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**December 9, 2013**

**Tutorials for "Automated Reasoning"**
**Exercise sheet 8**

**Exercise 8.1:** (*6 P*)
Let $\Sigma = (\Omega, \Pi)$ be a signature with $\Omega = \{a/0,\ f/1\}$ and $\Pi = \{P/1\}$. Suppose that the atom ordering $\succ$ is defined in such a way that $P(f^n(a)) \succ P(f^m(a))$ if and only if $n > m \geq 0$. Let $N$ be the following set of clauses:

$$P(a) \vee P(f(f(a)))$$
$$\neg P(x) \vee P(f(x))$$

(a) Sketch how the set $G_\Sigma(N)$ of all ground instances of clauses in $N$ looks like. How is it ordered with respect to the clause ordering $\succ_C$?

(b) Construct the candidate interpretation $I^{\succ}_{G_\Sigma(N)}$ of the set of all ground instances of clauses in $N$.

**Exercise 8.2:** (*6 P*)
Consider the following formulae:

- $F_1 := \forall x(S(x) \rightarrow \exists y(R(x,y) \wedge P(y)))$
- $F_2 := \forall x(P(x) \rightarrow Q(x))$
- $F_3 := \exists x S(x)$
- $G := \exists x \exists y(R(x,y) \wedge Q(y))$

Use ordered resolution to prove that $\{F_1, F_2, F_3\} \models G$. (Choose some selection function and ordering on ground atoms).

**Exercise 8.3:** (*6 P*)
Theorem 3.48 states that the set of all formulas on a maximal open path of a propositional

tableau is satisfiable. The proof of Theorem 3.48, as given in the slides, works only for clausal tableaux. Give a proof for arbitrary propositional tableaux. (Hint: Prove first that the set of all atoms and negated atoms occurring on a maximal open path is satisfiable; then use induction. Omit the subproof for $\rightarrow$.)

Submit your solution during the tutorial on December 17 or 18 or in lecture hall E1.3, Room 001 during the lecture on November 18. Please write your name and the date of your tutorial group (Tue, Wed) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.

*Note that there is no lecture on December 11 and 16.*