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**Tutorials for “Automated Reasoning”**  
**Exercise sheet 12**

**Exercise 12.1:** (2 P)

Let  $\Omega = \{f/1, a/0, b/0\}$  and  $a \succ f \succ b$  and we use the LPO. Let  $E$  be the following set of equations:

$$f(a) \approx a \quad (1)$$

$$f(b) \approx a \quad (2)$$

Show that the Knuth-Bendix Completion procedure without the simplification rules cannot be used to transform  $E$  to an equivalent convergent finite set of rewrite rules. How can it be fixed (the KBC procedure must remain as above)?

**Exercise 12.2:** (3 P)

Let  $\Sigma$  be an arbitrary first-order signature. Define the ordering  $\succ$  on  $T_{\Sigma}(X)$  by

$$s \succ s' \text{ if and only if } |s| > |s'|,$$

where  $|t|$  is the size of  $t$ , that is, the cardinality of  $pos(t)$ . Is  $\succ$  a reduction ordering? Give a proof or a counterexample.

**Exercise 12.3:** (3 P)

Let  $\Omega = \{f/2, h/1, a/0\}$  and  $h \succ f \succ a$ . Compare the following pairs of terms with respect to the lexicographic path ordering  $\succ_{lpo}$  induced by  $\succ$ . Justify your answer.

- a)  $f(x, a)$  and  $x$ ;
- b)  $f(x, y)$  and  $f(x, z)$ ;
- c)  $h(f(x, y))$  and  $f(h(y), h(x))$ ;
- d)  $f(x, f(y, z))$  and  $f(h(x), y)$ ;
- e)  $h(x)$  and  $h(f(x, y))$ ;
- f)  $f(x, f(y, z))$  and  $f(f(x, y), z)$ .

**Exercise 12.4:** (4 P)

Apply the Knuth-Bendix procedure to the set of equations

$$\{ f(f(x)) \approx g(x), f(a) \approx b \}$$

and transform it into a finite convergent term rewrite system; use the Knuth-Bendix ordering with weight 1 for all function symbols and variables and the precedence  $g > f > a > b$ .

**Challenge Problem:** (2 Bonus Points)

Let  $\sqsupset$  be as defined on page 101 of the lecture script. Prove that  $\sqsupset$  is a strict partial ordering. *Hint.* Show that  $\sqsupset$  is irreflexive and transitive.

Submit your solution in lecture hall 002 during the lecture on July 13. Please write your name and the date of your tutorial group (Tue, Wed, Fri) on your solution.

**Note:** Joint solutions are not permitted (work in groups is encouraged).