may keep the same value!

Abstract CFA



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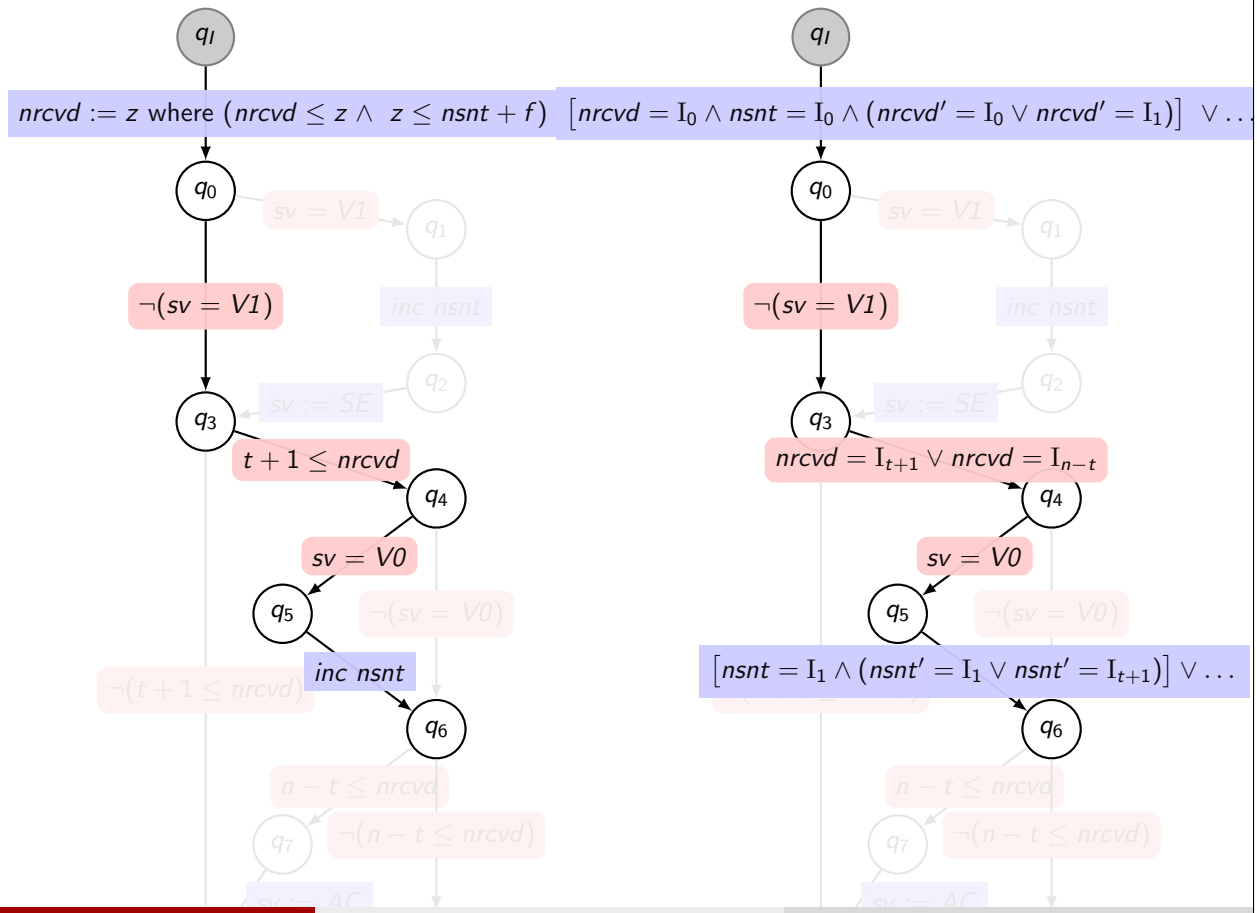

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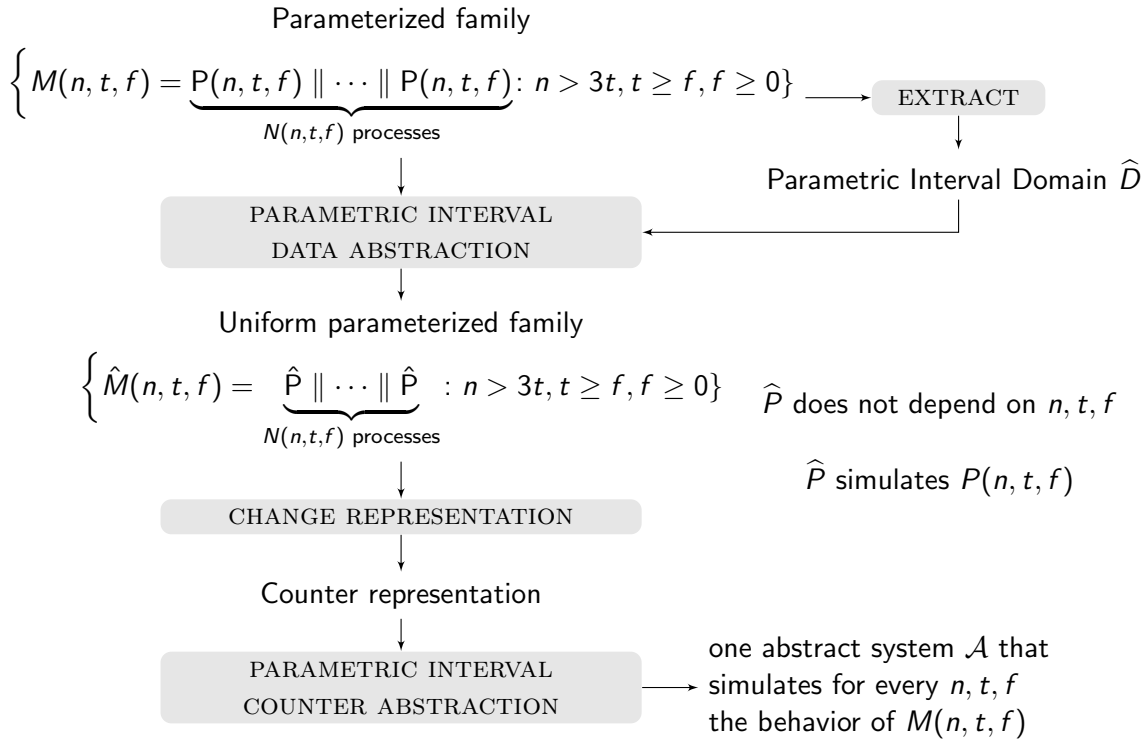

Counter abstraction

Classic $(0, 1, \infty)$-counter abstraction

Pnueli, Xu, and Zuck (2001) introduced $(0, 1, \infty)$-counter abstraction:

- finitely many local states,
  e.g., $\{N, T, C\}$.
- based on counter representation:
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- finitely many local states,
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- based on counter representation:
  for each local states count how many processes are in it
- abstract the number of processes in every state,
  e.g., $K: \quad C \mapsto \mathbf{0}, \quad T \mapsto \mathbf{1}, \quad N \mapsto$ **"many"**.
- perfectly reflects mutual exclusion properties
  e.g., $\mathbf{G}\,(K(C) = \mathbf{0} \vee K(C) = \mathbf{1})$.

Limits of $(0, 1, \infty)$-counter abstraction

Our parametric data + counter abstraction:

- we require finer counting of processes:
  - $t + 1$ processes in a specific state can force global progress,
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- mapping t, $t + 1$, and $n - t$ to **"many"** is too coarse.

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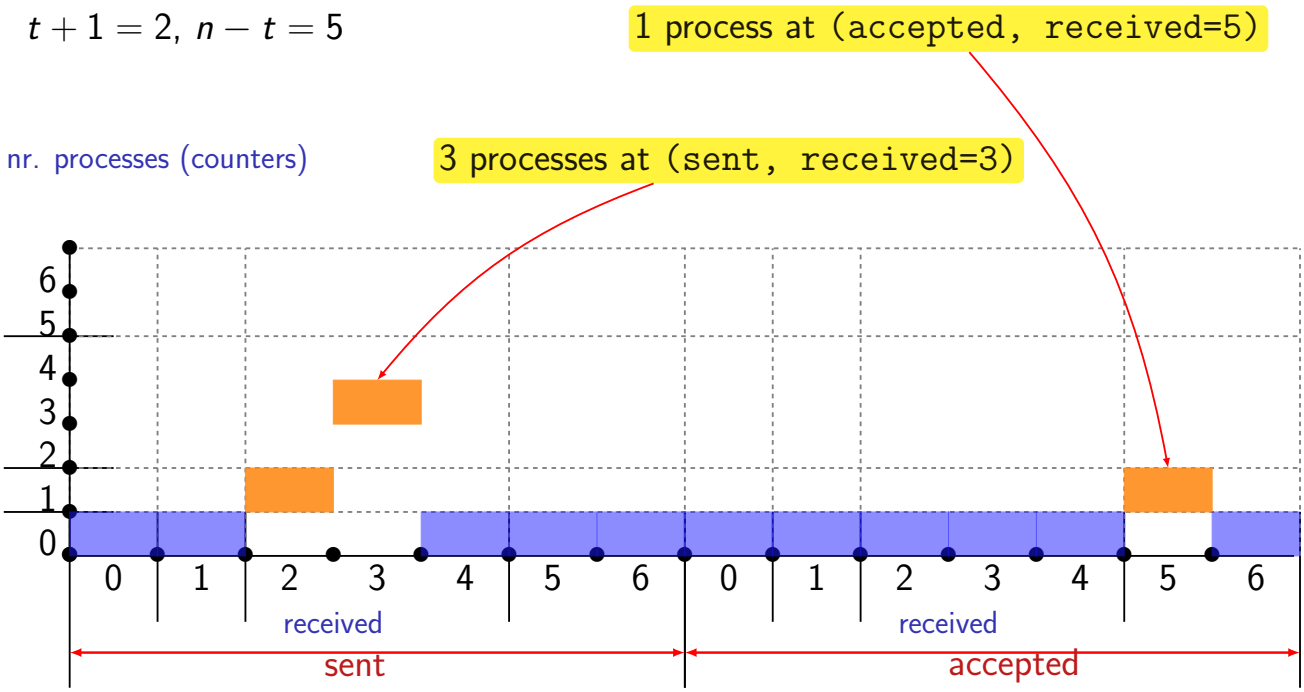
starting point of our approach...

