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Tutorials for "Automated Reasoning II"
Exercise sheet 2

Exercise 2.1:
The quantifier elimination algorithms for linear rational arithmetic can also be applied to non-linear formulas, provided that all the bound variables occur only linearly. That is, the atoms can have the form $0 \sim \sum_i s_i(\vec{z}) \cdot x_i$ where the coefficients $s_i(\vec{z})$ are terms that may contain arbitrary arithmetic operations, say $(z_1 + z_3^2)$ or even $(\sin z_2 + e^{z_5} + 3)$, but no bound variables. There is one additional problem, though. Why? How can you solve it?

Exercise 2.2:
(1) Use the nondeterministic Nelson–Oppen method to show that the following formula is unsatisfiable in the combination of EUF and linear integer arithmetic:

$$\exists x, y\, (x + y \approx 0 \;\wedge\; f(x) + f(-y) \approx 1)$$

(If you choose the equations to split cleverly, the proof is quite short.)

(2) Use the deterministic Nelson–Oppen method for the same problem.

Exercise 2.3:
Find a simple example that demonstrates that the deterministic Nelson-Oppen combination procedure is incomplete if one of the theories is not convex.

Exercise 2.4:
Is the theory of abelian groups stably infinite? Give an explanation.

Exercise 2.5:
Is the theory described by the following set of axioms stably infinite? Give an explanation.

$$\forall x\, (x * 0 \approx 0)$$
$$\forall x\, (x * 1 \approx x)$$

Bring your solution (or solution attempt) to the tutorial on April 25.