Exercise 5.1:
Prove that the multiset extension of a reduction ordering is stable under substitutions
(which implies that the literal ordering defined on page 50 of the lecture notes is stable
under substitutions). Note: There are several ways to characterize a multiset ordering, see
e.g. the lecture notes from the previous semester or the book by Baader and Nipkow. You
may pick the most convenient one for this purpose.

Exercise 5.2:
On page 50 of the lecture notes it is stated that the ordering restrictions of the inference
rules of the superposition calculus must be satisfied after applying the mgu to the premises.
Give a simple example that shows that a literal may be maximal in a clause, but that the
maximality requirement may be violated after applying the mgu.

Exercise 5.3:
Let $D = D' \lor t \approx t'$ and $C[u]$ be two clauses such that there is a (positive or negative)
superposition inference between $D$ and $C$ with conclusion $C_0 = (D' \lor C[t'])\sigma$, where $\sigma$ is the
mgu of $t$ and $u$. Suppose that $t\sigma$ occurs at least once in $C[t']\sigma$. Let $C'_0$ be the clause that we
obtain from $C_0$ if every occurrence of $t\sigma$ within $C[t']\sigma$ is replaced by $t'\sigma$. (As an example,
consider $D = g(x) \neq g(y) \lor f(x, y) \approx f(y, x), C = h(f(g(b), z)) \approx f(g(b), z), t = f(x, y),
t\sigma = f(g(b), z), C_0 = g(g(b)) \neq g(z) \lor h(f(z, g(b))) \approx f(g(b), z), C'_0 = g(g(b)) \neq g(z) \lor
h(f(z, g(b))) \approx f(z, g(b))).$

(a) $C'_0$ is entailed by $D$ and $C_0$. Why?

(b) $C_0$ is not redundant w.r.t. $\{D, C'_0\}$. Why?

(c) The inference that produces $C_0$ is redundant w.r.t. $\{D, C'_0\}$. Why?

Hint 1: Read the definitions of redundant inferences and instances of inferences re-
ally carefully. Hint 2: The ordering restrictions are an integral part of the definition of
superposition inferences.
Exercise 5.4:
Compute $R_\infty$ for the clause set \{\(f(x) \approx a\)\} and the signature $\Sigma = (\{f/1, g/1, a/0\}, \emptyset)$; use the LPO with $g > f > a$.

Bring your solution (or solution attempt) to the tutorial on May 22.