Data + counter abstraction over parametric intervals

$n = 6,\ t = 1,\ f = 1$

$t + 1 = 2,\ n - t = 5$



Local state is $(sv, nrcvd)$,
where $sv \in \{sent, accepted\}$ and $0 \leq rcvd \leq n$

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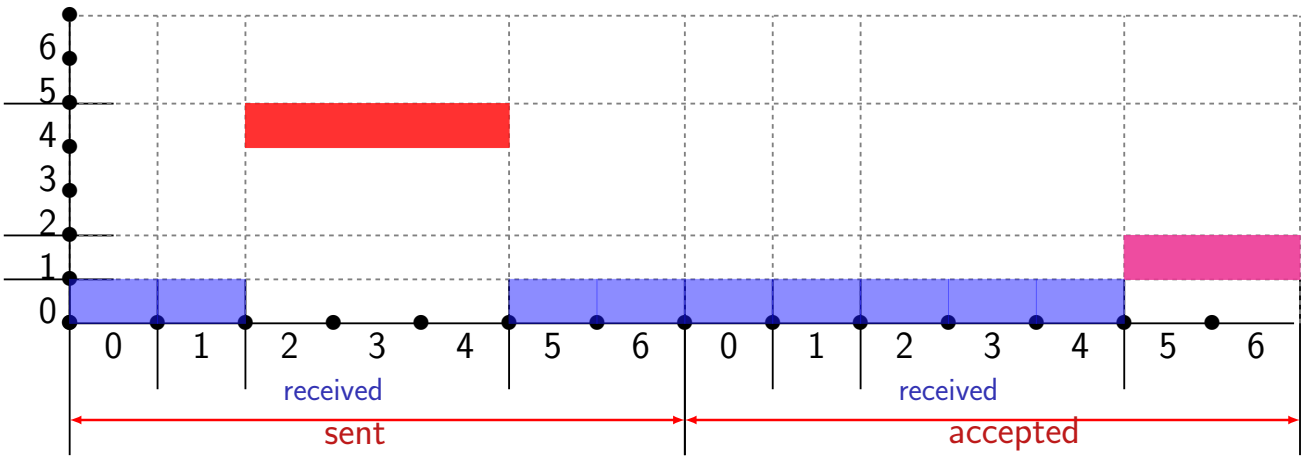

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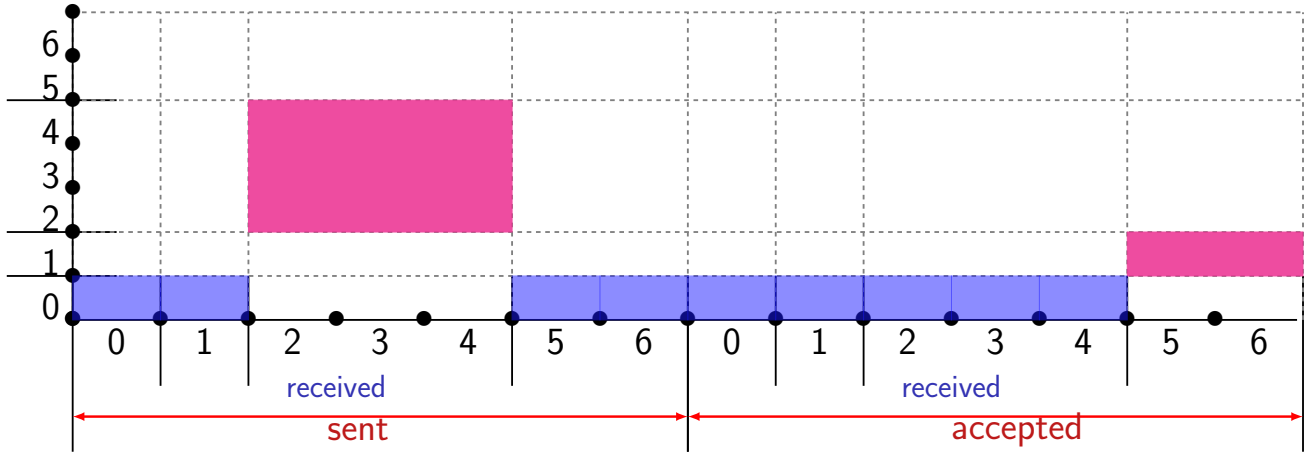

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Data + counter abstraction over parametric intervals

~~$n = 6,\ t = 1,\ f = 1$~~

$n > 3 \cdot t \wedge t \geq f$

Parametric intervals:

$\mathrm{I}_0 = [0, 1) \quad \mathrm{I}_1 = [1, t + 1)$

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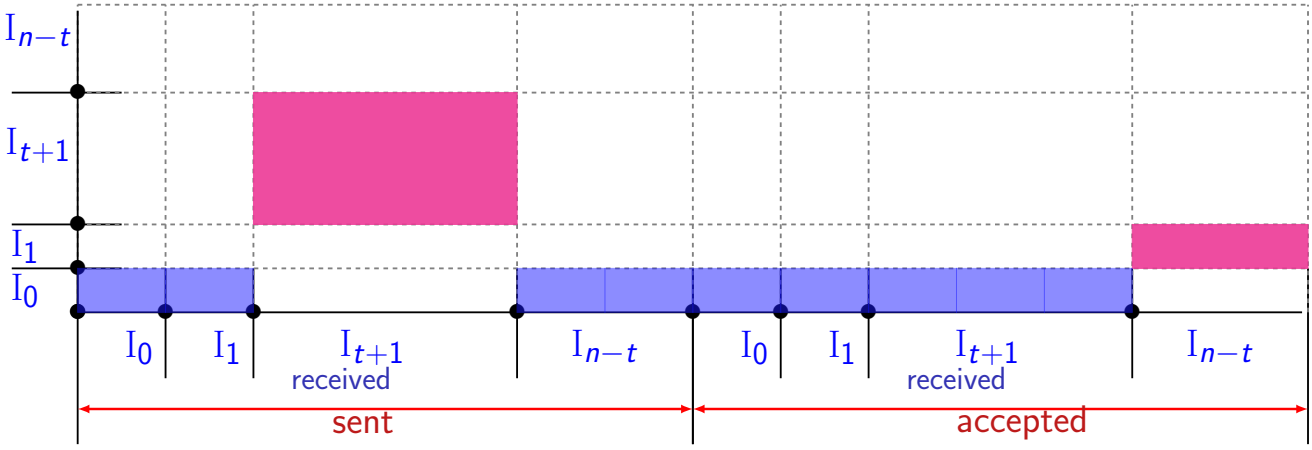

