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**Tutorials for "Automated Reasoning II"**
**Exercise sheet 5**

**Exercise 5.1:**
In many applications of CDCL or CDCL(T), one does not only want a yes/no answer, but also an explanation for it. In the case of an unsatisfiable input, this explanation is an "unsatisfiable core", i.e., a (small) subset of the input clauses that is already sufficient to show $\mathcal{T}$-inconsistency. How can we get an unsatisfiable core from a CDCL(T) proof?

**Exercise 5.2:**
Many decision procedures detect the unsatisfiability of a set of literals by iteratively deriving new literals from given literals; if an inconsistent literal is derived at the end, the input is unsatisfiable. Examples include Gaussian elimination or the Fourier-Motzkin procedure.

For such decision procedures, it is easy to generate explanations for unsatisfiability. We associate a set $E(L)$ of input literals to each literal $L$: for input literals $L$, $E(L) := \{L\}$; for literals $L$ derived from ancestor literals $L_1, \ldots, L_n$, $E(L) := E(L_1) \cup \cdots \cup E(L_n)$. When an inconsistent literal $L_0$ is derived at the end, $E(L_0)$ yields the explanation.

However, the explanations computed in this way are not always minimal. Consider the following set of equations in linear rational arithmetic:

$$\begin{array}{rl} x - 2z = 1 & (1) \\ -x + y - 3w = 3 & (2) \\ z - 2w = 0 & (3) \\ 2x - 2y + 3z = 5 & (4) \end{array}$$

If we use equation (1) to eliminate $x$ from the other equations, then (2) to eliminate $y$, then (3) to eliminate $z$, equation (4) is turned into $0 = 11$. All four equations were involved in this derivation; still $\{(1), (2), (3), (4)\}$ is *not* a minimal explanation for the contradiction. How could one efficiently find a smaller explanation? (Hint: Think about linear combinations.)

**Exercise 5.3:**
Prove that the Equality Factoring rule (page 47 of the script) is correct.

**Exercise 5.4:**
Refute the following set of equational clauses by superposition:

$$f(x) \not\approx a \vee f(x) \approx b \quad (1)$$
$$f(f(x)) \approx x \quad (2)$$
$$a \not\approx b \quad (3)$$

Choose an appropriate ordering and perform only inferences that satisfy the ordering restrictions.

Bring your solution (or solution attempt) to the tutorial on June 20.