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Tutorials for "Automated Reasoning"
Exercise sheet 6

Exercise 6.1: (*4 P*)
Consider our Superposition Theorem Prover (STP, notation: $\Rightarrow_{STP}$) from the lecture. Let $N = \{R \lor S, R \lor P \lor \neg Q, \neg Q \lor \neg S, \neg P \lor Q \lor \neg R, \neg Q \lor \neg S \lor \neg R, P \lor Q, \neg R \lor S, \neg P \lor \neg S\}$. Show a derivation of the empty clause from N using $\Rightarrow_{STP}$, i.e. show that $(N; \emptyset; \emptyset) \Rightarrow^*_{STP} (N' \cup \{\bot\}; U; W)$. Use the ordering $P \succ Q \succ R \succ S$ on propositional variables. Do as many simplifications as possible, i.e. apply the Clause Processing Rule only when neither the Tautology Deletion rule neither any Subsumption Rule is applicable. Explicitly show which rule you are using on which clauses and what is the result of the rule application. When doing inferences mark explicitly involved clauses and literals.

Exercise 6.2: (*3 P*)
For every rule of the STP prove that is terminates.

Exercise 6.3: (*3 P*)
Let N be a finite set of propositional clauses and P a propositional variable. Assume that we don't have duplicate literals in clauses and that no clause contains Q and $\neg Q$ for any propositional variable Q. Let $P \lor C_1, \ldots, P \lor C_k$ be all clauses in N containing the literal P and $\neg P \lor D_1, \ldots, \neg P \lor D_l$ be all clauses in N containing literal $\neg P$. Define the set $\mathcal{E}(P, N) = (N - \{P \lor C_i \mid 1 \leq i \leq k\} - \{\neg P \lor D_j \mid 1 \leq j \leq l\}) \cup \{C_i \lor D_i \mid 1 \leq i \leq k, 1 \leq j \leq l\}$. Prove that if N is satisfiable then $\mathcal{E}(P, N)$ is satisfiable.

Exercise 6.4: (*4 Bonus Points*)
Prove that if $\mathcal{E}(P, N)$ is satisfiable then N is satisfiable.

Submit your solution in lecture hall 001 during the lecture **on May 24 or until May 25, 5 p.m.** to box on the doors of office 2.16 in building E1 7. Please write your name and the date of your tutorial group on your solution.

Note: Joint solutions are not permitted (work in groups is encouraged).