Lecture “Automated Reasoning II”  
(Winter Term 2012/13)  
Final Examination

Name: ..............................................................................................................

Student Number: ................................................................................................

Some notes:

• Things to do at the beginning:
  Put your student card and identity card (or passport) on the table.
  Switch off mobile phones.
  Whenever you use a new sheet of paper (including scratch paper), first
  write your name and student number on it.

• Things to do at the end:
  Mark every problem that you have solved in the table below.
  Stay at your seat and wait until a supervisor staples and takes your
  examination text.
  Note: Sheets that are accidentally taken out of the lecture room are
  invalid.

Sign here: ..........................................................................................................

Good luck!

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Problem 1 (Congruence Closure) (6 points)

Decide satisfiability of the below ground equations by congruence closure.

\[
\begin{align*}
&f(g(a), a) \approx h(b, b), f(g(b), c) \approx h(a, a), f(g(a), f(g(b), c)) \not\approx g(a), \\
&h(a, a) \approx a, g(a) \approx h(b, b)
\end{align*}
\]
Problem 2 (*Virtual Substitution*)

Eliminate the variable $x$ in the below formula by the virtual substitution method. Recall that there is a choice between considering the positive or negative limit.

$$
\exists x [y < x \land (x \leq 3 \lor 2x = 7) \land (x = 1 \lor 2y < x)]
$$
Problem 3 (CDCL(LA))

Decide satisfiability of the below clauses by CDCL(LA).

\[ x = 5 \lor y > 5 \]
\[ 3x + y < 7 \lor y < -1 \]
\[ y > 0 \lor x + y > 4 \]
Problem 4 (*Nelson-Oppen*) (6 points)

Check satisfiability of the below formula, a combination of EUF and LA, by the Nelson-Oppen combination method.

\[ g(x) \geq z \land g(g(x)) < y \land g(x) \approx x \land z - y \approx 0 \]