UNIVERSITÄT DES SAARLANDES
FR 6.2 – Informatik
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Lecture “Automated Reasoning”
(Summer Term 2012)

Final Examination

Name: .................................................................

Student Number: ................................................

Some notes:

• Things to do at the beginning:
  Put your student card and identity card (or passport) on the table.
  Switch off mobile phones.
  Whenever you use a new sheet of paper (including scratch paper), first
    write your name and student number on it.

• Things to do at the end:
  Mark every problem that you have solved in the table below.
  Stay at your seat and wait until a supervisor staples and takes your
    examination text.
  Note: Sheets that are accidentally taken out of the lecture room are
    invalid.

Sign here: Good luck!

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<table>
<thead>
<tr>
<th>Problem</th>
<th>1</th>
<th>2a</th>
<th>2b</th>
<th>2c</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>Σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Answered?</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Points</td>
<td></td>
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<td></td>
<td></td>
<td></td>
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</tbody>
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Problem 1 \((DPLL)\) (6 points)

Check via the rule-based CDCL calculus \(\Rightarrow_{DPLL} + \text{learning rule}\) whether the following clause set is satisfiable or not. Learn backjump clauses.

\[
\{\neg P_1 \lor Q \lor R, \neg P_2 \lor P_1 \lor R, P_2 \lor P_1 \lor R, \neg R \lor Q, \neg P_1 \lor R, \neg P_1 \lor \neg Q, \neg R \lor P_1\}
\]
Problem 2 (Superposition Model Building) (8 points)

Consider the following clause set $N$ with respect to an LPO where $g > f > b > a$.

$$N = \{ f(a, b) \approx b, b \approx a \vee b \approx g(a), b \not\approx g(b), f(a, g(a)) \approx g(b), b \not\approx a \}$$

(a) Compute $R_\infty$.
(b) Determine the minimal false clause.
(c) Compute the superposition inference out of (b), add it to the clause set $N$ compute the new respective $R_\infty$. 


Problem 3 (Unification) (6 points)

Solve the below unification problem using $\Rightarrow_{PU}$ and present the eventual unifier where $x, y, z, u, v$ are all variables.

$$E = \{ f(x, g(y)) \equiv f(f(z, y), u), g(u) \equiv v \}$$
Problem 4 (CNF)  

Apply the CNF algorithm of Section 3.6 from the lecture plus the eventual transformation to clauses to the below first-order formula. There is no beneficial subformula to rename.

$$\forall x \exists y (f(y, y) \approx y \lor (g(x) \approx x \land \forall y f(y, x) \approx g(y)))$$
Problem 5 (Completion) (8 points)

Apply completion ($\Rightarrow_{\text{KBC}}$) to the following set of equations with respect to a KBO where all signature symbols (and variables) have weight 1 and $f \succ g \succ b \succ a$.

$$N = \{ f(g(x), x) \approx f(x, x), f(g(a), b) \approx f(b, a), g(g(x)) \approx g(x) \}$$
Problem 6 (Saturation) (6 points)

Determine an ordering and a selection function such that for the below clause set no superposition inference is possible. As usual, $a$, $b$ are constants and $x$, $y$ are variables. Show the maximal/selected literals and argue why there is no inference.

\[ N = \{ g(a) \approx b, f(g(x), x) \approx x \lor f(x, g(x)) \approx x, f(a, a) \not\approx a \lor f(x, y) \approx g(y) \}\]
Problem 7 (Superposition Termination) (7 points)

Let $N$ be a finite set of first-order clauses all having the form $D \lor f(t_1, t_2) \approx a$ where $D$ might be empty or contains only disequations. The function $f$ does not occur in $t_1, t_2$. As usual $t_1, t_2$ are terms and $a$ is a constant.

Furthermore, with respect to some reduction ordering $\succ$, assume that for all clauses in $N$ the equation $f(t_1, t_2) \approx a$ is strictly maximal, $f(t_1, t_2) \succ a$, and $\text{vars}(D) \subseteq \text{vars}(f(t_1, t_2))$. Finally, for any variable $x$ occurring in $D$ and $f(t_1, t_2)$ for some clause $D \lor f(t_1, t_2) \approx a$, the depth of $x$ in a disequation in $D$ is smaller or equal than the depth of $x$ in $f(t_1, t_2)$.

Prove that for any ground disequation $s \not\approx t$ superposition terminates on $N \cup \{s \not\approx t\}$. 