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Tutorials for "Automated Reasoning II"
Exercise sheet 8

Exercise 8.1:
Let N be a set of equational clauses such that $\perp \notin N$. In Thm. 3.9, we have shown that whenever N is saturated up to redundancy, then every ground instance $C\theta \in G_\Sigma(N)$ is either productive or true in $R_{C\theta}$. The converse does not hold, not even for ground unit clauses: Give a (small) set of ground unit clauses N such that $\perp \notin N$ and every $C \in N$ is either productive or true in R_C, but N is not saturated up to redundancy.

Exercise 8.2:
Let $D = D' \vee t \approx t'$ and $C[u]$ be two clauses such that there is a (positive or negative) superposition inference between D and C with conclusion $C_0 = (D' \vee C[t'])\sigma$, where σ is the mgu of t and u. Suppose that $t\sigma$ occurs at least once in $C[t']\sigma$. Let C_0' be the clause that we obtain from C_0 if every occurrence of $t\sigma$ within $C[t']\sigma$ is replaced by $t'\sigma$. (As an example, consider $D = g(x) \not\approx g(y) \vee f(x,y) \approx f(y,x)$, $C = h(f(g(b),z)) \approx f(g(b),z)$, $t = f(x,y)$, $t\sigma = f(g(b),z)$, $C_0 = g(g(b)) \not\approx g(z) \vee h(f(z,g(b))) \approx f(g(b),z)$, $C_0' = g(g(b)) \not\approx g(z) \vee h(f(z,g(b))) \approx f(z,g(b))$.)

  (a) C_0' is entailed by D and C_0. Why?

  (b) C_0 is not redundant w. r. t. $\{D, C_0'\}$. Why?

  (c) The inference that produces C_0 is redundant w. r. t. $\{D, C_0'\}$. Why?

    Hint 1: Read the definitions of redundant inferences and instances of inferences really carefully. Hint 2: The ordering restrictions are an integral part of the definition of superposition inferences.

Exercise 8.3:
How would you redefine the fairness of a run if saturation is defined using redundant inferences? Try to find the easiest possible definition. Reprove Lemma 3.16 for the new definitions of saturation and fairness.

Bring your solution (or solution attempt) to the tutorial on July 11.