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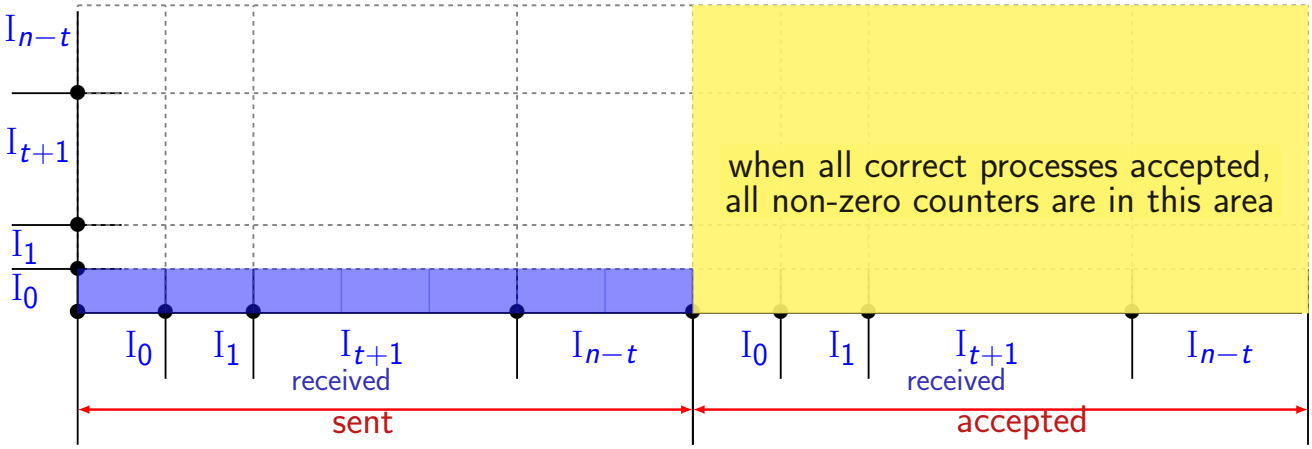

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Abstraction refinement

Spurious behavior

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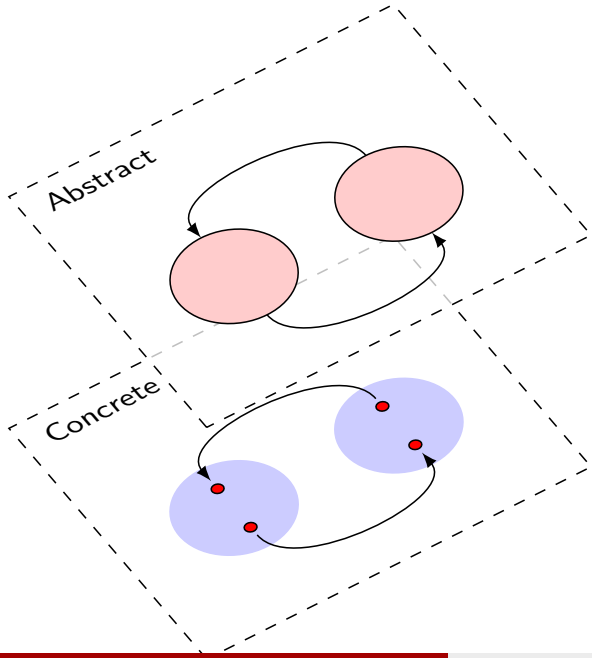
. . . and a new abstraction phenomenon

Parametric abst. refinement — uniformly spurious paths

Classic case:



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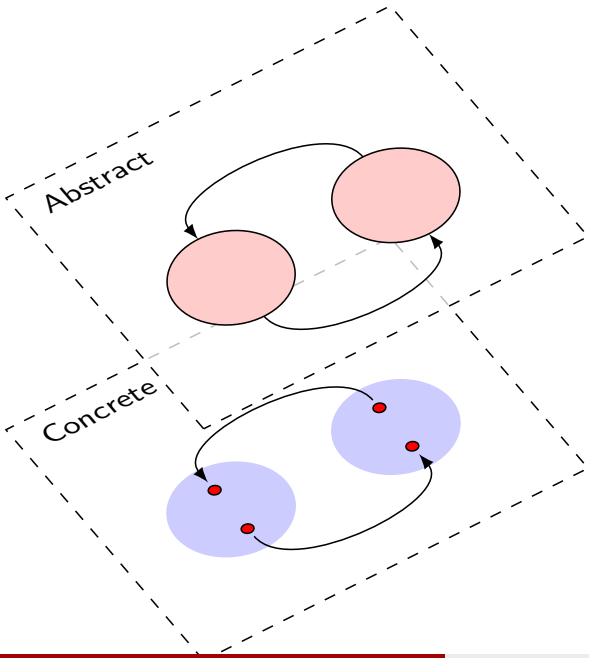
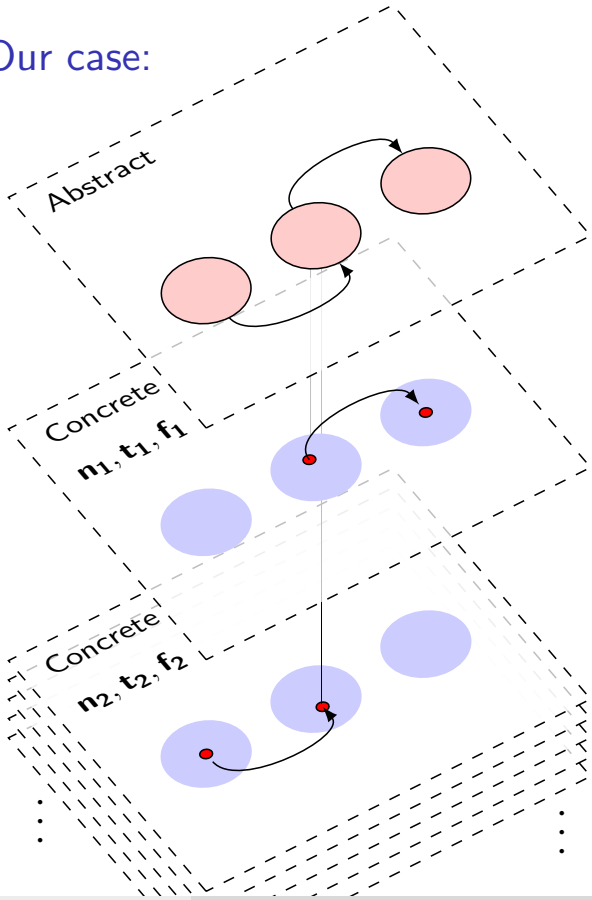
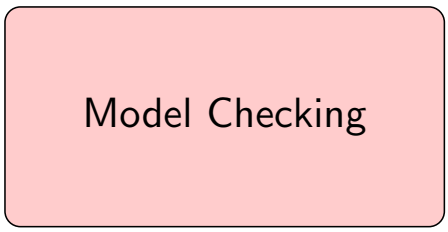


Our case:



CEGAR — automated workflow



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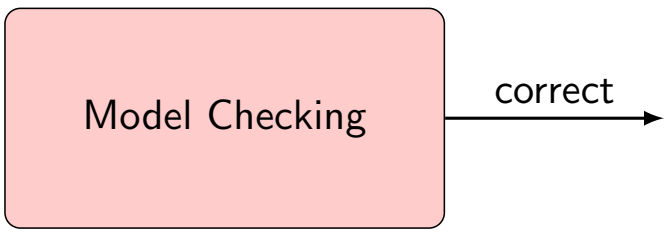

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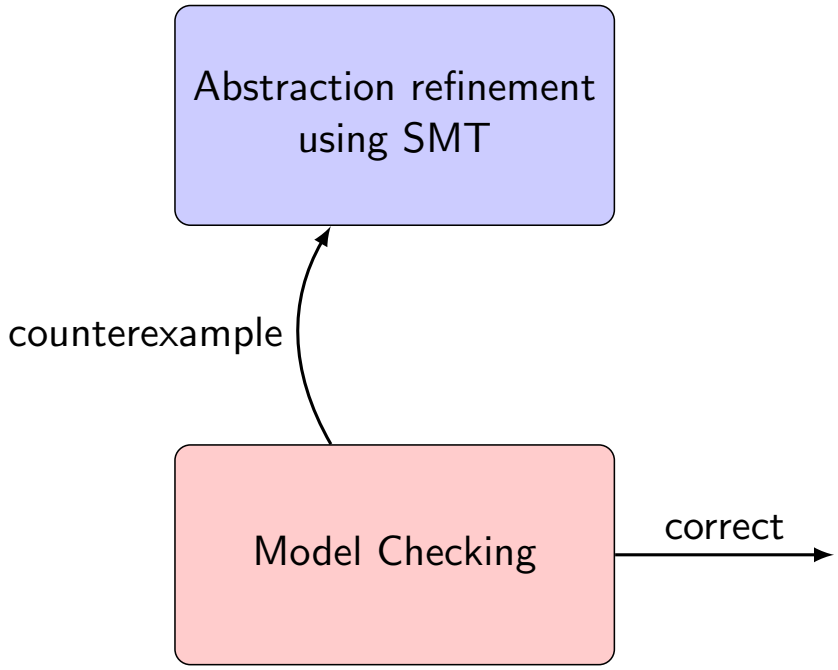

