Lecture “Automated Reasoning”  
(Summer Term 2008)

Midterm Examination

Name: .................................................................................................

Student Number: ..................................................................................

Some notes:

- Things to do at the beginning:
  Put your student card and identity card (or passport) on the table.
  Switch off mobile phones.
  Whenever you use a new sheet of paper (including scratch paper), first
  write your name and student number on it.

- Things to do at the end:
  Mark every problem that you have solved in the table below.
  Stay at your seat and wait until a supervisor staples and takes your
  examination text.
  Note: Sheets that are accidentally taken out of the lecture room are
  invalid.

Sign here: ........................................................................................

Good luck!

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Problem 1 (DPLL) (10 points)

Consider the propositional clause set

\[ N' = N \cup \{ \neg A_1 \lor \neg A_4 \lor A_6, \neg A_1 \lor \neg A_4 \lor \neg A_6 \} \]

During a DPLL-derivation, we have reached the state \( A_1^d \land A_2^d \land \neg A_3 \land A_4^d \land A_6 \| N' \).

Give two different backjump clauses that can be used in this situation and give the successor state with respect to \( \Rightarrow_{\text{DPLL}} \) for each of these backjump clauses.

Problem 2 (Algebras) (6 + 6 = 12 points)

Let \( \Sigma = (\Omega, \Pi) \), where \( \Omega = \{a, b, c\} \) and \( \Pi = \{P\} \). Let \( N \) be the set of formulas \( \{ \forall x \exists y P(x, y), \neg P(a, b), \neg P(a, c) \} \).

Part (a) Give a \( \Sigma \)-algebra that is a model of \( N \).

Part (b) Does \( N \) have a model over the universe \( \{1, 2\} \)? If yes, present the appropriate \( \Sigma \)-algebra. If no, prove why such a model cannot exist.

Problem 3 (CNF) (10 points)

Transform the formula

\[ \forall x \exists y \forall z (R(x, x) \lor (P(y) \land R(x, y) \land Q(z))) \]

into CNF using miniscoping.

Problem 4 (Unification) (10 points)

Transform the following unification problem into solved form using either \( \Rightarrow_{SU} \) or \( \Rightarrow_{PU} \):

\[ E = \{ f(x, g(h(y, z))), g(g(b))) = f(g(h(a, g(y))), x, g(z)) \} \].
Problem 5 (Model Construction) \hspace{1cm} (6 + 6 = 12 points)

Consider the following ground clause set $N$

\begin{align*}
P(a, a) \\
\neg Q(a) \lor \neg P(a, a) \\
R(a) \\
\neg R(a) \lor Q(g(a)) \\
\neg P(a, g(a)) \lor P(g(a), a)
\end{align*}

with atom ordering $R(a) \succ P(g(a), a) \succ P(a, g(a)) \succ P(a, a) \succ Q(g(a)) \succ Q(a)$.

Part (a) Construct $I_N$.

Part (b) Determine the minimal clause not satisfied by $I_N$ and perform one ordered ground resolution step with that clause generating a smaller clause not satisfied by $I_N$.

Problem 6 (Resolution) \hspace{1cm} (10 points)

Refute the following clause set via general resolution.

\begin{align*}
P(a, b) & \hspace{1cm} (1) \\
\neg P(x, y) \lor P(y, x) & \hspace{1cm} (2) \\
\neg P(x, y) \lor P(f(x), y) & \hspace{1cm} (3) \\
\neg P(b, f(f(a))) & \hspace{1cm} (4)
\end{align*}

For each inference give the parent clause numbers and the resulting clause.

Problem 7 (Clause Sets) \hspace{1cm} (10 points)

A clause is called positive if it consists of positive literals only, i.e., atoms. Let $N$ be a first-order clause set that does not contain a positive clause. Prove that $N$ is satisfiable.
Problem 8 (Terms) (10 points)

Let \( \# : T_\Sigma \rightarrow \mathbb{N} \) be a function mapping ground terms to the number of symbols occurring in the term, e.g., \( \#(g(a)) = 2 \), \( \#(h(a, g(b))) = 4 \). Furthermore, let \( \triangleright\triangleright \) be a total ordering on \( \Omega \). Now consider the binary relation \( \triangleright \subset T_\Sigma \times T_\Sigma \) defined by \( t \triangleright s \) where \( t = f(t_1, \ldots, t_n), s = g(s_1, \ldots, s_m) \) iff

1. \( \#(t) > \#(s) \) or
2. \( \#(t) = \#(s) \) and \( f \triangleright\triangleright g \) or
3. \( \#(t) = \#(s), f = g \) and \( (t_1, \ldots, t_n) \triangleright_{\text{lex}} (s_1, \ldots, s_m) \)

Prove by structural induction on the ground terms that \( \triangleright \) is total.