## Tutorials for "Automated Reasoning II" Exercise sheet 9

## Exercise 9.1:

Show that the following set of equational clauses can be finitely saturated up to redundancy, if the parameters of the superposition calculus and the strategy are chosen appropriately:

$$
\begin{align*}
h(y) \not \approx y \vee h(f(y)) & \approx f(y)  \tag{1}\\
g(g(x)) & \approx x  \tag{2}\\
f(b) & \approx c  \tag{3}\\
g(g(c)) \approx f(c) \vee g(g(c)) & \approx f(b) \tag{4}
\end{align*}
$$

## Exercise 9.2:

Let $N$ be the set of constrained clauses

$$
\begin{align*}
f(g(x)) & \approx x \llbracket 丁 \rrbracket  \tag{1}\\
h(b) & \approx c \llbracket 丁 \rrbracket \tag{2}
\end{align*}
$$

Are the following clauses redundant w.r.t. $N$, if we define redundancy as on page 64 of the lecture notes?

$$
\begin{align*}
f(h(x)) & \approx f(c) \llbracket x=b \rrbracket  \tag{3}\\
h(f(g(b))) & \approx c \quad \llbracket \top \rrbracket  \tag{4}\\
h(f(g(x))) & \approx h(x) \llbracket \top \rrbracket \tag{5}
\end{align*}
$$

## Exercise 9.3:

Refute the following set of clauses by hierarchic superposition; use linear rational arithmetic as base specification. The constants $b$ and $c$ are assumed to be Skolem constants of the base signature.

$$
\begin{align*}
f(f(x+1)) & \approx x  \tag{1}\\
f(b) & \approx c  \tag{2}\\
f(c) & \approx b+1 \tag{3}
\end{align*}
$$

## Exercise 9.4:

Compute minimal complete sets of unifiers for the following equality problems. (There is no need to construct and solve diophantine equation systems; the solutions are relatively obvious.)
(1) $\{x+y \approx a+b\}$ w.r.t. ACU.
(2) $\{x+y \approx a+b\}$ w.r.t. AC.
(3) $\{x+y \approx x\}$ w.r.t. ACU.
(4) $\{x+y \approx x\}$ w.r.t. AC.
(5) $\{x+y+a \approx z+b\}$ w.r.t. ACU.
(6) $\{x+y+a \approx z+z\}$ w.r.t. ACU.
(7) $\{a+x+x \approx y+b\}$ w.r.t. A.

Bring your solution (or solution attempt) to the tutorial on July 25.