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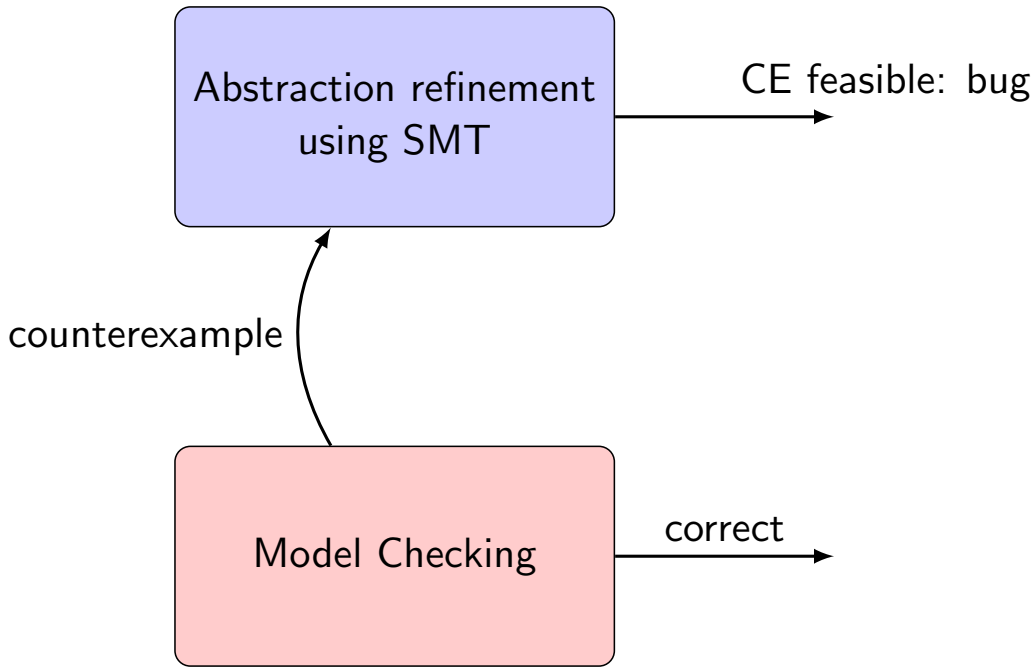

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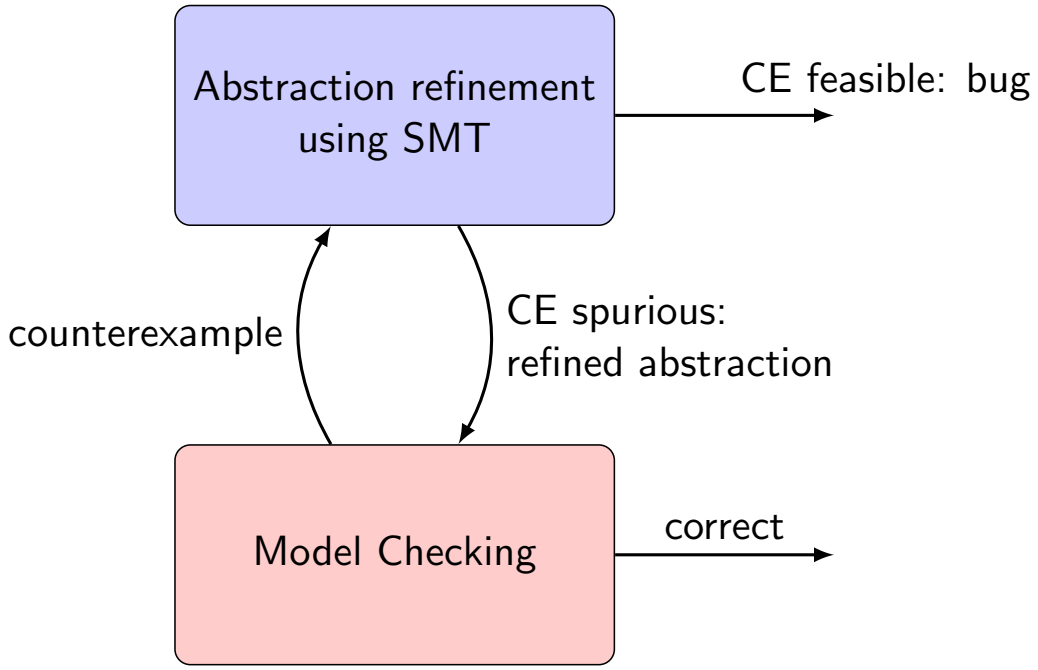

What is SMT?

recall SAT:

- given a Boolean formula, e.g., $(\neg a \vee \neg b \vee c) \wedge (\neg a \vee b \vee d \vee e)$
- is there an assignment of TRUE and FALSE to variables a, b, c, d, e such that the formula evaluates to TRUE?

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Satisfiability Modulo Theories (SMT) :

- here just linear arithmetics
- given a formula, e.g.,

$$x = y \ \wedge \ y = z \wedge u \neq x \wedge (x + y \leq 1 \wedge 2x + y = 1) \vee 3x + 2y \geq 3$$

- is there an assignment of values to u, x, y, z such that formula evaluates to TRUE?
- practically efficient tools: YICES, Z3

Counter example: losing processes

Output of data abstraction: 16 local states: $L = \{(sv, nr\hat{c}vd)$
with $sv \in \{v0, v1, sent, accepted\}$ and $r\hat{c}vd \in \{\mathrm{I}_0, \mathrm{I}_1, \mathrm{I}_{t+1}, \mathrm{I}_{n-t}\}\}$

An abstract global state is $(\hat{k}, n\hat{s}nt)$,
where $n\hat{s}nt \in \{\mathrm{I}_0, \mathrm{I}_1, \mathrm{I}_{t+1}, \mathrm{I}_{n-t}\}$ and $\hat{k} : L \to \{\mathrm{I}_0, \mathrm{I}_1, \mathrm{I}_{t+1}, \mathrm{I}_{n-t}\}$

Consider an abstract trace:

$n\hat{s}nt_1 = \mathrm{I}_0$
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Encode the last state in SMT as a conjunction T of the constraints:

resilience condition	$n > 3t \wedge t \geq f \wedge f \geq 0$
zero counters	$(i \neq 4 \wedge i \neq 8) \to 0 \leq k_3[i] < 1$
non-zero counters	$n - t \leq k_3[4] \wedge t + 1 \leq k_3[8] < n - t$
system size	$n - f = k_3[0] + k_3[1] + \cdots + k_3[15]$

UNSAT

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Remove transitions

- We ask the SMT solver:
  is there a satisfiable assignment for T?
- if yes,

  then the state is OK, may be part of a real counterexample
- if not, then the state is spurious

  remove transitions to that state in the abstract system

Liveness

- distributed algorithm requires reliable communication
- every message sent is eventually received
- $\neg in\_transit \equiv [\forall i.\ nrcvd_i \geq nsnt]$
- fairness $\mathbf{F}\,\mathbf{G}\,\neg in\_transit$ necessary to verify liveness,
  e.g., $\Big(\mathbf{F}\,\mathbf{G}\,\neg in\_transit \rightarrow \big(\mathbf{G}\,([\forall i.\ sv_i = v1] \rightarrow \mathbf{F}\,[\forall i.\ sv_i = accept])\big)\Big)$

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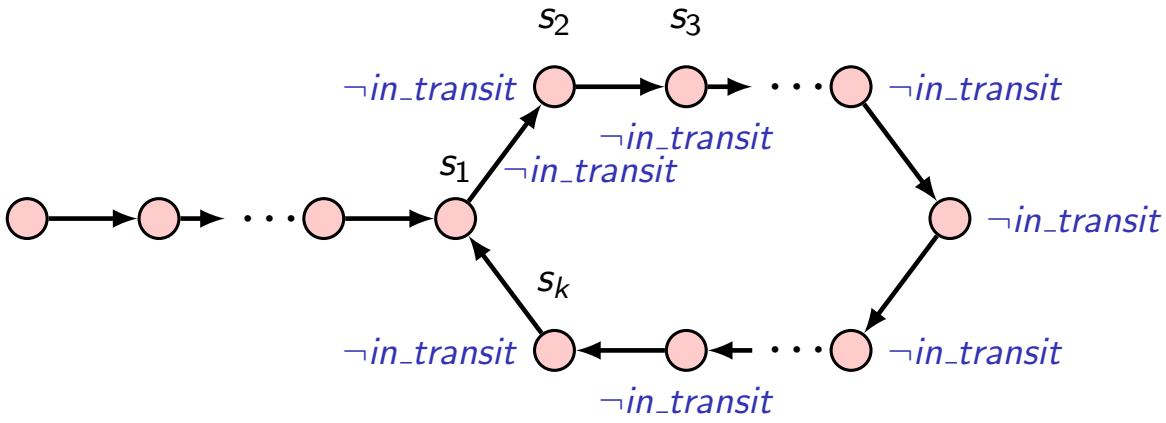
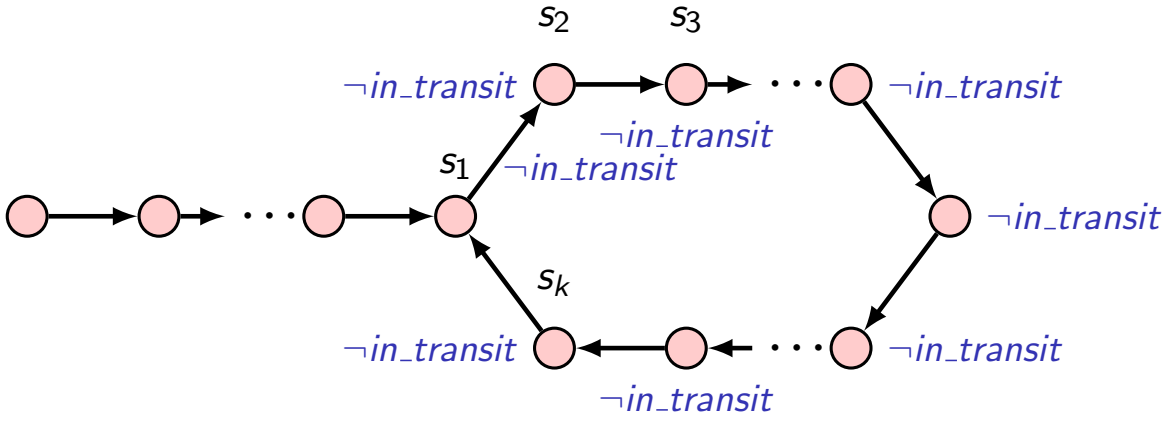
counter example (lasso):

