max planck institut
informatik

Universität
des
Saarlandes
FR Informatik

Evgeny Kruglov
Christoph Weidenbach

May 25, 2010

Tutorials for "Automated Reasoning"
Exercise sheet 6

Exercise 6.1: *(4 P)*
Prove or refute the following statements:

a) If F and G are first-order formulas, F is valid, and $F \rightarrow G$ is valid, then G is valid.

b) If F and G are first-order formulas, F is satisfiable, and $F \rightarrow G$ is satisfiable, then G is satisfiable.

c) If F and G are first-order formulas and x is a variable, then $\forall x\,(F \wedge G) \models (\forall x\, F) \wedge (\forall x\, G)$ and $(\forall x\, F) \wedge (\forall x\, G) \models \forall x\,(F \wedge G)$.

d) If F and G are first-order formulas and x is a variable, then $\exists x\,(F \wedge G) \models (\exists x\, F) \wedge (\exists x\, G)$ and $(\exists x\, F) \wedge (\exists x\, G) \models \exists x\,(F \wedge G)$.

Exercise 6.2: *(2 P)*
Prove that $(QxF) \rightarrow G \models\!\mid \overline{Q}y(F[y/x] \rightarrow G),$ where y fresh.

Exercise 6.3: *(8 P)*
Compute the standard and optimized clausal normal forms of the following first-order formulas:

a) $\exists x\, \forall y\, (\forall z\, (P_1(y,\, z) \vee \neg f(x,y) \approx y) \rightarrow (\forall z\, (P_2(y,\, z) \wedge \neg P_3(x,\, y))))$;

b) $\forall x\, (\forall y\, ((\forall z\, (P(x,w,z)) \rightarrow \exists w\, (Q(x,y,w))) \rightarrow R(x)) \rightarrow S(y))$.

Hint: use the transformation sequences $(\Rightarrow_P^* \Rightarrow_S^* \Rightarrow_K^*)$ and $(\Rightarrow_{NNF}^* \Rightarrow_{MS}^* \Rightarrow_{SK}^*)$, respectively.

Challenge Problem: (*2 Bonus Points*)
Prove Theorem 3.10 from the script of the lecture. Show that converse of the theorem is not true in general.

Submit your solution in lecture hall 002 during the lecture on June 1. Please write your name and the date of your tutorial group (Tue, Wed, Fri) on your solution.

Note: Joint solutions are not permitted (work in groups is encouraged).