Exercise 10.1: (4 P)
Let \( a, b, c \) be constants, \( f \) binary function symbol and \( P \) and \( Q \) binary predicates. Apply the resolution calculus on the following two clauses:

\[
C_1 = \neg Q(f(x, z), f(y, z)) \lor Q(x, y) \lor Q(z, c) \lor \neg P(x, y)
\]

\[
C_2 = \neg Q(f(a, c), b) \lor \neg P(a, x) \lor Q(a, b).
\]

How many different possibilities to apply the resolution calculus are there? Show all resulting clauses with respective substitutions.

Exercise 10.2: (2 P)
Let \( \Sigma = \{ a/0, b/0, f/1, g/1; P/1, Q/1 \} \) be a signature (\( P, Q \) are predicates, the other symbols are function symbols). Find some Knuth-Bendix ordering (i.e. define weight function and precedence) in such a way that the following will hold:

\[
Q(f(b)) \lor P(g(b)) \gg_{kbo} P(f(a)) \lor P(f(a)) \lor \neg P(g(b)) \gg_{kbo} P(g(b)) \lor P(a) \lor P(g(b)) \lor Q(b) \gg_{kbo} \neg P(f(a)) \lor Q(b) \gg_{kbo} \neg P(a).
\]

Note that for the ordering we consider the usual lifting of \( \gg_{kbo} \) to literals (negative larger) and clauses (multiset extension). Justify your definitions.

Exercise 10.3: (3 P)
Let \( w \) be a weight function defined as follows on function symbols: \( w(f) = 3, w(g) = 2, w(a) = 1 \). On predicate symbols the function \( w \) is defined as \( w(P) = 5, w(Q) = 4 \). Weight of every variable is 1. Which literals in the following clauses are maximal when we consider Knuth-Bendix ordering induced by the weight function \( w \)? Mark all the maximal literals and justify your choice.

1. \( \neg P(g(a)) \lor P(f(f(a))) \)
2. \( P(f(x)) \lor P(g(y)) \)
3. \( \neg P(a) \lor \neg P(f(a)) \lor Q(f(a), f(f(a))) \)
4. \( \neg Q(f(x), y) \lor Q(f(f(x)), y) \lor P(x) \)

5. \( \neg P(x) \lor P(f(x)) \)

6. \( Q(f(z)) \lor \neg P(f(g(x))) \lor \neg Q(g(f(f(y)))) \)

**Exercise 10.4:** (2 P)
Show some selection function sel such that the following set of clauses \( N \) is saturated with respect to the calculus \( Sup_{sel} \equiv \) for some ordering \( \succ \). \( N = \{ \neg P(x) \lor Q(f(a)) \lor P(f(x)), P(g(x)) \lor P(g(f(x))) \lor \neg P(a), P(g(x)) \lor Q(y) \lor P(f(f(y))) \lor \neg Q(f(y)), P(g(y)) \lor \neg Q(f(a)), Q(f(z)) \lor P(f(g(f(z)))) \lor \neg Q(f(g(f(z)))) \} \). Justify your definition of the selection function.

**Exercise 10.5:** (3 P)
Prove Lemma 3.28.

**Exercise 10.6:** (3 Bonus Points)
Consider a signature \( \Sigma \). Let \( N \) be some set of ground clauses. Prove that for any ground clause \( C \) over the signature \( \Sigma \), \( N \models C \) if and only if \( N \vDash_{Res} D \) for some clause \( D \) such that \( D \subseteq C \).

Submit your solution in lecture hall 001 during the lecture on June 25. Please write your name and the date of your tutorial group on your solution.

**Note:** Joint solutions are not permitted (work in groups is encouraged).