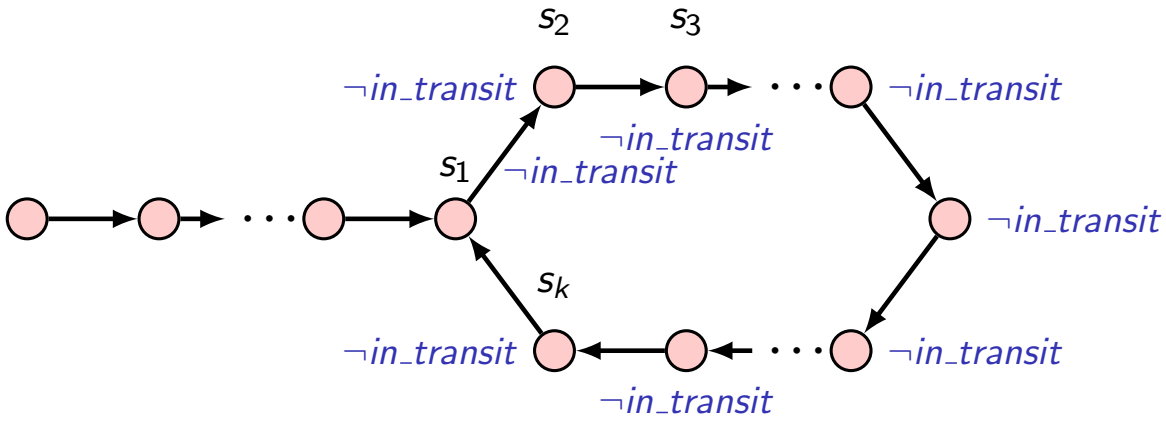


Liveness — fairness suppression



if there is a spurious s_j (all its concretizations violate $\neg in\_transit$), then the loop is spurious.

Liveness — fairness suppression



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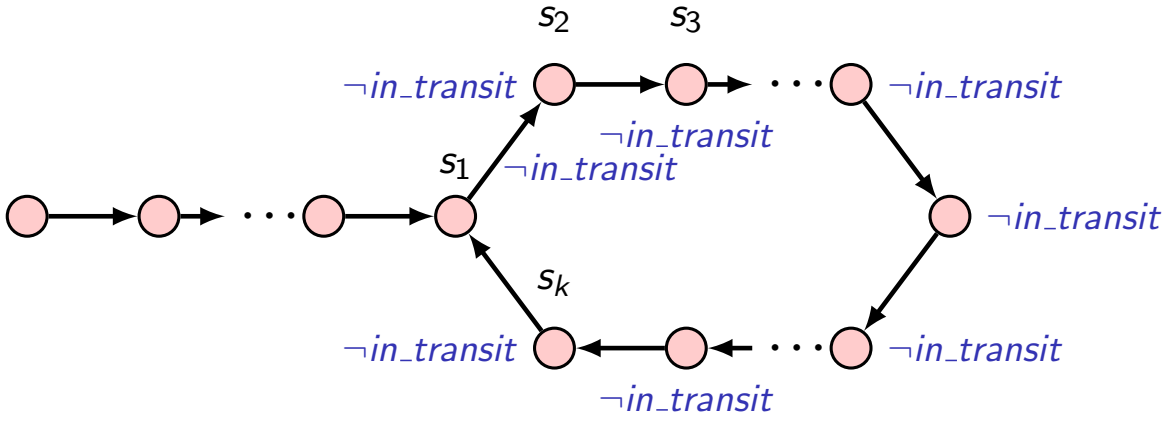

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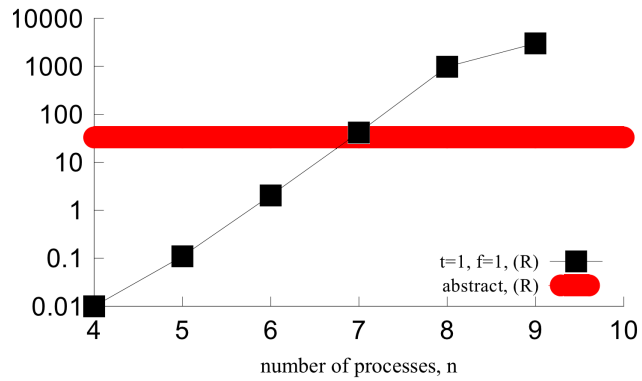
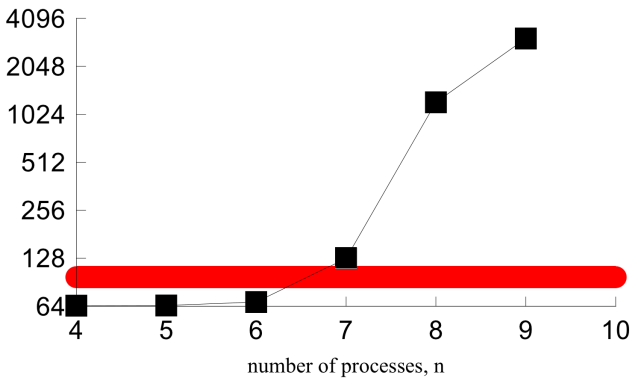
experimental evaluation

Concrete vs. parameterized (Byzantine case)

Time to check relay (sec, logscale)



Memory to check relay (MB, logscale)



- Parameterized model checking performs well (the red line).
- Experiments for fixed parameters quickly degrade ($n = 9$ runs out of memory).
- We found counter-examples for the cases $n = 3t$ and $f > t$, where the resilience condition is violated.

Experimental results at a glance

Algorithm	Fault	Resilience	Property	Valid?	#Refinements	Time
ST87	Byz	$n > 3t$	**U**	✓	0	4 sec.
ST87	Byz	$n > 3t$	**C**	✓	10	32 sec.
ST87	Byz	$n > 3t$	**R**	✓	10	24 sec.
ST87	Symm	$n > 2t$	**U**	✓	0	1 sec.
ST87	Symm	$n > 2t$	**C**	✓	2	3 sec.
ST87	Symm	$n > 2t$	**R**	✓	12	16 sec.
ST87	Omit	$n > 2t$	**U**	✓	0	1 sec.
ST87	Omit	$n > 2t$	**C**	✓	5	6 sec.
ST87	Omit	$n > 2t$	**R**	✓	5	10 sec.
ST87	Clean	$n > t$	**U**	✓	0	2 sec.
ST87	Clean	$n > t$	**C**	✓	4	8 sec.
ST87	Clean	$n > t$	**R**	✓	13	31 sec.
CT96	Clean	$n > t$	**U**	✓	0	1 sec.
CT96	Clean	$n > t$	**A**	✓	0	1 sec.
CT96	Clean	$n > t$	**R**	✓	0	1 sec.
CT96	Clean	$n > t$	**C**	✗	0	1 sec.

When resilience condition is wrong...

Algorithm	Fault	Resilience	Property	Valid?	#Refinements	Time
ST87	Byz	$n > 3t \wedge f \leq t{+}1$	**U**	✗	9	56 sec.
ST87	Byz	$n > 3t \wedge f \leq t{+}1$	**C**	✗	11	52 sec.
ST87	Byz	$n > 3t \wedge f \leq t{+}1$	**R**	✗	10	17 sec.
ST87	Byz	$n \geq 3t \wedge f \leq t$	**U**	✓	0	5 sec.
ST87	Byz	$n \geq 3t \wedge f \leq t$	**C**	✓	9	32 sec.
ST87	Byz	$n \geq 3t \wedge f \leq t$	**R**	✗	30	78 sec.
ST87	Symm	$n > 2t \wedge f \leq t{+}1$	**U**	✗	0	2 sec.
ST87	Symm	$n > 2t \wedge f \leq t{+}1$	**C**	✗	2	4 sec.
ST87	Symm	$n > 2t \wedge f \leq t{+}1$	**R**	✓	8	12 sec.
ST87	Omit	$n \geq 2t \wedge f \leq t$	**U**	✓	0	1 sec.
ST87	Omit	$n \geq 2t \wedge f \leq t$	**C**	✗	0	2 sec.
ST87	Omit	$n \geq 2t \wedge f \leq t$	**R**	✗	0	2 sec.

