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Tutorials for "Automated Reasoning"
Exercise sheet 11

Exercise 11.1: (*5 P*)
For each of the following term rewriting systems give, if possible, an example of a lexicographic path ordering, a multiset path ordering, and a Knuth-Bendix ordering that proves its termination.

$$
\begin{aligned}
(1) \qquad g(f(x)) &\rightarrow f(f(g(x)));\\
(2) \qquad g(f(x)) &\rightarrow f(h(g(x)));\\
(3) \qquad g(f(x)) &\rightarrow g(h(f(x)));\\
(4) \qquad f(f(x)) &\rightarrow g(x),\\
g(g(x)) &\rightarrow g(f(x));\\
(5) \quad h(g(x),g(x),y) &\rightarrow h(x,y,y);\\
(6) \qquad h(x,b,y) &\rightarrow h(y,c,x);\\
(7) \qquad h(g(x),y,y) &\rightarrow h(y,x,x);\\
(8) \qquad h(x,f(x),y) &\rightarrow h(g(x),x,g(x));\\
(9) \qquad h(x,y,y) &\rightarrow h(b,y,f(y));\\
(10) \qquad (x+y)+z &\rightarrow x+(y+z),\\
0+x &\rightarrow x.
\end{aligned}
$$

Exercise 11.2: (*3 P*)
Apply the Knuth-Bendix procedure to the set of equations $\{\, f(f(x)) \approx x,\ f(a) \approx b \,\}$ using the Knuth-Bendix ordering with weight 1 for all function symbols and variables and the precedence $g > f > a > b$.

Exercise 11.3: (*2 P*)
If the set R in the Knuth-Bendix procedure contains two rules whose left-hand sides are equal up to variable renaming, then none of them can be simplified using the other one. In practice, this is not a problem. Why?

Submit your solution in lecture hall 003 during the lecture on July 10. Please write your name and the date of your tutorial group (Mon, Thu, Fri) on your solution.

Note: Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.