max planck institut
informatik

Universität
des
Saarlandes
FR Informatik

Sophie Tourret
Uwe Waldmann

April 25, 2018

Tutorials for "Automated Reasoning II"
Exercise sheet 3

Exercise 3.1:
Let $\Sigma = (\Omega, \emptyset)$ be a signature without predicate symbols (except built-in equality). For two Σ-algebras $\mathcal{A}$ and $\mathcal{B}$, we define the product $\mathcal{A} \times \mathcal{B}$ as the Σ-algebra whose universe is the cartesian product of the universes of $\mathcal{A}$ and $\mathcal{B}$, and where $f_{\mathcal{A}\times\mathcal{B}}((a_1, b_1), \dots, (a_n, b_n)) = \big(f_{\mathcal{A}}(a_1, \dots, a_n), f_{\mathcal{B}}(b_1, \dots, b_n)\big)$.

A Σ-theory $\mathcal{T}$ is called closed under products, if the product of any two models of $\mathcal{T}$ is again a model of $\mathcal{T}$.

Prove: If $\mathcal{T}$ is closed under products, then it is convex.

Exercise 3.2:
Prove: If the axioms of the Σ-theory $\mathcal{T}$ are unversally quantified equational Horn clauses (that is, clauses where all atoms are equations and at most one of the literals is positive), then $\mathcal{T}$ is convex. (You may use the previous exercise.)

Exercise 3.3:
Use the CDCL(EUF) calculus to determine whether the following set of clauses is satisfiable or not:

$$f(a,b) \not\approx f(a',b') \qquad (1)$$

$$g(g(c)) \not\approx c \qquad (2)$$

$$g(d) \approx c \ \lor\ g(g(c)) \approx c \qquad (3)$$

$$a \approx a' \ \lor\ c \approx d \qquad (4)$$

$$b \approx b' \ \lor\ c \approx d \qquad (5)$$

Exercise 3.4:
Normalization of the input literals is an important part of the pre-processing that take place in an SMT solver before running the actual CDCL(T) algorithm. How would you normalize literals in linear integer arithmetic?

Bring your solution (or solution attempt) to the tutorial on May 3.