Summary of results

- Abstraction tailored for distributed algorithms
  - threshold-based
  - fault-tolerant
  - allows to express different fault assumptions
- Verification of threshold-based fault-tolerant algorithms
  - with threshold guards that are widely used
  - Byzantine faults (and other)
  - for all system sizes

Related work: non-parameterized

Model checking of the small size instances:

- clock synchronization [Steiner, Rushby, Sorea, Pfeifer 2004]
- consensus [Tsuchiya, Schiper 2011]
- asynchronous agreement, folklore broadcast, condition-based consensus [John, Konnov, Schmid, Veith, Widder 2013]
- and more...

Related work: parameterized case

Regular model checking of fault-tolerant distributed protocols:

[Fisman, Kupferman, Lustig 2008]

- "First-shot" theoretical framework.
- No guards like $x \geq t + 1$, only $x \geq 1$.
- No implementation.
- Manual analysis applied to folklore broadcast (crash faults).

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Backward reachability using SMT with arrays:

[Alberti, Ghilardi, Pagani, Ranise, Rossi 2010-2012]

- Implementation.
- Experiments on Chandra-Toueg 1990.
- No resilience conditions like $n > 3t$.
- Safety only.

Our current work

	Discrete synchronous	Discrete partially synchronous	Discrete asynchronous	Continuous synchronous	Continuous partially synchronous
One instance/ finite payload	**one-shot broadcast, c.b.consensus**: core of {ST87, BT87, CT96}, MA06 (common), MR04 (binary)				
Many inst./ finite payload					
Many inst./ unbounded payload					
Messages with reals					

Future work: threshold guards + orthogonal features

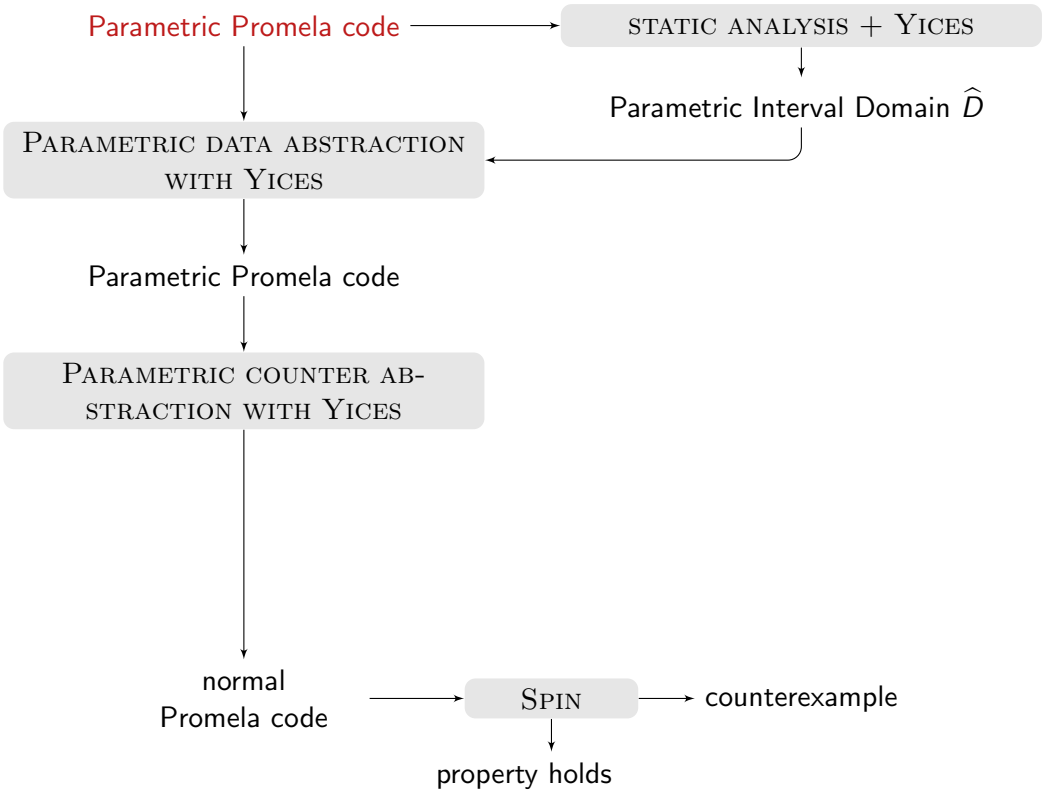
	Discrete synchronous	Discrete partially synchronous	Discrete asynchronous	Continuous synchronous	Continuous partially synchronous
One instance/ finite payload	**one-shot broadcast, c.b.consensus**: core of {ST87, BT87, CT96}, MA06 (common), MR04 (binary)				
Many inst./ finite payload	DHM12	CT96 (failure detecto			**clock sync**: FLPS13, FSFK06
Many inst./ unbounded payload	ST87	DLS86, ..06, L98 (Paxos)	**broadcast**: ST87, ..87, CT96, ... with failure-detectors	WS07	WS09
Messages with reals	AK00		**approx. agreement**: DLPSW86	ST87 (JACM)	

Thank you!

[http://forsyte.at/software/bymc]

the implementation

Tool Chain: ByMC



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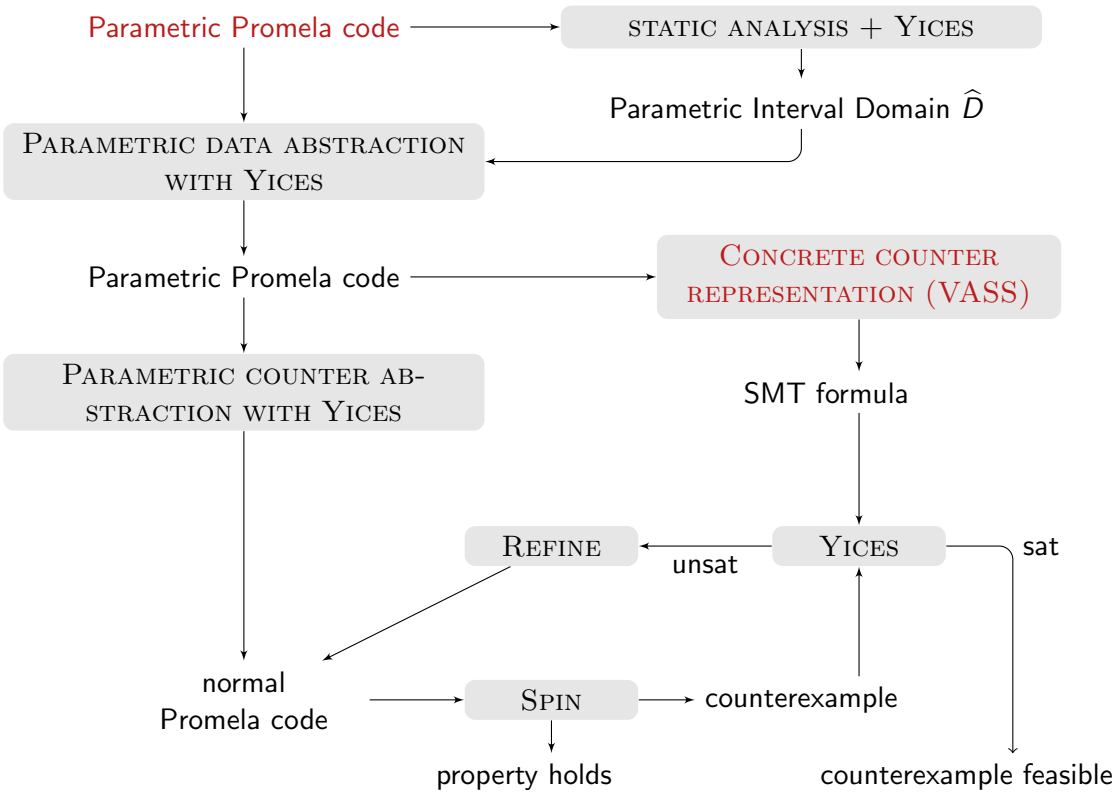

