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Uwe Waldmann **October 24, 2013**

Tutorials for "Automated Reasoning"
Exercise sheet 2

Exercise 2.1: (*4 P*)
The sudoku puzzle presented in the lecture

	1	2	3	4	5	6	7	8	9
1								1	
2	**4**								
3		2							
4					5		4		7
5			8				3		
6			1		9				
7	3			4			2		
8		5		1					
9				8		6			

has a unique solution. If we replace the 4 in column 1, row 2, by some other digit, this need no longer hold. Use a SAT solver to find values in column 1, row 2, such that the puzzle has

(1) no solution,
(2) more than one solution.

Explain how you found the values. Hint: The perl script at

`http://www.mpi-inf.mpg.de/departments/rg1/teaching/autrea-ws13/gensud`

produces an encoding of the sudoku above in DIMACS CNF format, which is accepted by most SAT solvers.

Exercise 2.2: (*4 P*)
Let F and G be propositional formulas. Prove or refute the following statements:

(1) If F is valid and $F \to G$ is valid then G is valid.

(2) If F is satisfiable and $F \to G$ is satisfiable then G is satisfiable.

Exercise 2.3: (*6 P*)
Let F, G, H be propositional formulas, let p be a position in H, and let $\mathcal{A}$ be a valuation. Prove: If $\mathrm{pol}(H,p) = 1$ and $\mathcal{A}(F) \le \mathcal{A}(G)$, then $\mathcal{A}(H[F]_p) \le \mathcal{A}(H[G]_p)$. If $\mathrm{pol}(H,p) = -1$ and $\mathcal{A}(F) \le \mathcal{A}(G)$, then $\mathcal{A}(H[F]_p) \ge \mathcal{A}(H[G]_p)$. (You need an induction to prove this property. It is sufficient if you consider the boolean connectives $\wedge$ and $\neg$; the other cases are proved analogously.)

Exercise 2.4: (*4 P*)
Use the previous exercise to prove Prop. 2.10 of the lecture:

Let P be a propositional variable not occurring in $H[F]_p$. Then $H[F]_p$ is satisfiable if and only if $H[P]_p \wedge \mathrm{def}(H,p,P,F)$ is satisfiable, where $\mathrm{def}(H,p,P,F)$ equals

- $(P \rightarrow F)$, if $\mathrm{pol}(H,p) = 1$,
- $(F \rightarrow P)$, if $\mathrm{pol}(H,p) = -1$,
- $(P \leftrightarrow F)$, if $\mathrm{pol}(H,p) = 0$.

There will be no lecture on October 30. Submit your solution during the tutorial on October 29 or 30 or put it into the box in front of Björn Borowski's office (E1.7, Room 636) until October 30, 17:00. Please write your name and the date of your tutorial group (Tue, Wed) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.