Exercise 10.1: (2 P)
Prove Theorem 3.42(ii).

Exercise 10.2: (4 P)
Is the rewrite system
\[
\{ f(a) \rightarrow f(b), f(b) \rightarrow f(c), f(c) \rightarrow f(a), f(x) \rightarrow x \}
\]
(i) terminating, (ii) normalizing, (iii) locally confluent, (iv) confluent? Give a brief explanation.

Exercise 10.3: (3 P)
Let \( \Sigma = (\Omega, \emptyset) \) with \( \Omega = \{a/0, b/0, f/1\} \). Given \( E = \{f(f(x)) \approx a\} \) derive \( E \vdash f(a) \approx a \). How many elements does the universe of \( T_\Sigma(\emptyset)/E \) have? How do they look like?

Exercise 10.4: (2 P)
Let \( E \) be a set of equations, let \( \theta : X \rightarrow T_\Sigma(X) \) be a substitution. Prove that \( E \vdash t \approx t' \) implies \( E \vdash t\theta \approx t'\theta \) for all terms \( t, t' \) over \( \Sigma \).
**Challenge Problem: (2 Bonus Points)**

Let $\rightarrow$ be a relation, such that if $y \leftarrow x \rightarrow z$ and $y \neq z$, then there is an element $u$ such that $y \rightarrow u \leftarrow z$.

Show that if an element $a$ has a normal form, then there is no infinite reduction sequence starting from $a$.

Submit your solution in lecture hall 002 during the lecture on June 29. Please write your name and the date of your tutorial group (Tue, Wed, Fri) on your solution.

**Note:** Joint solutions are not permitted (work in groups is encouraged).