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Tutorials for "Automated Reasoning"
Exercise sheet 8

Exercise 8.1: (*4 P*)
Which of the following closed formulas are valid, satisfiable, unsatisfiable? Justify, i.e. either prove that the formula is valid or unsatisfiable, or give examples to show that they are satisfiable but not valid.

1. $\forall x P(x) \rightarrow \exists x P(x)$
2. $\forall x(P(x) \vee Q(x)) \rightarrow [\forall x P(x) \vee \forall x Q(x)]$
3. $\forall x \exists y P(x,y) \rightarrow \exists x \forall y P(x,y)$
4. $\forall x\,(P(x) \rightarrow P(f(x))) \rightarrow \forall x P(x)$

Exercise 8.2: (*4 P*)
Let φ and σ be a first-order formula and a substitution, respectively, as given below. Compute $\varphi\sigma$ and identify the free and bound occurences of variables in φ and $\varphi\sigma$.

1. $\varphi = \neg\,(\forall x P(x,z) \wedge \neg\exists y(Q(x,y) \vee \forall z \neg R(y,z))) \rightarrow (\exists z Q(x,z) \wedge \forall u P(f(u,u),z))$,
   $\sigma = \{x \mapsto f(x,y), y \mapsto a, z \mapsto g(u), u \mapsto b\}$
2. $\varphi = \forall x \exists y R(f(x,y),c) \rightarrow \exists z\,(S(y,z) \vee R(z,g(u)))$,
   $\sigma = \{x \mapsto f(x,c), y \mapsto b, z \mapsto g(f(u,c)), u \mapsto x\}$.

Exercise 8.3: (*3 P*)
Prove or disprove the following statement: For all terms t, u, v and distinct variables x, y it holds that $(t\{x \mapsto u\})\{y \mapsto v\} = t\{x \mapsto u, y \mapsto v\}$. With this notation we mean substitution applied to respective term.

Exercise 8.4: (*4 Bonus Points*)
Show a closed first-order formula having *only* models of infinite cardinality. Precisely define

the language of the formula (function symbols and predicate symbols with their arities) and prove that the formula fulfills the property.

Submit your solution in lecture hall 001 during the lecture **on June 11**. Please write your name and the date of your tutorial group on your solution.

Note: Joint solutions are not permitted (work in groups is encouraged).