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Tutorials for "Automated Reasoning"
Exercise sheet 11

Exercise 11.1: (*2 P*)
A relation $\to$ is *semi-confluent* iff

$$y_1 \leftarrow x \to^* y_2 \Rightarrow y_1 \downarrow y_2.$$

Prove: A relation $\to$ is semi-confluent iff it is confluent.

Exercise 11.2: (*1 + 2 + 1 + 2 P*)
Compute all critical pairs for each of the following systems:

a) $R = \{f(g(f(x))) \to x,\ f(g(x)) \to g(f(x))\}$;

b) $R = \{0 + y \to y,\ s(x) + y \to s(x + y),\ x + 0 \to x,\ x + s(y) \to s(x + y)\}$;

c) $R = \{f(x, x) \to a,\ f(x, g(x)) \to b\}$;

d) $R = \{f(f(x, y), z) \to f(x, f(y, z)),\ f(x, 1) \to x\}$.

Note: recall that variables in rewrite systems are universally quantified, hence they can be renamed such that no two rules of the same system share a variable.

Exercise 11.3: (*2 P*)
Show that the following system is locally confluent:

$$\begin{array}{ll} f(f(x)) \to f(x), & f(g(x)) \to g(x), \\ g(g(x)) \to f(x), & g(f(x)) \to g(x). \end{array}$$

Exercise 11.4: *(1 + 1 P)*
Show that the following TRSs are terminating:

a) $R = \{f(f(x)) \to g(g(f(x)))\}$;

b) $R = \{s(x) + y \to s(x + y), s(0 + x) \to s(x)\}$.

Hint: provide an appropriate ordering (an instance of LPO or KBO) for each of the TRSs, proving their termination.

Challenge Problem: *(2 Bonus Points)*
Give example of a signature Σ, containing at least one constant symbol, a set E of equations over Σ, two terms $s, t \in T_\Sigma(X)$, with the property that $T_\Sigma(\{x_1\})/E \models \forall\vec{x}\,(s \approx t)$, but $T_\Sigma(\{x_1, x_2\})/E \not\models \forall\vec{x}\,(s \approx t)$, where $\vec{x}$ consists of all the variables occurring in s and t. Clarify your hypothesis.

Submit your solution in lecture hall 002 during the lecture on July 6. Please write your name and the date of your tutorial group (Tue, Wed, Fri) on your solution.

Note: Joint solutions are not permitted (work in groups is encouraged).