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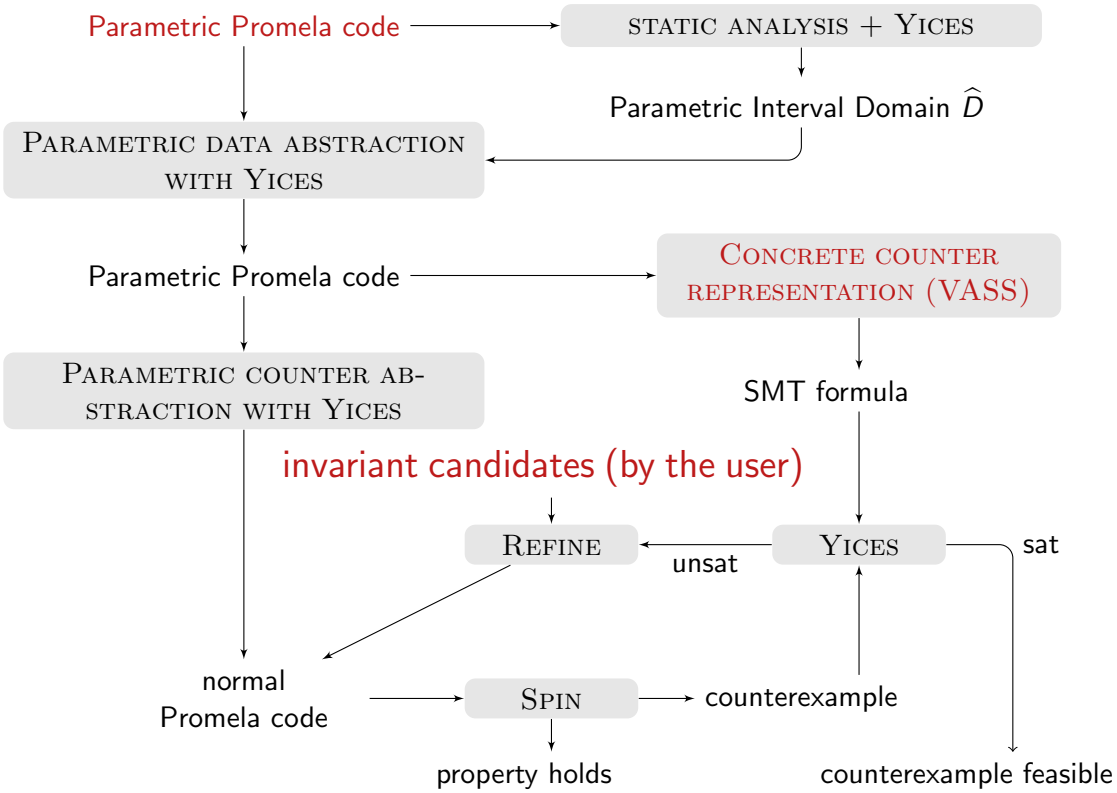
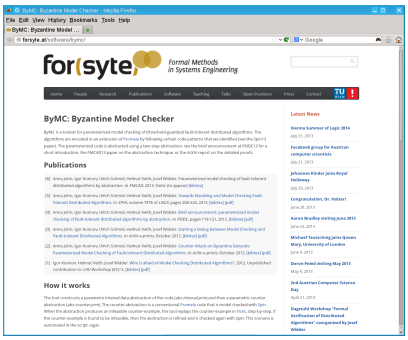
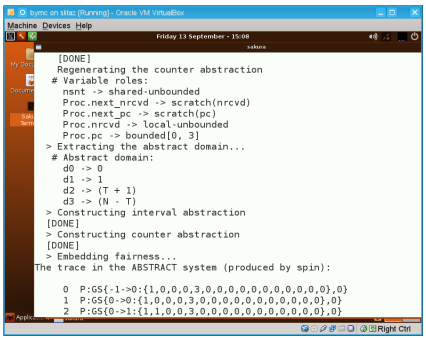
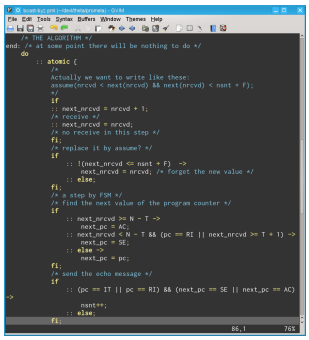


Experimental setup







The tool (source code in OCaml),
the code of the distributed algorithms in Parametric Promela,
and a virtual machine with full setup

are available at: http://forsyte.at/software/bymc

Running the tool — concrete case

- user specifies parameter value
- useful to check whether the code behaves as expected
- `$bymc/verifyco-spin "N=4,T=1,F=1" bcast-byz.pml relay`
  - model checking problem in directory "`./x/spin-bcast-byz-relay-N=4,T=1,F=1`"
  - in `concrete.prm`
    - parameters are replaced by numbers
    - process prototype is replaced with $N - F = 3$ active processes

Running the tool — parameterized model checking

- PIA data and counter abstraction
- finite-state model checking on abstract model
- `$bymc/verifypa-spin bcast-omit.pml relay`
  - model checking problem in directory "`./x/bcast-byz-relay-yymmdd-HHMM.*`"
  - directory contains
    - `abs-interval.prm`: result of the data abstraction;
    - `abs-counter.prm`: result of the counter abstraction;
    - `abs-vass.prm`: auxiliary abstraction for abstraction refinement;
    - `mc.out`: the last output by SPIN;
    - `cex.trace`: the counterexample (if there is one);
    - `yices.log`: communication log with YICES.

Fairness, Refinement, and Invariants

- In the Byzantine case we have $in\_transit : \forall i.\,(nrcvd_i \geq nsnt)$ and $\mathbf{G}\,\mathbf{F}\,\neg in\_transit$.
- In this case communication fairness implies computation fairness.
- But in the abstract version *nsnt* can deviate from the number of processes who sent the echo message.
- In this case the user formulates a simple state invariant candidate, e.g., $nsnt = K([sv = SE \vee sv = AC])$ (on the level of the original concrete system).
- The tool checks automatically, whether the candidate is actually a state invariant.
- After the abstraction the abstract version of the invariant restricts the behavior of the abstract transition system.

Parametric abstraction refinement — justice suppression

justice $\mathbf{G}\,\mathbf{F}\,\neg in\_transit$ necessary to verify liveness

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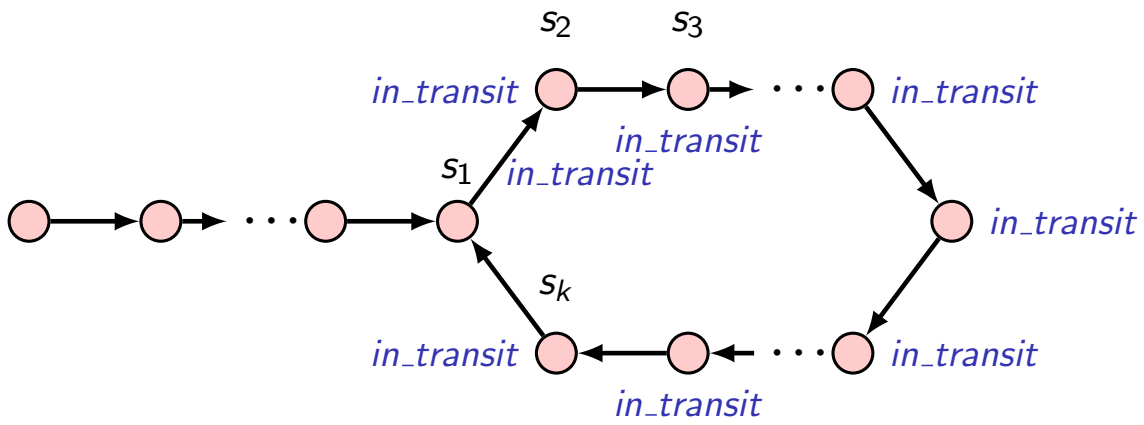

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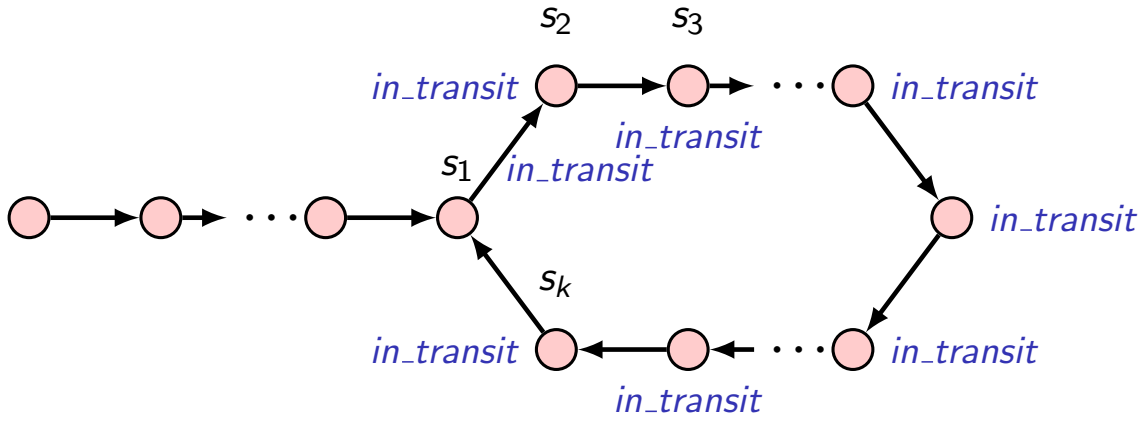

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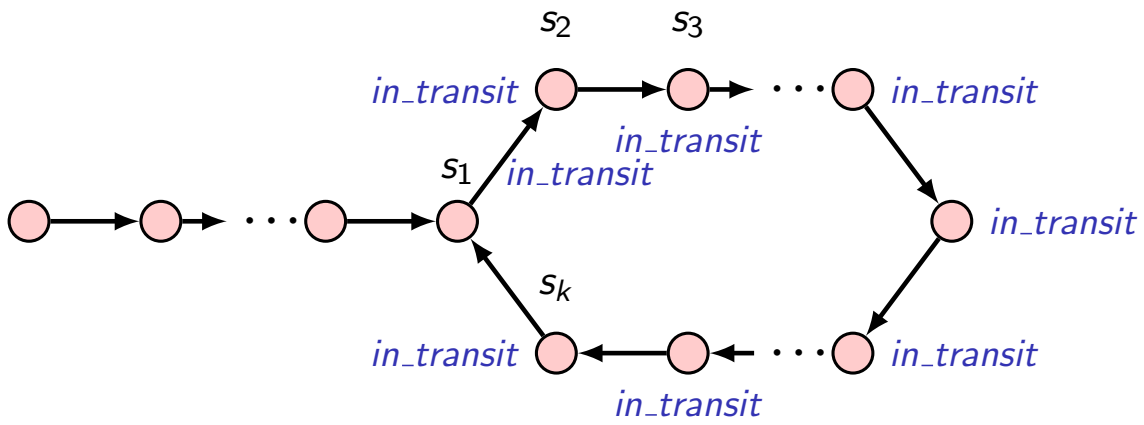

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. . . we use unsat cores to refine several loops at once

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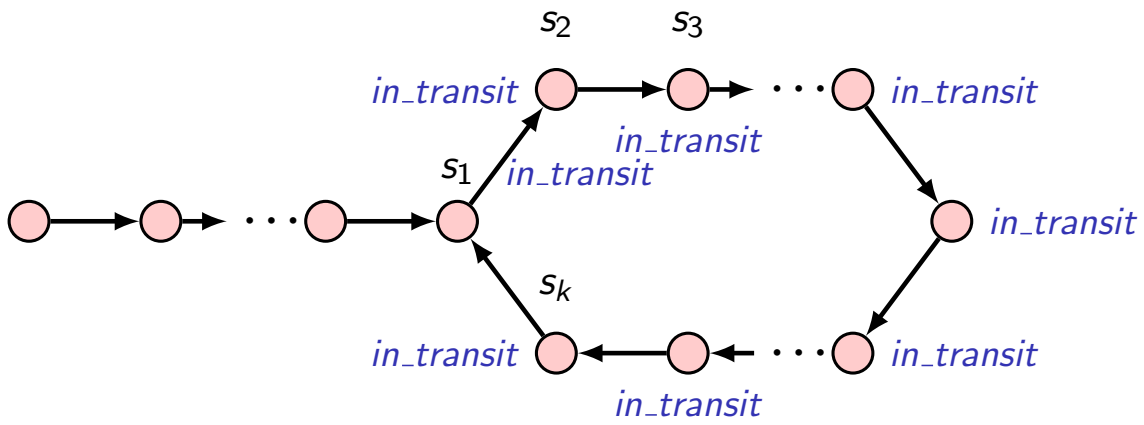

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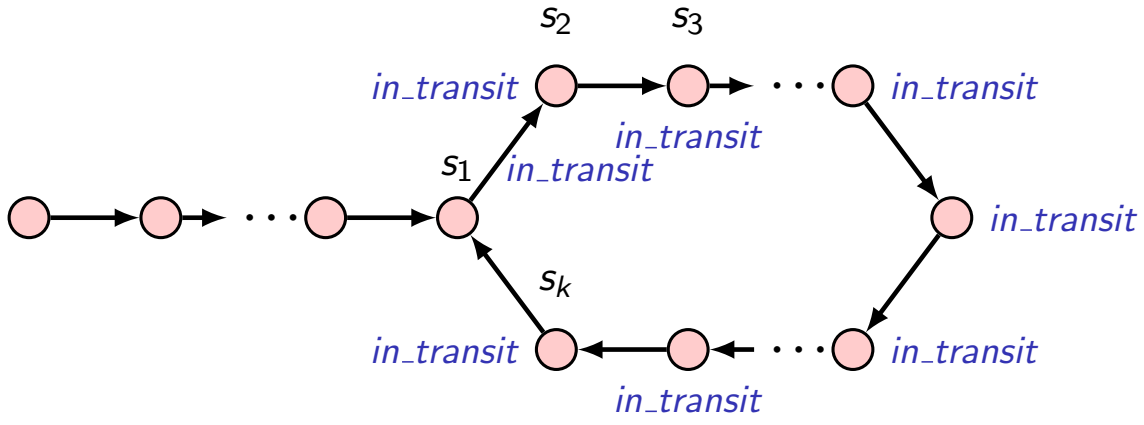

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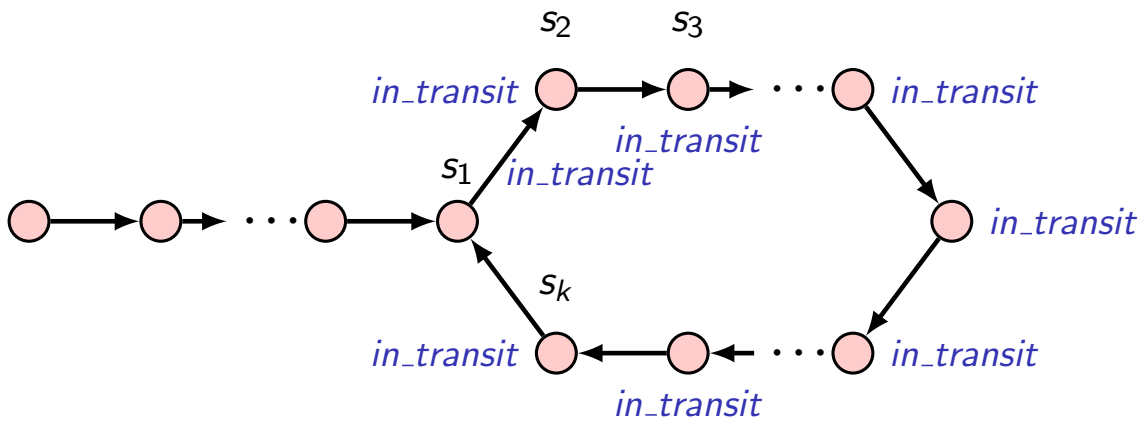

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asynchronous reliable broadcast (srikanth & toueg 1987)

the core of the classic broadcast algorithm from the da literature.
it solves an agreement problem depending on the inputs v_i.

Variables of process i
v_i: {0, 1} **init with** 0 **or** 1
$accept_i$: {0, 1} **init with** 0

An indivisible step:
if $v_i = 1$
then send (echo) to all;

if received (echo) from at least
t + 1 distinct processes
and not sent (echo) before
then send (echo) to all;

if received (echo) from at least
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then $accept_i := 1$;

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correct if $n > 3t$
resilience condition rc

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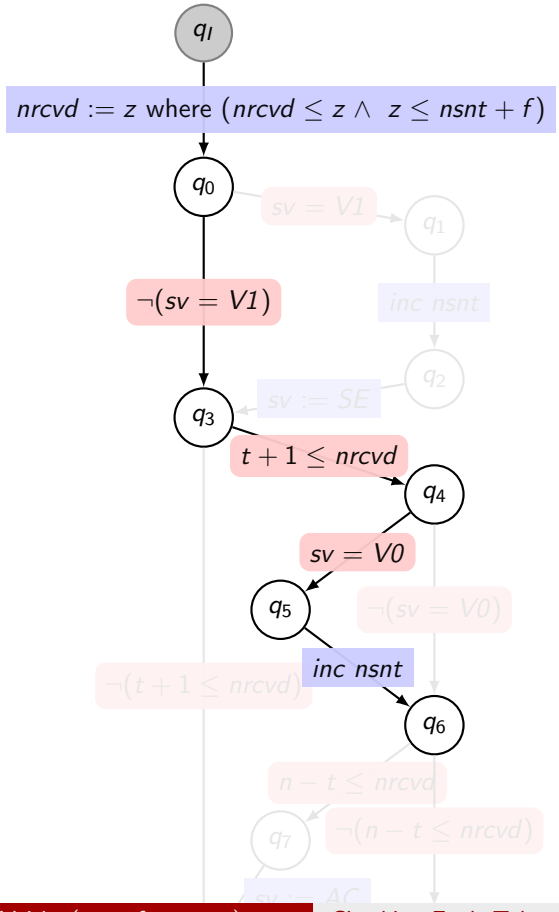
parameterized process
skeleton $p(n, t)$

Abstract CFA



Abstract CFA